Abstract

- Extension of Roughgarden et al. [RVW18] framework for proving lower bounds for Massively Parallel Computation (MPC) to capture adaptivity and promise problems.
- First unconditional lower bound on the promise problem 1v2-CYCLE of distinguishing between a cycle of length \( n \) versus two cycles of length \( n/2 \), matching upper bound of Behnezhad et al. [BDE+19].
- Optimal lower bounds on the query complexity and approximate certificate complexity of 1v2-CYCLE.

Massively Parallel Computation (MPC) Model

- Captures central features of recent parallel computation platforms for large-scale data
- Computation in rounds, complexity = number of rounds.
- Machine has restricted I/O capacity: total size of messages received/sent in each round \( \leq S \) (\( S \leq \text{input size} \)).

Adaptive MPC (AMPC) Model

- Behnezhad et al. [BDE+19], inspired by practical low-latency remote direct memory-access framework.
- Machines adaptively query inputs from shared memory containing messages from previous round.
- I/O capacity in AMPC: each machine can query/write \( \leq S \) messages from/to the shared memory in each round.
- Significantly improved round complexities for central graph problems [BDE+19].

MPC Lower Bounds via Polynomial Method

- Power and generality of MPC expressing challenging to prove lower bounds
  - input-dependent communication pattern among machines
  - unbounded computational power of machines
- Roughgarden et al. [RVW18]: relates MPC round complexity to Boolean function complexity:
  - Theorem [RVW18]: \( g: \{0,1\}^n \rightarrow \{0,1\} \) computed by \( R \)-round MPC algorithm, then \( \deg(g) \leq S^R \).
- Application to undirected connectivity on \( n \) vertices:
  - Corollary for CONNECTIVITY: Any MPC algorithm for CONNECTIVITY using machines with I/O capacity \( S = n^k \) requires \( \Omega(1/\epsilon) \) rounds.

The 1v2-CYCLE Problem

- Widely believed: CONNECTIVITY requires \( \Omega(\log n) \) MPC rounds
- Stronger conjecture: \( \Omega(\log n) \) rounds even for simpler promise problem 1v2-CYCLE:
  - \( 1 \text{-v2-CYCLE} \): is the input graph a cycle of length \( n \) or two cycles of length \( n/2 \)?
  - 1v2-CYCLE is represented by a partial Boolean function
    - Logarithmic round conjecture: Any MPC algorithm for 1v2-CYCLE using machines with I/O capacity \( S = n^k \) requires \( \Omega(\log n) \) rounds.

This Work: Polynomial Method for AMPC

- Conjecture does not hold in the more powerful adaptive setting:
  - [BDE-19]'s adaptive algorithm for 1v2-CYCLE: \( O(1/\epsilon) \)-round randomized AMPC algorithm for 1v2-CYCLE using machines with I/O capacity \( S = n^k \).

Deterministic AMPC Lower Bound for 1v2-CYCLE via Query Complexity

- Deterministic query complexity of \( g \): \( D(g) \): minimum number of queries made by deterministic query algorithm for \( g \)
- 1v2-CYCLE has asymptotically optimal lower bound:
  - Theorem 4: \( D(1v2-Cycle) = \Omega(n^2) \).
- Our adversary strategy:
  - reveals a YES edge for every \( (n) \) NO edges revealed
  - keeps the subgraph induced by YES edges a disjoint union of paths
  - whether the final graph is 1-Cycle or 2-Cycle remains ambiguous
- Corollary 5: Any deterministic AMPC algorithm using machines with I/O capacity \( S = n^k \) requires \( \Omega(1/\epsilon) \) rounds.

Randomized AMPC Lower Bound for 1v2-CYCLE via Certificate Complexity

- Certificate of \( g \) on input \( x \): subset of bits of \( x \) that certifies \( g(x) \)
- Certificate complexity of \( g \):
  - \( C(g) = \max_x \text{minimum size of certificate of } g \text{ on } x \)
- Easy: \( C(1v2-CYCLE) = n/2 \)
- For randomized computations, need to bound \( \delta \)-approximate certificate complexity of \( g \), \( C_\delta(g) \): minimum certificate complexity of any \( f \) that is \( \delta \)-close to \( g \)
  - Theorem 6: \( C_\delta(1v2-CYCLE) = \Omega(n^2) \).
- Corollary 7: Any randomized AMPC algorithm for 1v2-CYCLE that makes error \( \leq 1/6 \) and uses machines with I/O capacity \( S = n^k \) requires \( \Omega(1/\epsilon) \) rounds, matching upper bound of [BDE+19].

References