1. Let \( n, i \geq 1 \) be two integers, and \( 0 \leq a_1, a_2, \ldots, a_n \leq 9 \) be digits, with \( a_1 \neq 0 \). Prove that

\[ 81 \mid a_1 a_2 \cdots a_{n-1} a_n a_n \cdots a_2 a_1 \]

if and only if

\[ 81 \mid a_1 a_2 \cdots a_{n-1} a_n \underbrace{0 \cdots 0}_{2i} a_n a_n \cdots a_2 a_1, \]

where

\[ d_k d_{k-1} \cdots d_1 d_0 = d_k \cdot 10^k + d_{k-1} \cdot 10^{k-1} + \cdots + d_0 \cdot 10^0. \]

2. Given that

\[ 2 \sec^{-1} a + 4 \tan^{-1} b = \pi, \quad a, b \in \mathbb{Q}^+, \]

find the minimum possible value of \( a + b \).

3. In \( \triangle ABC \), \( \angle A = \frac{4\pi}{7} \) and \( \angle B = \frac{\pi}{7} \). Let \( I \) be the incenter of \( \triangle ABC \) so that \( CI \) meets \( AB \) at \( D \). Also let the circumcircles of \( \triangle AID \) and \( \triangle BDC \) meet at \( Q \). If \( BQ \) intersects \( CD \) at \( P \), and \( QI \) intersects \( BC \) at \( R \), prove that

\[ \frac{DQ}{DP} = \frac{IC}{IR}. \]
4. Find all functions $f : \mathbb{Z} \rightarrow \mathbb{Z}$ such that

$$(x + 1)f(x) + xf(x + 1) = f(x)f(x + 1)$$

for all integers $x$.

5. Given $a, b, c$ the sides of a non-obtuse triangle, prove that

$$a^2(41a^2 - 16b^2 - 7c^2 + 40bc - 30ca - 24ab) + b^2(34b^2 + 7c^2 - 9a^2 + 30ca - 24ab - 40bc) + c^2(25c^2 + 9a^2 + 16b^2 + 24ab - 40bc - 30ca) \geq 0.$$

6. There are $n$ points in the coordinate plane, all distinct from the origin, such that no two of the points are collinear with the origin. Determine the maximum possible number of triangles completely containing the origin that can be formed using the $n$ points as vertices.