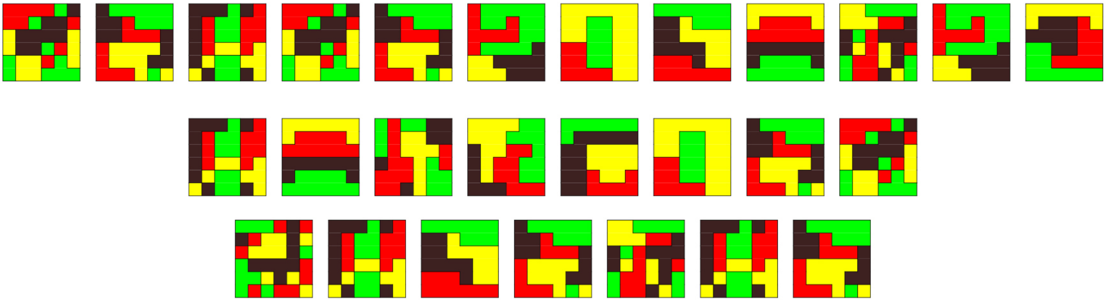
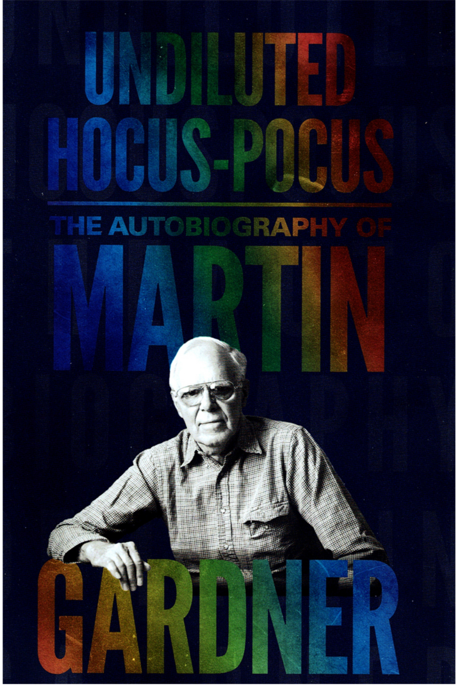
RECREATIONAL COMPUTER SCIENCE



Don Knuth, 6 January 2024

International conference on Fun With Algorithms

First	FUN 1996 (Alba)
...	
Fifth	FUN 2010 (Ischia)
Sixth	FUN 2012 (Venice)
...	
Twelfth	FUN 2024 (La Maddalena, 4–8 June)



ming, have made him the world's best-known computer scientist. Because Knuth likes to include in those books as much recreational material as he can cram in, he once visited me at my home in North Carolina. At that time my library and files were in a condominium that I rented solely to house them. The apartment had a kitchen and bathroom. Knuth stayed there for a week going through my files, leaving a stack of papers he wanted copied and sent to him. He cooked his own

MARTIN GARDNER
10 EUCLID AVENUE
HASTINGS-ON-HUDSON
NEW YORK, 10706

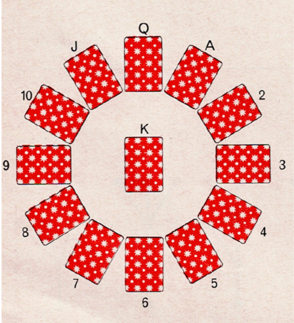
Dear Don: 14 Dec 1967

Page proofs of your wonderful Vol.I arrived yesterday, just in the nick of time. I had phoned Norman Stanton, at ~~Wiley,~~ Add-Wesley and after he promised me the pages before my Dec. 15 deadline, I went ahead with plans to devote the Feb. column to trees. As the deadline came ominously close, however, I had to protect myself against the possibility that the proofs might not arrive before the 15th, so I typed out a tentative column, including your surprising discovery about clock solitaire which you had been good enough to explain in a letter. I spent last evening browsing through the volume and today am typing out the final copy in which I will give your book an enthusiastic plug. I explain all this because had I gotten my hands on the pages a bit earlier I could have drawn more from the book. On the other hand, it may be just as well that I focus on this one item, holding off other gems to mention in later columns and thus provide excuses for additional plugs.

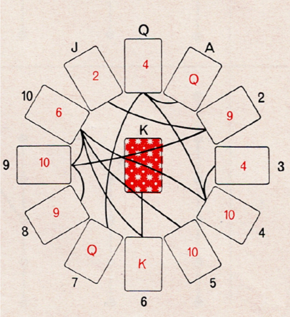
I am overwhelmed by the wealth of exciting and fresh material you have managed to pack into the book, especially in view of the fact that it is only the first of seven volumes! "Monumental" is the only word for it, and I predict the series will become a classic, like Dickson's history of the theory of numbers. Moreover, it is written with a grace and humor that is, as you know, exceedingly rare in books on mathematics. I greatly enjoyed your dedication, your flow-chart for reading the series, your notes on the exercises; above all, your choice of illustrative material throughout and the clarity and brevity with which you explain everything. My congratulations on getting the series off to such a fine start.

tions research, game strategy and all kinds of combinatorial problems. The most striking example I know of the unexpected applicability of tree diagrams to a combinatorial problem (in this case a game of card solitaire) is given in a discussion of tree theory in *Fundamental Algorithms*, just published by Addison-Wesley as the first volume of a projected seven-volume series titled *The Art of Computer Programming*. The author is Donald E. Knuth, a mathematician at the California Institute of Technology.

The solitaire game is best known as "clock," although it also goes by such names as "travelers," "hidden cards" and "four of a kind." The pack is dealt into 13 face-down piles of four cards each, the piles arranged as shown in the illustration at the left at the bottom of page 120 to correspond to the numbers on a clock face. The 13th (king) pile goes in the center. Turn over the top card of the king pile, then slide it face up under whichever pile corresponds to the card's value. For example, if it is a four, put it under the four o'clock pile; if a jack, under the 11 o'clock pile, and so on. Now turn up the top card of the pile under which you just placed the card and do the same thing with the new card. The play continues in this way. If you turn a card that matches the pile it is in, slide it face up under that pile and turn the next top card. If you place a card under a pile and there is no face-down card on it (the pile consisting of four face-up cards of the same value), then move to the next-higher pile clockwise. The game is won if you get all 52 cards face up. If you turn a fourth king before this happens, the play is blocked and the game lost.



Starting position for clock solitaire



Bottom cards and their tree connections

Playing clock is purely mechanical, demanding no skill. Knuth proves in his book that the chances of winning are exactly 1/13 and that in the long run the average number of cards turned up per game is 42.4. Even more astonishing is Knuth's delightful discovery of a simple way to know in advance, merely by checking the bottom card of each pile, whether the game will be won or lost. Draw another clock-face diagram, but this time indicate on each pile the value of the bottom card of that pile—except for the center, or king, pile, the bottom card of which remains unknown. Now draw a line from each of the 12 bottom-card values to the pile with the corresponding number [*see illustration at right at bottom of next page*]. (No line is drawn if the card's value matches its own pile.) Redraw the resulting graph to reveal its tree structure [*see top illustration on page 121*]. If and only if the graph is a tree that includes all 13 piles will the game be won. The arrangement of the 40 unknown cards is immaterial!

The illustrated game, as the tree graph reveals, will be won. The reader is invited to draw a similar diagram for another starting position [*see bottom illustration on page 121*] to determine whether it is a win or a loss, and then to check the result by actually playing the game. A proof that the tree test always works will be found in Knuth's fascinating and charmingly written 634-page book. In addition to being the introductory volume of what will surely be a monumental survey of computer science, it is crammed with fresh material that is of great interest to recreational mathematicians.

▶ **16.** [*M24*] In a popular solitaire game called "clock," the 52 cards of an ordinary deck of playing cards are dealt face down into 13 piles of four each; 12 piles are arranged in a circle like the 12 hours of a clock and the thirteenth pile goes in the center. The solitaire game now proceeds by turning up the top card of the center pile, and then if its face value is k, by placing it next to the kth pile. (The numbers $1, 2, \ldots, 13$ are equivalent to $\mathrm{A}, 2, \ldots, 10, \mathrm{J}, \mathrm{Q}, \mathrm{K}$.) Play continues by turning up the top card of the kth pile and putting it next to *its* pile, etc., until we reach a point where we cannot continue since there are no more cards to turn up on the designated pile. (The player has no choice in the game, since the rules completely specify what to do.) The game is won if all cards are face up when play terminates. [*Reference:* E. D. Cheney, *Patience* (Boston: Lee & Shepard, 1870), 62–65; the game was called "Travellers' Patience" in England, according to M. Whitmore Jones, *Games of Patience* (London: L. Upcott Gill, 1900), Chapter 7.]

Show that the game will be won if and only if the following directed graph is an oriented tree: The vertices are $V_1, V_2, \ldots, V_{13}$; the arcs are $e_1, e_2, \ldots, e_{12}$, where e_j goes from V_j to V_k if k is the *bottom* card in pile j after the deal.

(In particular, if the bottom card of pile j is a "j", for $j \neq 13$, it is easy to see that the game is certainly lost, since this card could never be turned up. The result proved in this exercise gives a much faster way to play the game!)

17. [*M32*] What is the probability of winning the solitaire game of clock (described in exercise 16), assuming the deck is randomly shuffled? What is the probability that exactly k cards are still face down when the game is over?

beautiful general procedure, discovered in 1934 by Monroe H. Martin of the University of Maryland, that covers minimum-length bracelets showing all n-tuplets for beads of m different colors. For example, if there are three colors, 0, 1, 2, and we want a bracelet showing all 27 triplets, we start with 000 and proceed to add digits, always selecting the highest digit that will not duplicate a triplet that has already been formed. The result is 000222122021121020120011101. The procedure is given in exercise No. 17, page 33, of the second volume of Donald E. Knuth's monumental continuing series, *The Art of Computer Programming* (Addison-Wesley, 1969). The book is as rich in recreational material and little-known historical sidelights as last year's first volume, and I recommend it highly. In Volume I (answer to exercise No. 23, page 379) Knuth gives a remarkable formula (due to N. G. de Bruijn of Holland) that provides the number of minimum-length bracelets of n-tuplets and m colors, including reversals as different. Knuth tells me that when reversals are *not* considered different (as in the problem given here), the formula is

$$\frac{\frac{1}{2}(m!)^{m^{n-1}}}{m^n}.$$

If reversals are considered different, the formula is simply doubled. It is not hard to prove, Knuth adds, that no bracelets are symmetrical in the sense that they are identical with their reversals. The only exception is the four-bead doublet bracelet of two colors; in

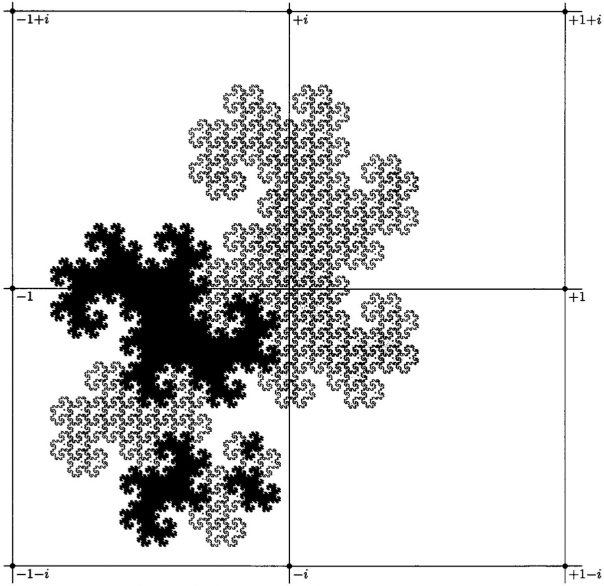


Fig. 1. The fractal set S called the "twindragon."

Another "binary" complex number system may be obtained by using the base $i-1$, as suggested by W. Penney [*JACM* **12** (1965), 247–248]:

$$\begin{aligned}(\ldots a_4a_3a_2a_1a_0.a_{-1}\ldots)_{i-1}\\ = \cdots - 4a_4 + (2i{+}2)a_3 - 2ia_2 + (i{-}1)a_1 + a_0 - \tfrac{1}{2}(i{+}1)a_{-1} + \cdots.\end{aligned}$$

In this system, only the digits 0 and 1 are needed. One way to demonstrate that every complex number has such a representation is to consider the interesting set S shown in Fig. 1; this set is, by definition, all points that can be written as $\sum_{k\geq 1} a_k(i-1)^{-k}$, for an infinite sequence a_1, a_2, a_3, … of zeros and ones. It is also known as the "twindragon fractal" [see M. F. Barnsley, *Fractals Everywhere*, second edition (Academic Press, 1993), 306, 310]. Figure 1 shows that S can be decomposed into 256 pieces congruent to $\frac{1}{16}S$. Notice that if the diagram of S is rotated counterclockwise by 135°, we obtain two adjacent sets congruent to

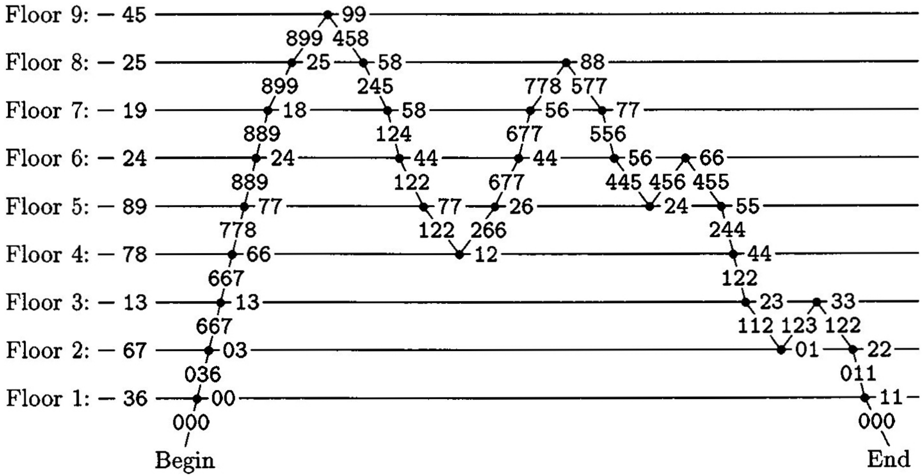


Fig. 88. An optimum way to rearrange people using a small, slow elevator. (People are each represented by the number of their destination floor.)

```
2024 = 1234-5+6+789
     = (((.12+3.4)*5)/.6)*(78-9)
     = (((1/(.2-.3))/.4)/.5-.6)/(.7/.8-.9)
     = -1+(2/.3)*45*6.78-9
     = -((((1+.2)/.3)/.4)*5+.6)/(.7/.8-.9)
```

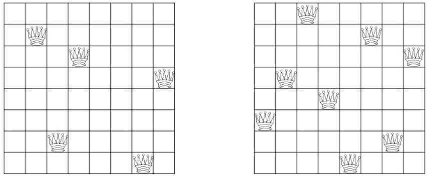
— Dr. I. J. Matrix

88/3 times 69 is 2024
2/.3 times 45 times 6.78 is 2034

there are 1329 representations starting without -
there are 1679 representations starting with -

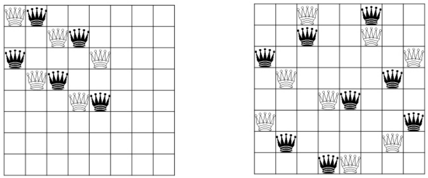
QUEENS

Maximal independent sets:

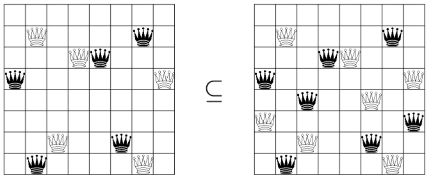


$$728z^5 + 6912z^6 + 2456z^7 + 92z^8$$

Maximal induced bipartite subgraphs:

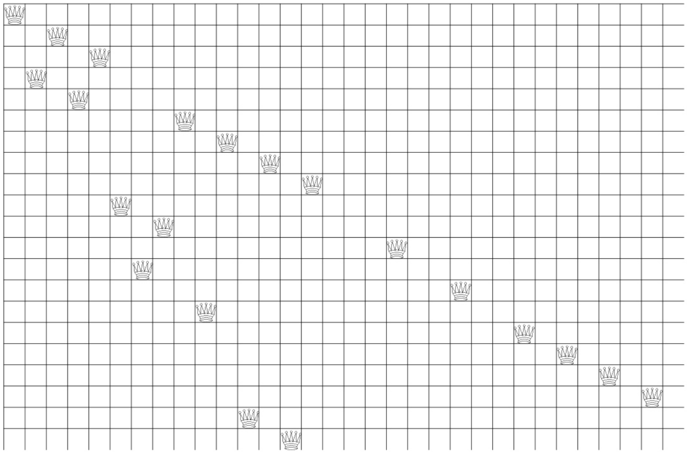


$$109894z^{10} + 2561492z^{11} + 13833474z^{12} + 9162232z^{13} + 1799264z^{14} + 99408z^{15} + 1626z^{16}$$



Maximal induced tripartite subgraphs?

Lexicographically smallest solution to the infinite queens problem

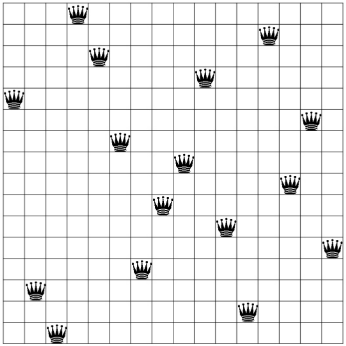


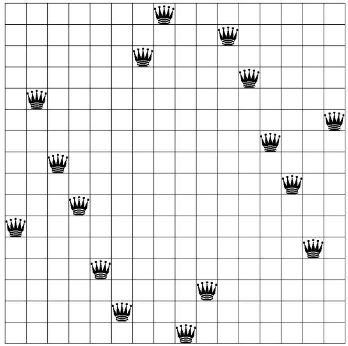
(n-th queen in column q_n; A065188)

$$
\begin{aligned}
q_{999999997} &= 618033989 \\
q_{999999998} &= 1618033985 \\
q_{999999999} &= 618033988 \\
q_{1000000000} &= 1618033988 \\
q_{1000000001} &= 1618033990 \\
q_{1000000002} &= 1618033992 \\
q_{1000000003} &= 1618033994 \\
q_{1000000004} &= 618033991
\end{aligned}
$$

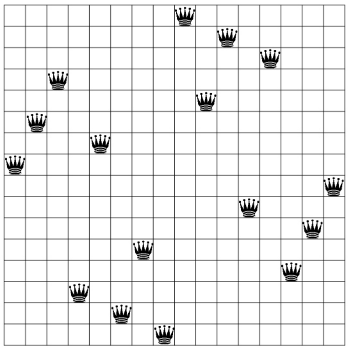
q_n can be computed with about 5.726 memory accesses

N queens as close or as far from the center as possible

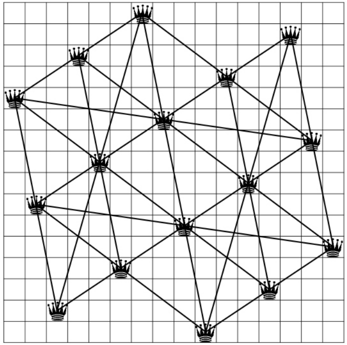




Minimizing the sum of distance^{M} for large M



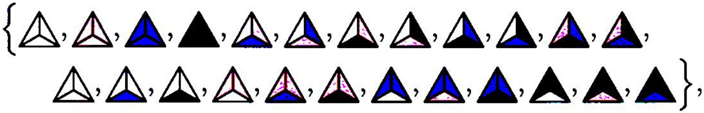
N queens with as many 3-in-a-line as possible



(Dudeney's "Orchard Problem")

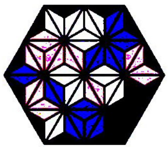
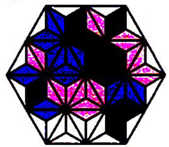
MacMahon's tiles

The great combinatorialist P. A. MacMahon introduced several families of colorful geometric patterns that continued to fascinate him throughout his life. For example, in U.K. Patent 3927 of 1892, written with J. R. J. Jocelyn, he considered the 24 different triangles that can be made with four colors on their edges,



(58)

and showed two ways in which they could be arranged to form a hexagon with matching colors at adjacent edges and with solid colors on the outer boundary:

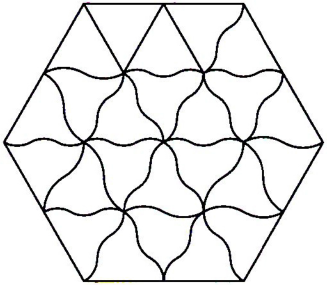
(a)  (b)  (59)

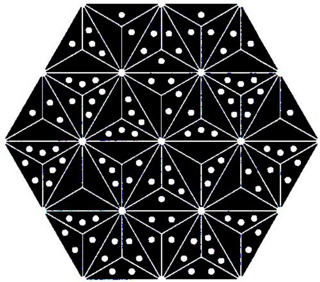
(Notice that chiral pairs, like  and  in (58), are considered to be distinct; MacMahon's tiles can be rotated, but they can't be "flipped over.")

Four suitable colours are black, white, red, and blue,
as they are readily distinguishable at night.

— P. A. MACMAHON, *New Mathematical Pastimes* (1921)

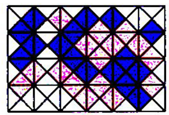
131. [*28*] (P. A. MacMahon, 1921.) Instead of using the colored tiles of (58), which yield (59), we can form hexagons from 24 different triangles in two other ways:





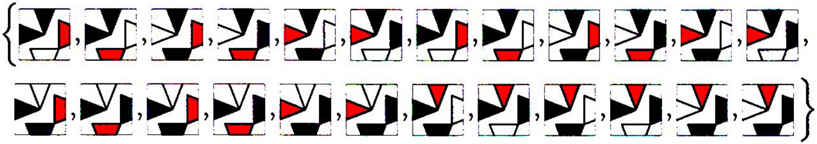
The left diagram shows a "jigsaw puzzle" whose pieces have four kinds of edges. The right diagram shows "triple three triominoes," which have zero, one, two, or three spots at each edge; adjacent triominoes should have a total of three spots where they meet.

a) In how many ways can that jigsaw puzzle make a hexagon? (All pieces are white.)
b) How many triomino arrangements have that pattern of dots at the edges?

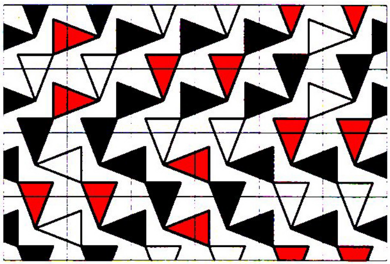


133. [*21*] (P. A. MacMahon, 1921.) A set of 24 *square* tiles can be constructed, analogous to the triangular tiles of (58), if we restrict ourselves to just three colors. For example, they can be arranged in a 4×6 rectangle as shown, with all-white border. In how many ways can this be done?

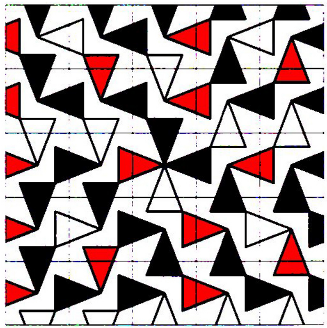
138. [*25*] (*Heads and tails.*) Here's a set of 24 square tiles that MacMahon missed(!):



They each show two "heads" and two "tails" of triangles, in four colors that exhibit all possible permutations, with heads pointing to tails. The tiles can be rotated, but not flipped over. We can match them properly in many ways, such as

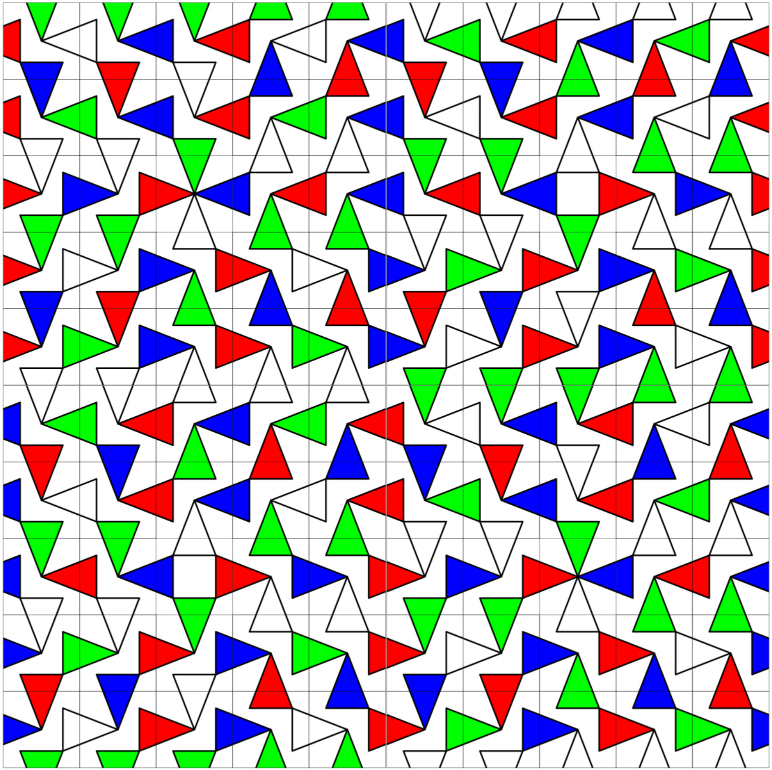


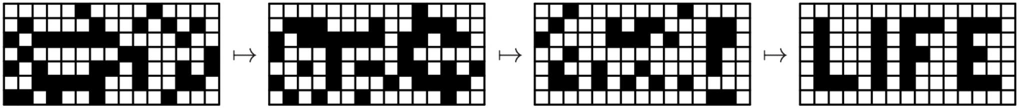
or

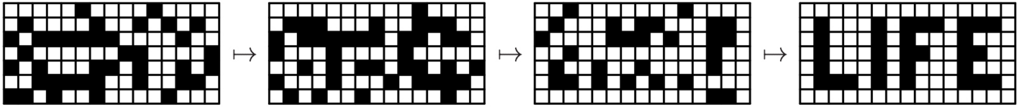


,

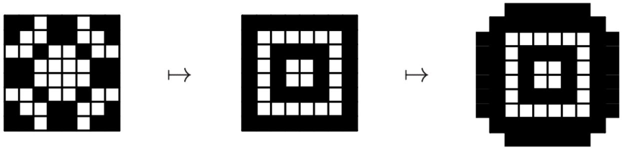
where the 4×6 arrangement will tile the plane; the 5×5 arrangement has a special "joker" tile in the middle, containing all four heads.

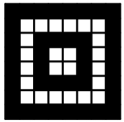


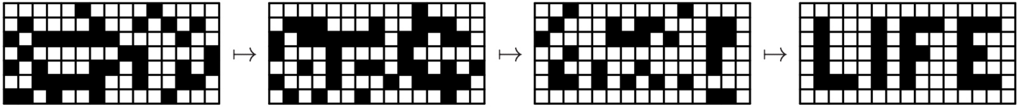




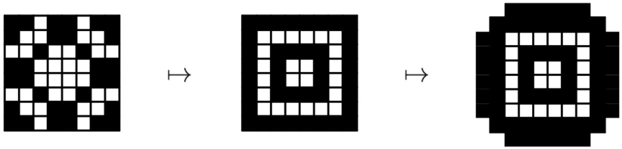
An 8-by-8 pattern is called *tame* if it stays inside its 8-by-8 box in one Life step. Otherwise it is *wild*.

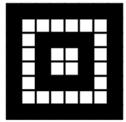


(Therefore  is tame and  is wild.)



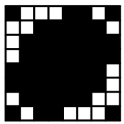
An 8-by-8 pattern is called *tame* if it stays inside its 8-by-8 box in one Life step. Otherwise it is *wild*.



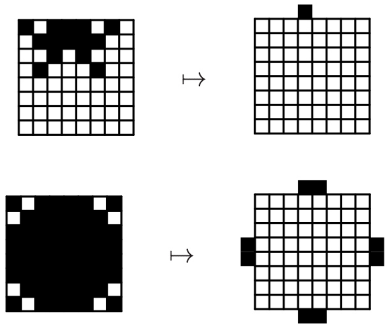
(Therefore  is tame and  is wild.)

Exactly 21,929,490,122 tame patterns vanish in one step.
(And exactly 5,530,201,631,127,973,447 on an 11-by-11 board.)
(And 4,080,967,796,136,376,032,811,105,207,453 on 15-by-15.)
(Tom Rokicki, 31 October 2010; A134963)

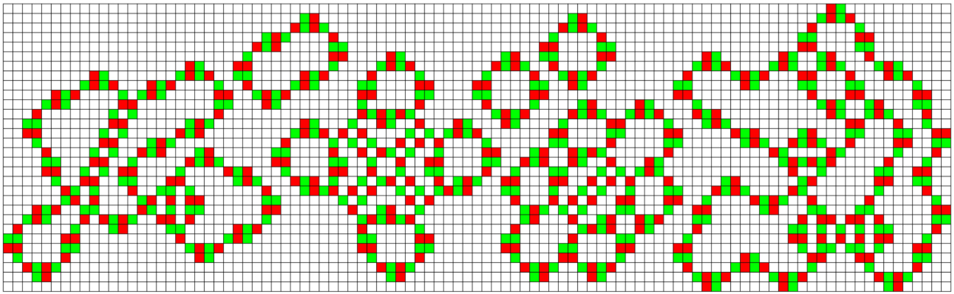
Here's one of the eight 8-by-8s that that have weight 46:



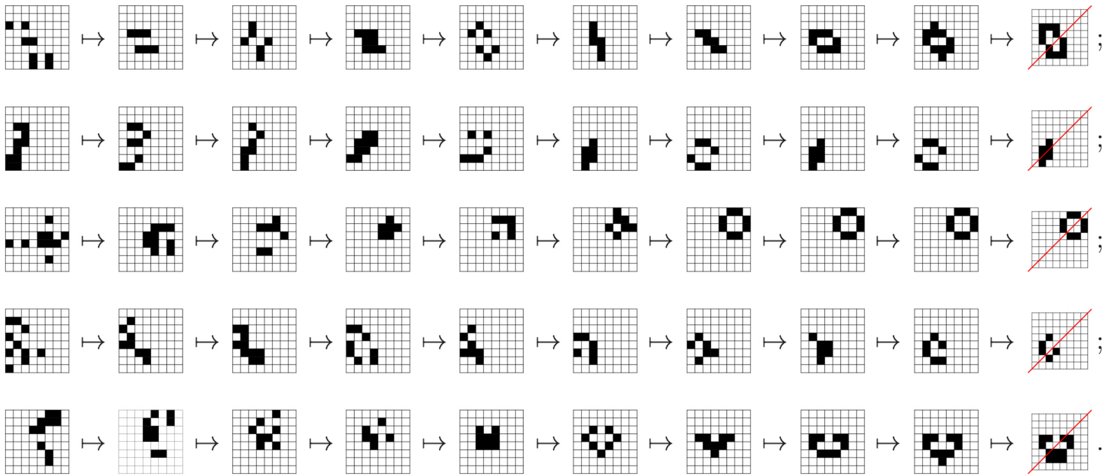
There are 12,942,036,750 wild patterns that vanish inside the 8-by-8:



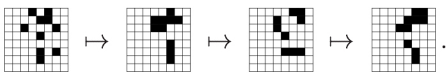
A *mobile path* has no cell alive more than four steps in a row.
A *mobile flipflop* has period 2.



A *gourmet* 8-by-8 mobile path always has between 6 and 10 live cells:



Problem: Is there a gourmet 8-by-8 mobile path of length 12?



Light Speed in Life

$$f(x,y) = x\,[x \geq 0] + y\,[y \geq 0] + (x+y)\,[x+y \geq 0]$$

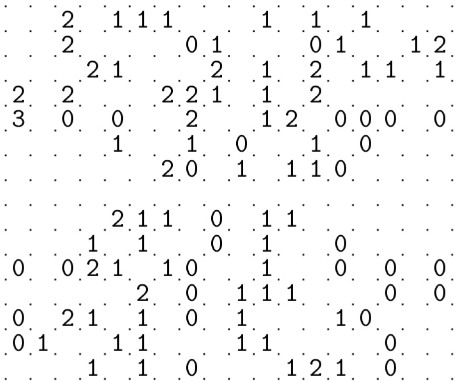
11	12	13	14	15	16	18	20	22	24	26	28	30	32
9	10	11	12	13	14	16	18	20	22	24	26	28	30
7	8	9	10	11	12	14	16	18	20	22	24	26	28
5	6	7	8	9	10	12	14	16	18	20	22	24	26
4	4	5	6	7	8	10	12	14	16	18	20	22	24
3	3	3	4	5	6	8	10	12	14	16	18	20	22
2	2	2	2	3	4	6	8	10	12	14	16	18	20
1	1	1	1	1	2	4	6	8	10	12	14	16	18
0	0	0	0	0	0	2	4	6	8	10	12	14	16
0	0	0	0	0	0	1	3	5	7	9	11	13	15
0	0	0	0	0	0	1	2	4	6	8	10	12	14
0	0	0	0	0	0	1	2	3	5	7	9	11	13
0	0	0	0	0	0	1	2	3	4	6	8	10	12
0	0	0	0	0	0	1	2	3	4	5	7	9	11

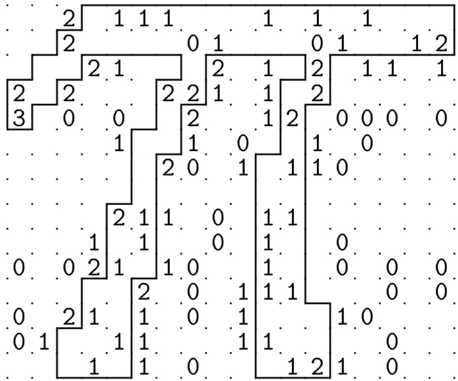
Light Speed in Life

$$f(x,y) = x\,[x \geq 0] + y\,[y \geq 0] + (x+y)\,[x+y \geq 0]$$

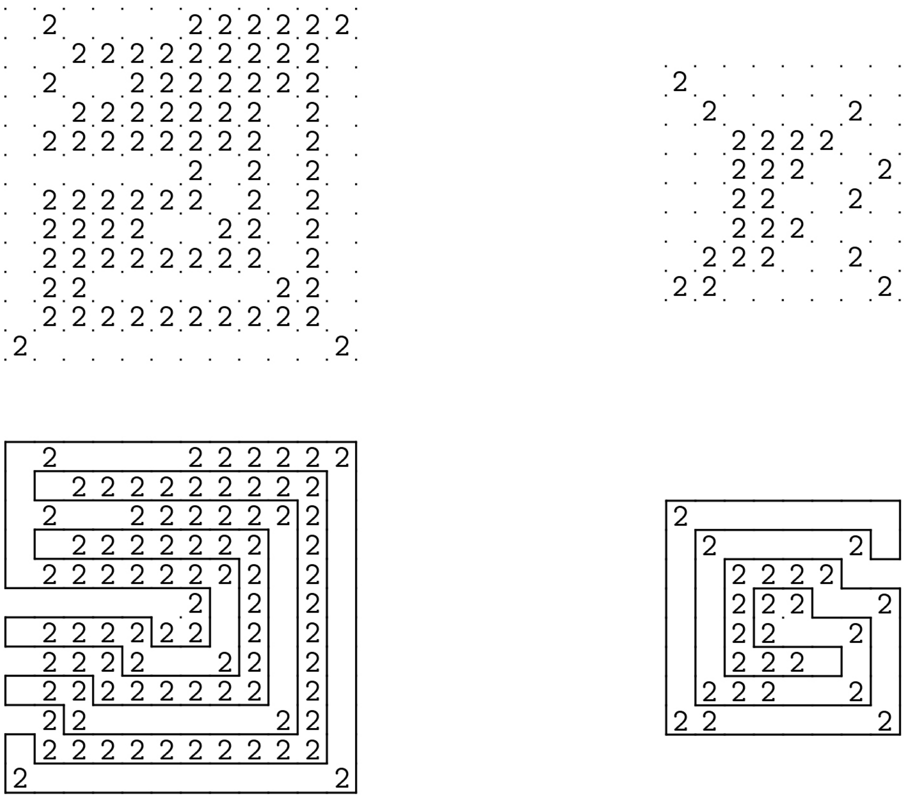
11	12	13	14	**15**	**16**	18	20	22	24	26	**28**	**30**	**32**
9	10	11	12	**13**	**14**	16	18	20	22	**24**	**26**	**28**	**30**
7	8	9	10	**11**	**12**	14	16	18	**20**	**22**	**24**	**26**	**28**
5	**6**	7	8	**9**	**10**	12	14	**16**	**18**	**20**	**22**	**24**	26
4	**4**	**5**	6	**7**	**8**	10	**12**	**14**	**16**	**18**	**20**	22	24
3	**3**	**3**	**4**	**5**	**6**	**8**	**10**	**12**	**14**	**16**	18	20	22
2	**2**	**2**	**2**	**3**	**4**	**6**	**8**	**10**	**12**	14	16	18	20
1	**1**	**1**	**1**	**1**	**2**	**4**	**6**	**8**	10	12	14	16	18
0	**0**	**0**	**0**	**0**	**0**	**2**	**4**	**6**	**8**	**10**	**12**	**14**	**16**
0	**0**	**0**	**0**	**0**	**0**	**1**	**3**	**5**	**7**	**9**	**11**	**13**	**15**
0	**0**	**0**	**0**	**0**	**0**	**1**	**2**	**4**	6	8	10	12	14
0	**0**	**0**	**0**	**0**	**0**	**1**	**2**	**3**	**5**	7	9	11	13
0	**0**	**0**	**0**	**0**	**0**	**1**	**2**	**3**	**4**	**6**	8	10	12
0	**0**	**0**	**0**	**0**	**0**	**1**	**2**	**3**	**4**	**5**	**7**	9	11

Slitherlink



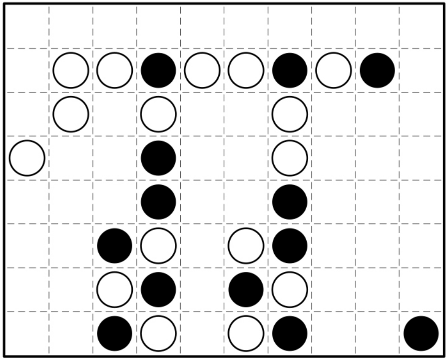


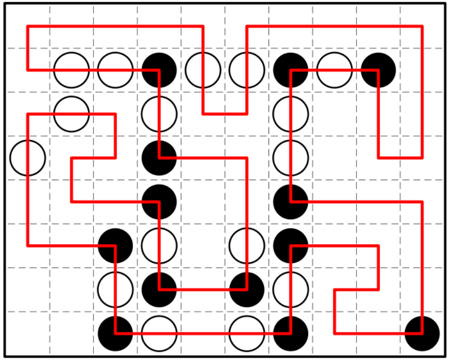
Slitherlink: Smallest density of all-2 clues
(Nikolai Beluhov; Palmer Mebane)



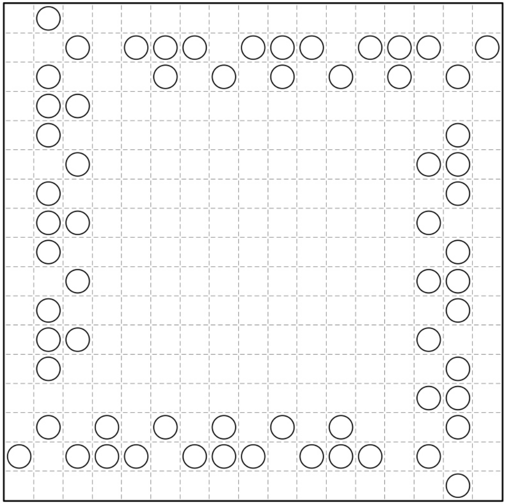
upper bound 11/16

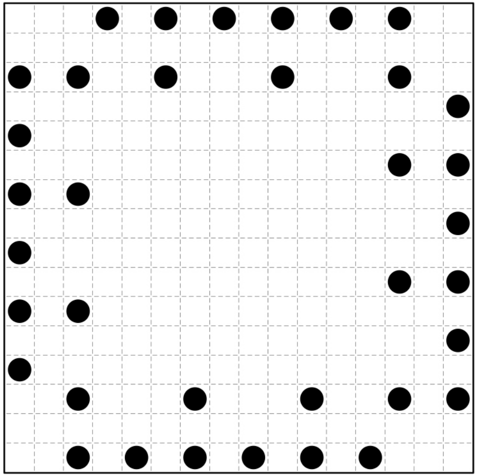
Masyu





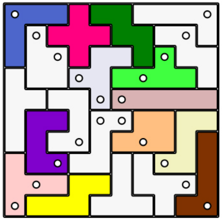
Masyu: Smallest density of all-white or all-black clues
(Nikolai Beluhov)

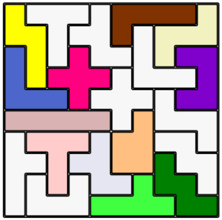


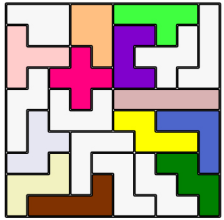


(The loop must pass through all four corner cells)

Packing Pentominoes in 10-by-10
(Aad van de Wetering)







12 colored pentominoes, 8 white “ghosts”
Every row has six nonghost cells, four ghost cells
Every column has six nonghost cells, four ghost cells

An amazingly graceful square
($K_4 \,\square\, K_4$)

$$\begin{pmatrix} 48 & 0 & 39 & 35 \\ 1 & 17 & 47 & 23 \\ 38 & 45 & 7 & 2 \\ 19 & 20 & 5 & 46 \end{pmatrix}$$

Some dazzling patterns arise when we consider "KP graphs" of the form $K_n \,\square\, P_r$, which consist of $r > 1$ cliques in a row, each of size $n > 2$. For example, here are two of the many graceful labelings of $K_4 \,\square\, P_{10}$ and $K_5 \,\square\, P_7$:

$$\begin{pmatrix} 0 & 96 & 4 & 93 & 5 & 90 & 11 & 88 & 22 & 84 \\ 1 & 3 & 13 & 65 & 89 & 14 & 62 & 25 & 81 & 58 \\ 91 & 9 & 87 & 7 & 77 & 50 & 18 & 72 & 51 & 69 \\ 95 & 28 & 73 & 12 & 55 & 17 & 82 & 33 & 68 & 27 \end{pmatrix}; \qquad \begin{pmatrix} 10 & 56 & 99 & 0 & 100 & 13 & 93 \\ 33 & 66 & 7 & 77 & 12 & 87 & 59 \\ 81 & 95 & 1 & 41 & 3 & 94 & 8 \\ 86 & 2 & 97 & 15 & 70 & 26 & 71 \\ 89 & 6 & 79 & 52 & 69 & 45 & 24 \end{pmatrix}. \qquad (39)$$

Each of the 10 columns on the left has six differences; in the first column they are $\{|0-1|, |0-91|, |0-95|, |1-91|, |1-95|, |91-95|\} = \{1, 91, 95, 90, 94, 4\}$. And each row also has nine differences between adjacent columns; in the first row they are $\{|0-96|, |96-4|, \ldots, |22-84|\} = \{96, 92, 89, 88, 85, 79, 77, 66, 62\}$. Those 60+36 differences are all distinct! And so are the 70+30 differences on the right!!

$K_n \,\square\, P_2$ is ungraceful for all $n > 5$

(computer-generated proof; 1.6 megamems)

$K_n \square P_2$ is ungraceful for all $n > 5$
(computer-generated proof; 1.6 megamems)

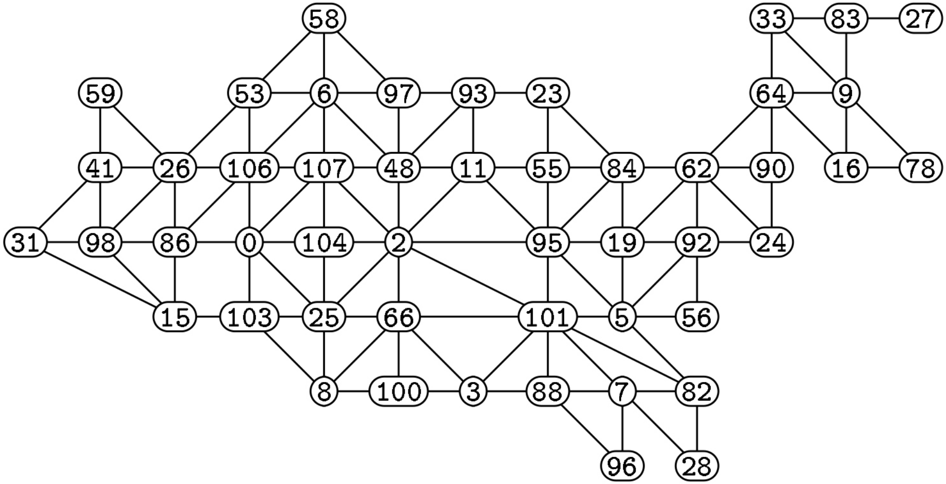
$K_n \square P_3$ is ungraceful for all $n > 6$
(computer-generated proof; 1.9 teramems)

Fig. 108. Some graceful gems: The unique labelings of $K_5 \,\square\, P_2$ and $K_6 \,\square\, P_3$. Also a (less rare) $K_6 \,\square\, P_4$ and $K_5 \,\square\, C_5$.

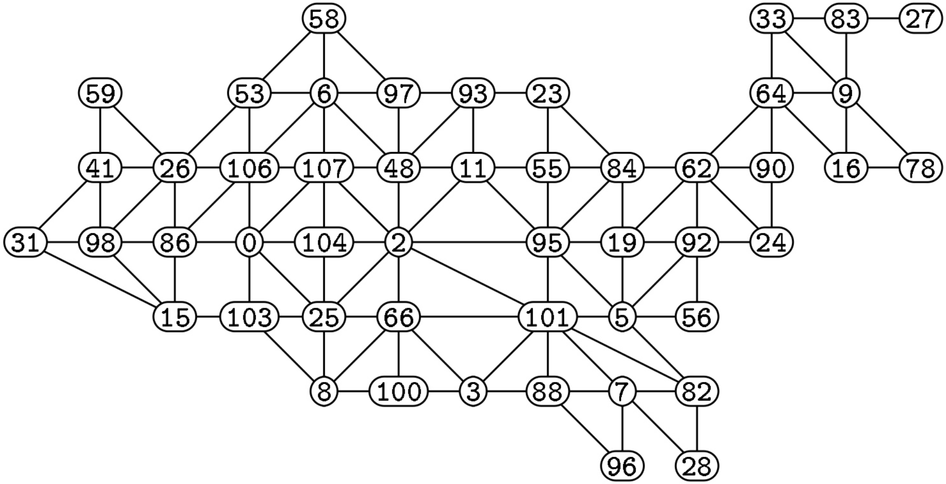
$$\begin{pmatrix} 0 & 24 \\ 6 & 22 \\ 7 & 19 \\ 21 & 11 \\ 25 & 2 \end{pmatrix} \quad \begin{pmatrix} 0 & 56 & 1 \\ 5 & 36 & 9 \\ 12 & 6 & 52 \\ 33 & 55 & 26 \\ 44 & 2 & 49 \\ 57 & 20 & 11 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 78 & 4 & 76 \\ 16 & 37 & 67 & 25 \\ 40 & 69 & 17 & 53 \\ 62 & 3 & 72 & 70 \\ 73 & 2 & 60 & 6 \\ 77 & 51 & 7 & 45 \end{pmatrix} \quad \begin{pmatrix} 0 & 62 & 6 & 64 & 75 \\ 3 & 18 & 69 & 10 & 33 \\ 41 & 70 & 23 & 59 & 20 \\ 73 & 9 & 43 & 24 & 51 \\ 74 & 2 & 71 & 14 & 8 \end{pmatrix}$$

A graceful miracle
(Tom Rokicki)



A graceful miracle
(Tom Rokicki)



	$98-86=12$	$33-9=24$	$92-56=36$	$64-16=48$	$86-26=60$	$98-26=72$	$95-11=84$	$101-5=96$
$107-106=1$	$101-88=13$	$25-0=25$	$48-11=37$	$97-48=49$	$84-23=61$	$92-19=73$	$88-3=85$	$100-3=97$
$64-62=2$	$19-5=14$	$90-64=26$	$62-24=38$	$83-33=50$	$78-16=62$	$83-9=74$	$86-0=86$	$101-3=98$
$107-104=3$	$41-26=15$	$53-26=27$	$97-58=39$	$56-5=51$	$66-3=63$	$82-7=75$	$92-5=87$	$101-2=99$
$97-93=4$	$31-15=16$	$90-62=28$	$95-55=40$	$58-6=52$	$66-2=64$	$95-19=76$	$103-15=88$	$106-6=100$
$58-53=5$	$25-8=17$	$84-55=29$	$66-25=41$	$106-53=53$	$84-19=65$	$82-5=77$	$96-7=89$	$107-6=101$
$101-95=6$	$59-41=18$	$92-62=30$	$48-6=42$	$82-28=54$	$90-24=66$	$103-25=78$	$95-5=90$	$104-2=102$
$16-9=7$	$101-82=19$	$64-33=31$	$62-19=43$	$64-9=55$	$98-31=67$	$104-25=79$	$97-6=91$	$103-0=103$
$96-88=8$	$106-86=20$	$55-23=32$	$55-11=44$	$83-27=56$	$92-24=68$	$106-26=80$	$100-8=92$	$104-0=104$
$11-2=9$	$28-7=21$	$59-26=33$	$93-48=45$	$98-41=57$	$78-9=69$	$88-7=81$	$95-2=93$	$107-2=105$
$41-31=10$	$84-62=22$	$100-66=34$	$48-2=46$	$66-8=58$	$93-23=70$	$93-11=82$	$101-7=94$	$106-0=106$
$95-84=11$	$25-2=23$	$101-66=35$	$53-6=47$	$107-48=59$	$86-15=71$	$98-15=83$	$103-8=95$	$107-0=107$

The 10-by-10 Strong Strong Queen Prime Attacking Problem
(G. L. Honacker Jr. and Peter Weigel)

11	34	99	96	**71**	06	75	94	**73**	82
98	**37**	10	33	**00**	95	72	**83**	76	93
35	12	**97**	70	**07**	18	**05**	74	81	84
38	09	36	**17**	**32**	**01**	80	85	92	77
13	**16**	**31**	**08**	**69**	**04**	**19**	78	**89**	86
30	39	14	**41**	**02**	**79**	58	87	20	91
15	42	**47**	68	**59**	66	**03**	90	57	88
48	**29**	40	45	**26**	63	60	**23**	54	21
43	46	27	50	**67**	24	65	52	**61**	56
28	49	44	25	**64**	51	62	55	22	**53**

closed knight's tour of length 100
every prime number is attacked by the red queen
every power of 2 is also attacked by the red queen
00 is in second row and attacked by the red queen
the digits of pi – **31**, **41**, **59**, **26** – appear in fixed places
exactly three solutions; 532 gigamems of exhaustive search

APPLAUSE