

# Game Theory and Mechanism Design

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# Sad (and happy!) introduction

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# Your First Game: Rock, Paper, Scissors

	Rock	Paper	Scissors
Rock	0, 0	-1, 1	1, -1
Paper	1, -1	0, 0	-1, 1
Scissors	-1, 1	1, -1	0, 0

# Nash Equilibrium: Prisoners' Dilemma

	Stay Silent	Confess
Stay Silent	$-1, -1$	$-10, 0$
Confess	$0, -10$	$-3, -3$



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# Why is Nash Equilibria Important?

## Nash's Theorem (Nash, 1951)

Every game with a finite number of players and actions has at least one Nash equilibrium.

# What is Mechanism Design?

- We get to design a game
- We want the game to have a certain outcome
- Let's design it so that the outcome we want is a **Nash equilibrium!**

# Auctions

- We have a good  $g$  that we want to sell
- We have  $n$  bidders, each of whom has a valuation  $v_i$  for the good
- They submit their bids  $b_i$  in sealed envelopes
- Who do we give the item to?

# The Vickrey Auction

- Instead of giving to the highest bidder for their bid, give to the highest bidder for the *second highest* bid.
- Claim: Vickrey auction induces an honest bidding Nash equilibrium.

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- Thought Experiment 2: What happens if I bid higher than my value?
- Conclusion: Honest bidding is a Nash equilibrium!

# The End