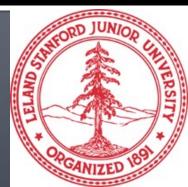
Introduction to Graph Neural Networks

Jiaxuan You, Stanford University

Adapted from Stanford CS 224W & CS 246



Many Types of Data are Graphs

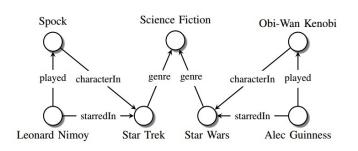


Image credit: <u>Maximilian Nickel et al</u>

Knowledge Graphs

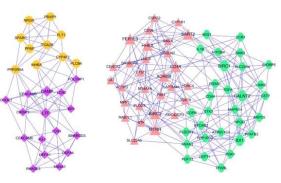


Image credit: <u>ese.wustl.edu</u>

Regulatory Networks

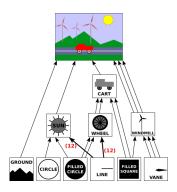
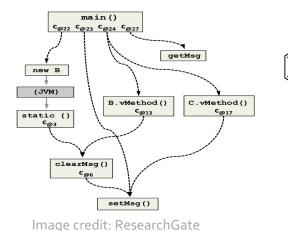


Image credit: math.hws.edu

Scene Graphs



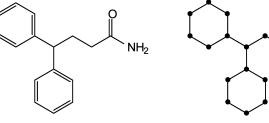


Image credit: MDPI

Molecules

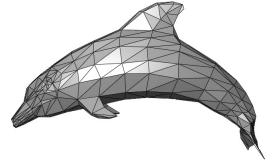
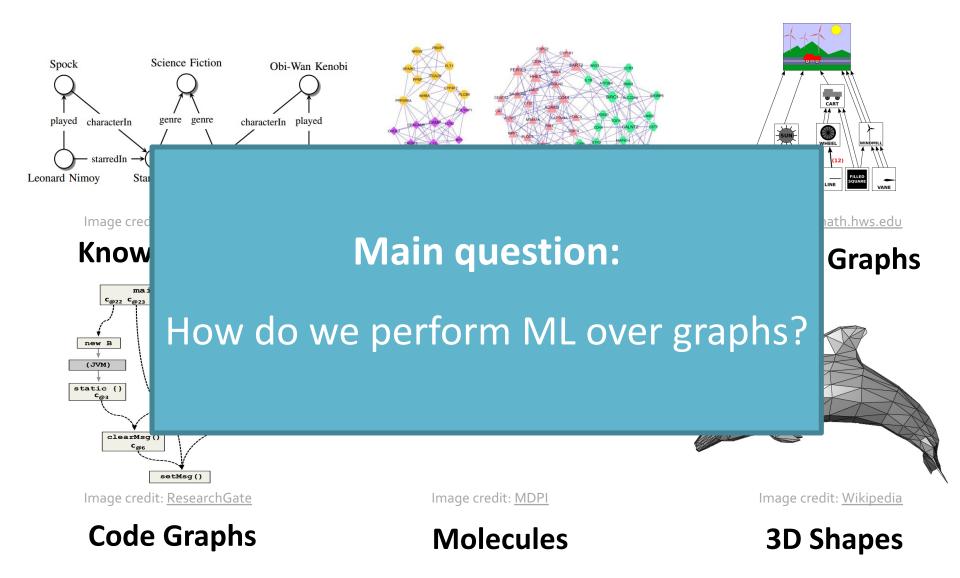


Image credit: Wikipedia

3D Shapes

Code Graphs

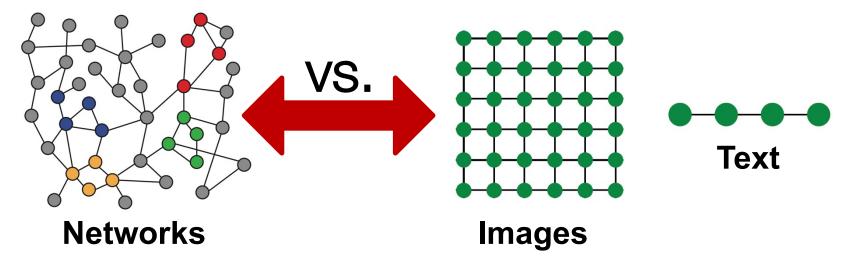
Many Types of Data are Graphs



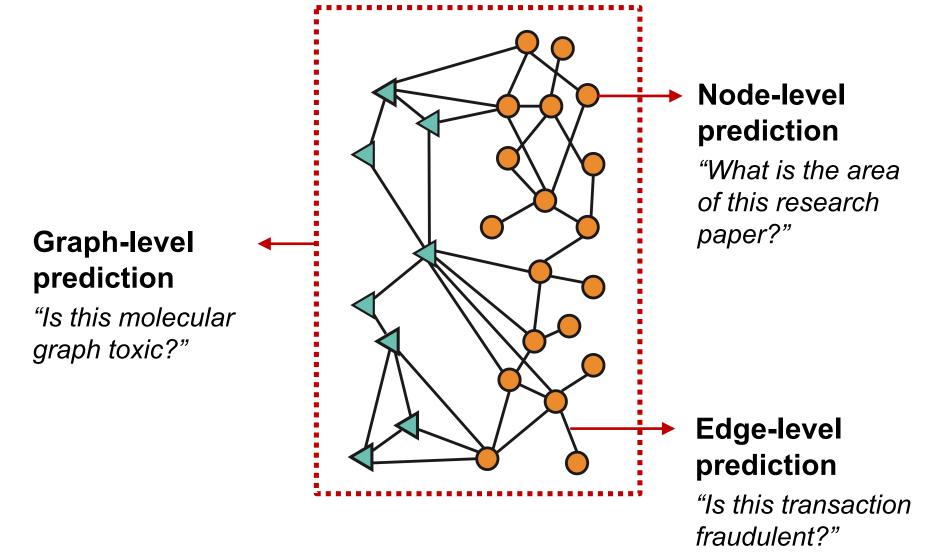
Why is it Hard?

Networks are complex.

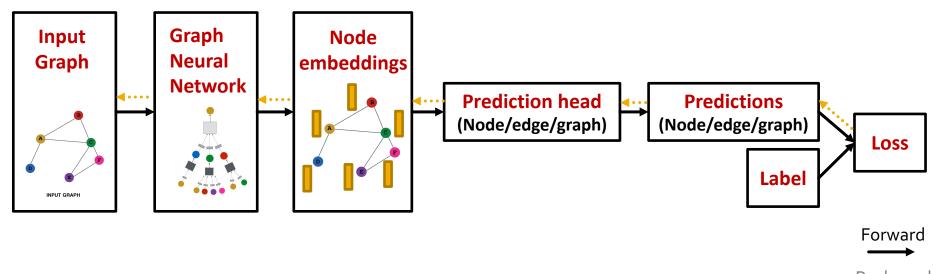
 Arbitrary size and complex topological structure (*i.e.*, no spatial locality like grids)



Learning From Graphs



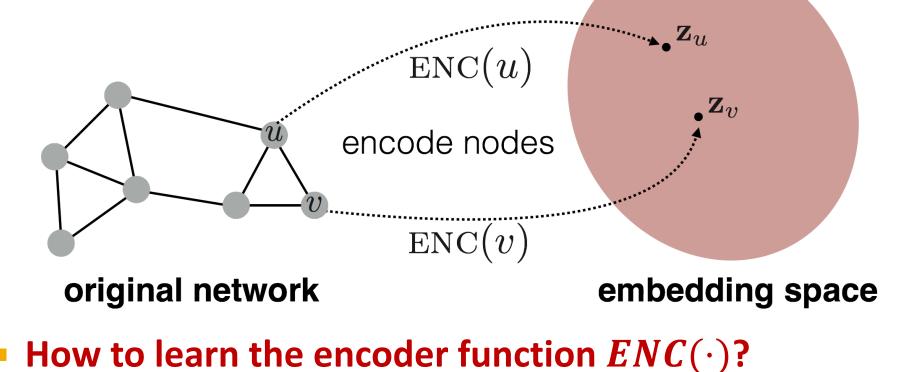
Deep Learning Pipeline for Graphs



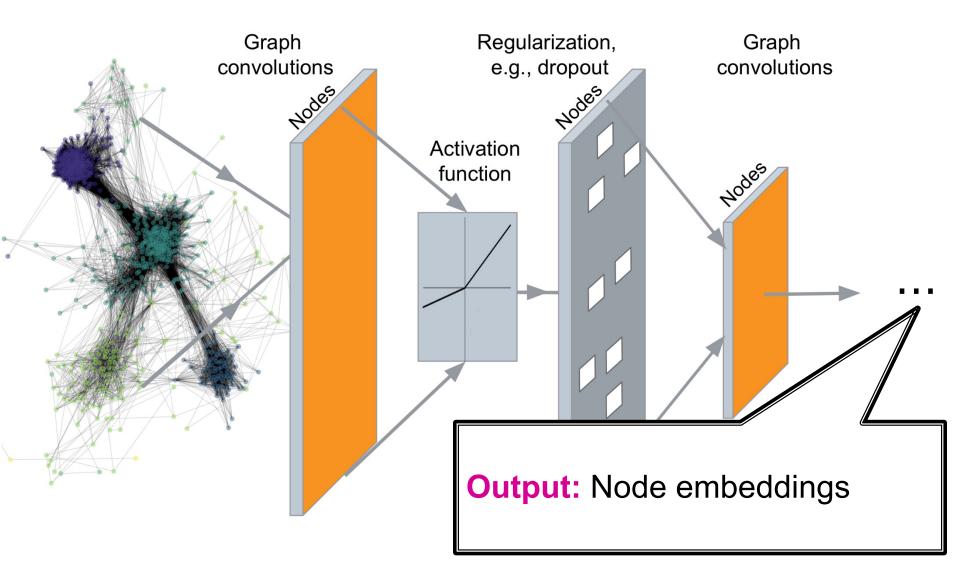
Backward

Key Concept: Node Embeddings

 Intuition: Map nodes to *d*-dimensional embeddings such that similar nodes in the graph are embedded close together



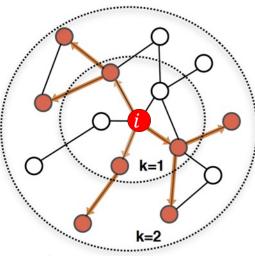
Deep Graph Encoders



Recap: Graph Neural Networks

Idea: Node's neighborhood defines a computation graph

 $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$



Determine node computation graph Propagate and transform information

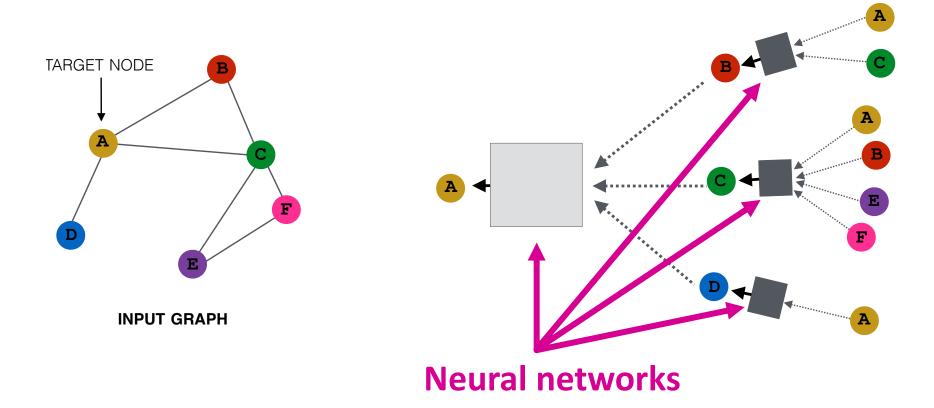
aggregator

aggregator

Learn how to propagate information across the graph to compute node features

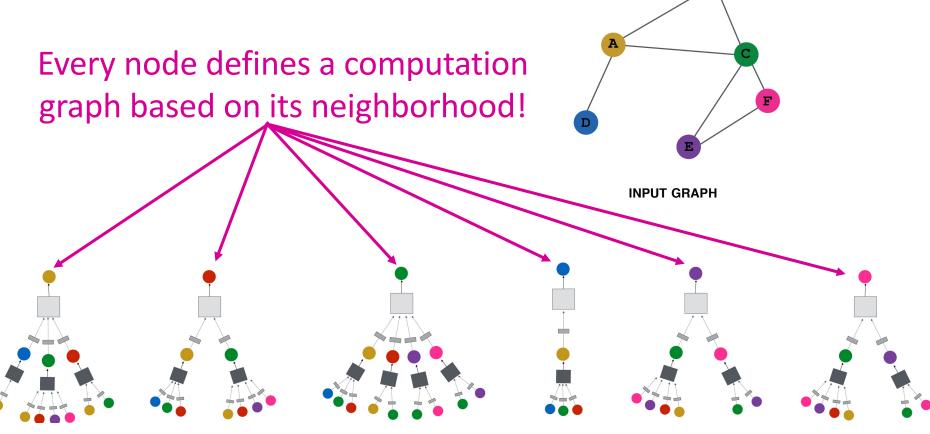
Recap: Aggregate from Neighbors

Intuition: Nodes aggregate information from their neighbors using neural networks



Recap: Aggregate Neighbors

 Intuition: Network neighborhood defines a computation graph



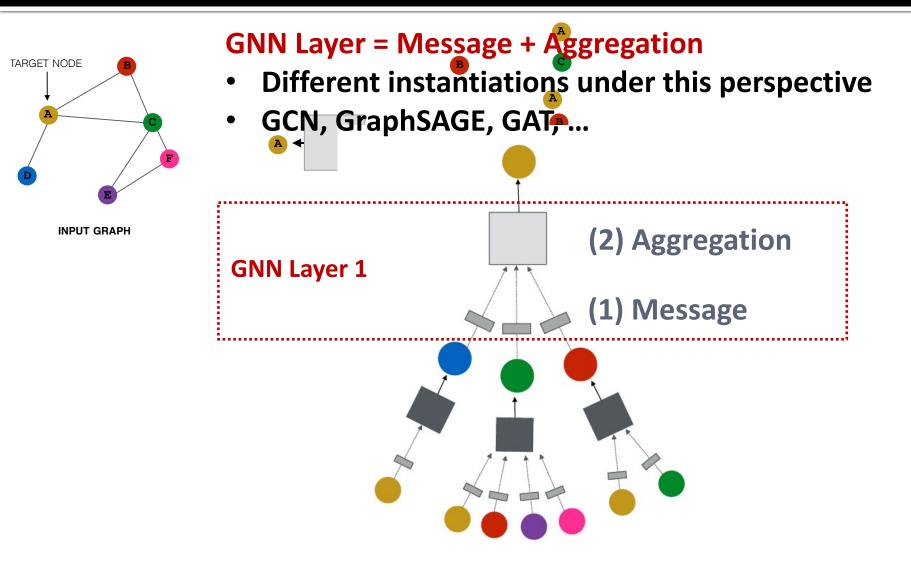
A General Perspective on Graph Neural Networks

Jiaxuan You, Stanford University

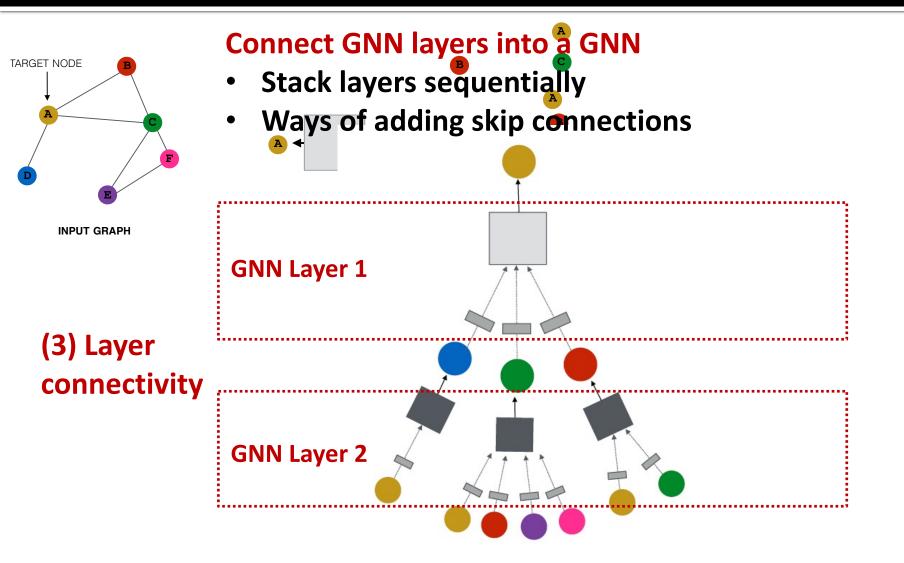
Adapted from Stanford CS 224W & CS 246



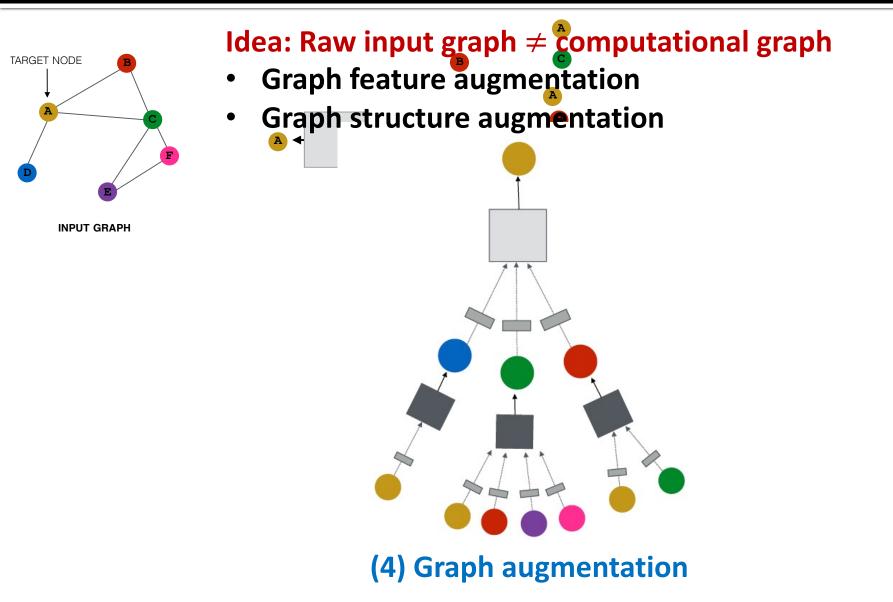
A General GNN Framework (1)



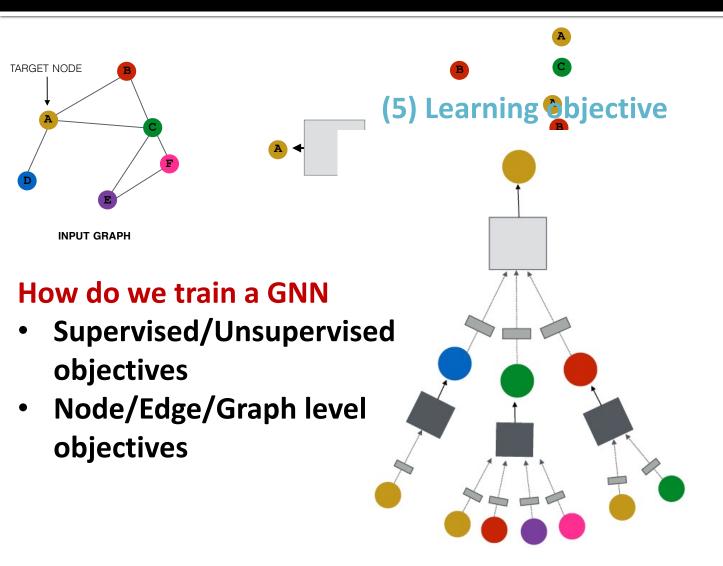
A General GNN Framework (2)



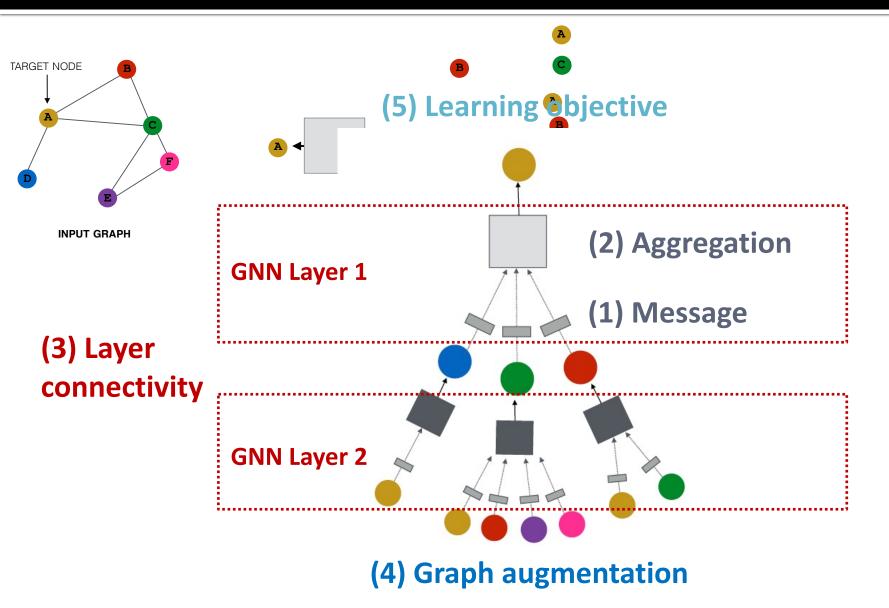
A General GNN Framework (3)



A General GNN Framework (4)



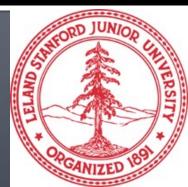
A General GNN Framework (5)



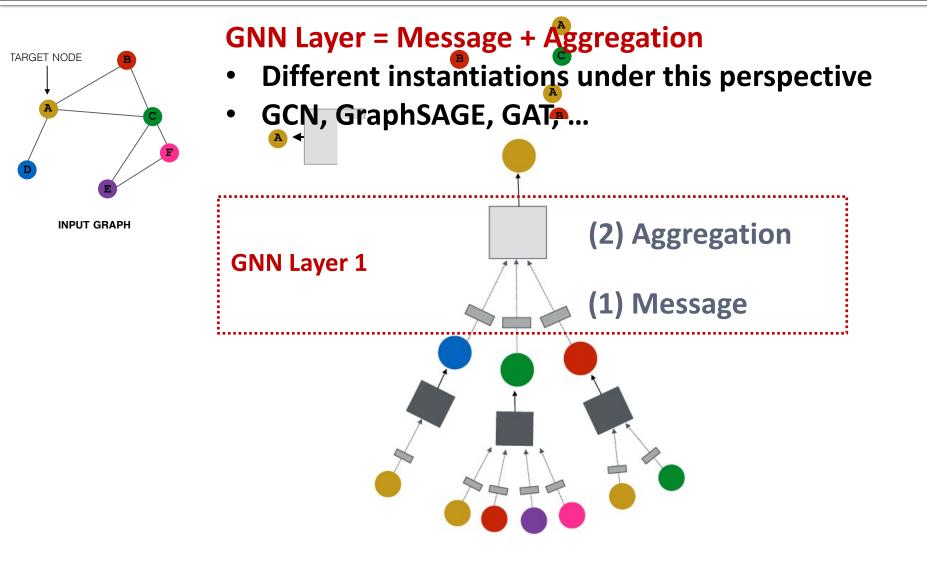
A Single Layer of a GNN

Jiaxuan You, Stanford University

Adapted from Stanford CS 224W & CS 246



A GNN Layer



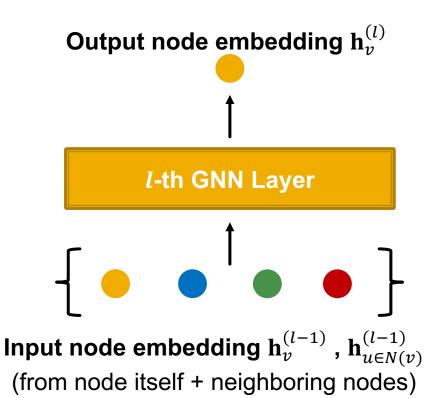
A Single GNN Layer

Idea of a GNN Layer:

- Compress a set of vectors into a single vector
- Two step process:
- (1) Message
 - (2) Aggregation

(2) Aggregation

(1) Message



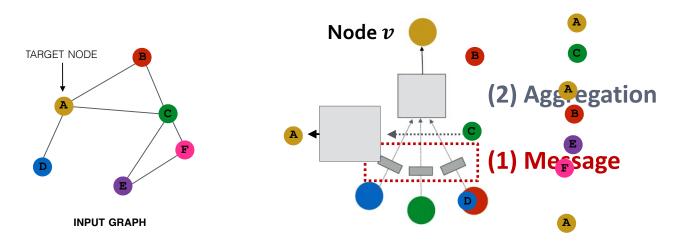
Node *v*

Message Computation

(1) Message computation

- Message function: $\mathbf{m}_{u}^{(l)} = MSG^{(l)}(\mathbf{h}_{u}^{(l-1)})$
 - Intuition: Each node will create a message, which will be sent to other nodes later
 - Example: A Linear layer $\mathbf{m}_{u}^{(l)} = \mathbf{W}^{(l)} \mathbf{h}_{u}^{(l-1)}$

Multiply node features with weight matrix $\mathbf{W}^{(l)}$



Message Aggregation

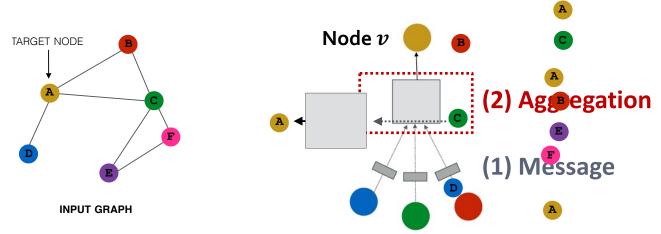
(2) Aggregation

Intuition: Each node will aggregate the messages from node v's neighbors

$$\mathbf{h}_{v}^{(l)} = \mathrm{AGG}^{(l)}\left(\left\{\mathbf{m}_{u}^{(l)}, u \in N(v)\right\}\right)$$

• **Example:** $Sum(\cdot)$, $Mean(\cdot)$ or $Max(\cdot)$ aggregator

•
$$\mathbf{h}_{v}^{(l)} = \operatorname{Sum}(\{\mathbf{m}_{u}^{(l)}, u \in N(v)\})$$



Message Aggregation: Issue

- Issue: Information from node v itself could get lost
 - Computation of $\mathbf{h}_v^{(l)}$ does not directly depend on $\mathbf{h}_v^{(l-1)}$
- Solution: Include $\mathbf{h}_{v}^{(l-1)}$ when computing $\mathbf{h}_{v}^{(l)}$
 - (1) Message: compute message from node v itself
 - Usually, a different message computation will be performed

$$\mathbf{m}_{u}^{(l)} = \mathbf{W}^{(l)} \mathbf{h}_{u}^{(l-1)} \qquad \mathbf{m}_{v}^{(l)} = \mathbf{B}^{(l)} \mathbf{h}_{v}^{(l-1)}$$

- (2) Aggregation: After aggregating from neighbors, we can aggregate the message from node v itself
 - Via concatenation or summation

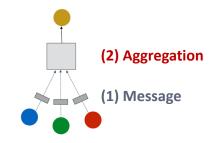
Then aggregate from node itself

$$\mathbf{h}_{v}^{(l)} = \text{CONCAT}\left(\text{AGG}\left(\left\{\mathbf{m}_{u}^{(l)}, u \in N(v)\right\}\right), \left[\mathbf{m}_{v}^{(l)}\right]\right)$$
First aggregate from neighbors

A Single GNN Layer

Putting things together:

- (1) Message: each node computes a message $\mathbf{m}_{u}^{(l)} = \mathrm{MSG}^{(l)} \left(\mathbf{h}_{u}^{(l-1)} \right), u \in \{N(v) \cup v\}$
- (2) Aggregation: aggregate messages from neighbors $\mathbf{h}_{v}^{(l)} = AGG^{(l)}\left(\left\{\mathbf{m}_{u}^{(l)}, u \in N(v)\right\}, \mathbf{m}_{v}^{(l)}\right)$
- Nonlinearity (activation): Adds expressiveness
 - Often written as $\sigma(\cdot)$: ReLU(\cdot), Sigmoid(\cdot), ...
 - Can be added to message or aggregation



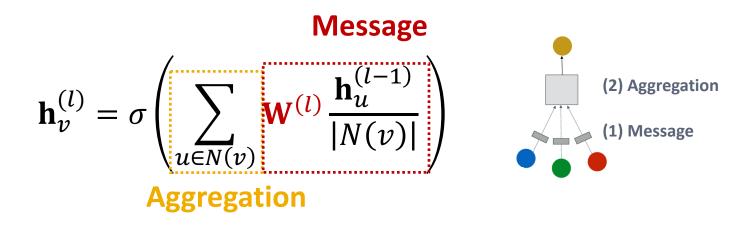
T. Kipf, M. Welling. Semi-Supervised Classification with Graph Convolutional Networks, ICLR 2017

Classical GNN Layers: GCN (1)

(1) Graph Convolutional Networks (GCN)

$$\mathbf{h}_{v}^{(l)} = \sigma \left(\mathbf{W}^{(l)} \sum_{u \in N(v)} \frac{\mathbf{h}_{u}^{(l-1)}}{|N(v)|} \right)$$

How to write this as Message + Aggregation?



Classical GNN Layers: GCN (2)

(1) Graph Convolutional Networks (GCN)

$$\mathbf{h}_{v}^{(l)} = \sigma\left(\sum_{u \in N(v)} \mathbf{W}^{(l)} \frac{\mathbf{h}_{u}^{(l-1)}}{|N(v)|}\right)$$
(2) Aggregation (1) Message

Message:

• Each Neighbor: $\mathbf{m}_u^{(l)} = \frac{1}{|N(v)|} \mathbf{W}^{(l)} \mathbf{h}_u^{(l-1)}$

Normalized by node degree

(In the GCN paper they use a slightly different normalization)

Aggregation:

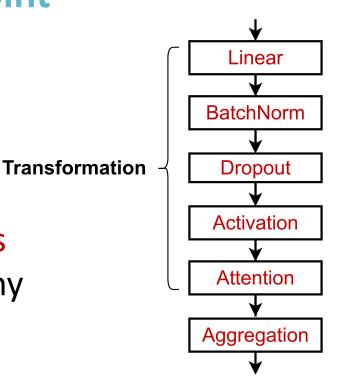
Sum over messages from neighbors, then apply activation

•
$$\mathbf{h}_{v}^{(l)} = \sigma\left(\operatorname{Sum}\left(\left\{\mathbf{m}_{u}^{(l)}, u \in N(v)\right\}\right)\right)$$

GNN Layer in Practice

- In practice, these classic GNN layers are a great starting point
 - We can often get better performance by considering a general GNN layer design
 - Concretely, we can include modern deep learning modules that proved to be useful in many domains





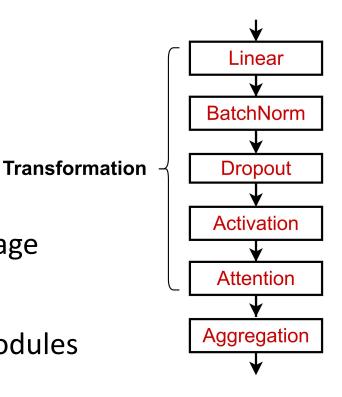
GNN Layer in Practice

- Many modern deep learning modules can be incorporated into a GNN layer
 - Batch Normalization:
 - Stabilize neural network training
 - Dropout:
 - Prevent overfitting
 - Attention/Gating:
 - Control the importance of a message

More:

Any other useful deep learning modules

A suggested GNN Layer



Batch Normalization

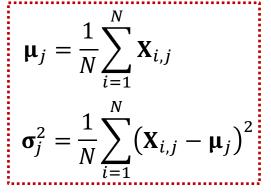
• **Goal**: Stabilize neural networks training

- Idea: Given a batch of inputs (node embeddings)
 - Re-center the node embeddings into zero mean
 - Re-scale the variance into unit variance

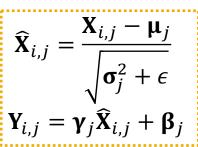
Input: $\mathbf{X} \in \mathbb{R}^{N \times D}$ *N* node embeddings

Trainable Parameters: $\gamma, \beta \in \mathbb{R}^D$

Output: $\mathbf{Y} \in \mathbb{R}^{N \times D}$ Normalized node embeddings Step 1: Compute the mean and variance over N embeddings

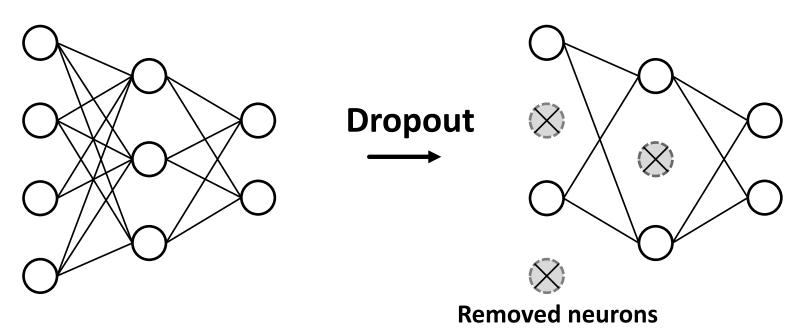


Step 2: Normalize the feature using computed mean and variance



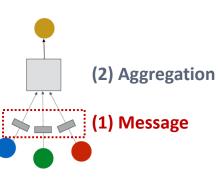
Dropout

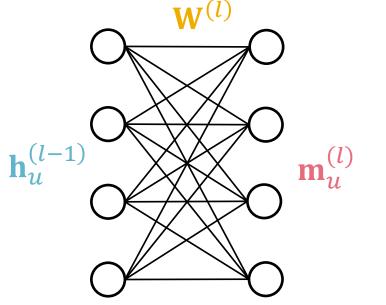
- Goal: Regularize a neural net to prevent overfitting.
 Idea:
 - During training: with some probability p, randomly set neurons to zero (turn off)
 - During testing: Use all the neurons for computation



Dropout for GNNs

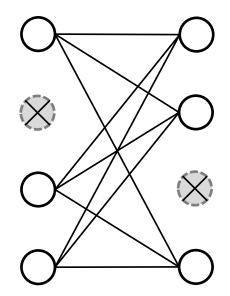
- In GNN, Dropout is applied to the <u>linear layer in the message function</u>
 - A simple message function with linear layer: $\mathbf{m}_{u}^{(l)} = \mathbf{W}^{(l)} \mathbf{h}_{u}^{(l-1)}$





Visualization of a linear layer

Dropout



Activation (Non-linearity)

Apply activation to *i*-th dimension of embedding **x**

- Rectified linear unit (ReLU)
 - $\text{ReLU}(\mathbf{x}_i) = \max(\mathbf{x}_i, 0)$
 - Most commonly used

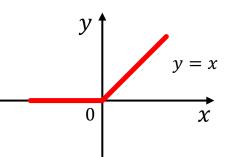
Sigmoid

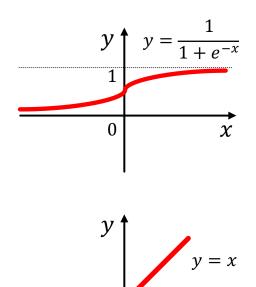
$$\sigma(\mathbf{x}_i) = \frac{1}{1 + e^{-\mathbf{x}_i}}$$

- Used only when you want to restrict the range of your embeddings
- Parametric ReLU

 $PReLU(\mathbf{x}_i) = \max(\mathbf{x}_i, 0) + \frac{a_i}{\min(\mathbf{x}_i, 0)}$

- a_i is a trainable parameter
- Empirically performs better than ReLU



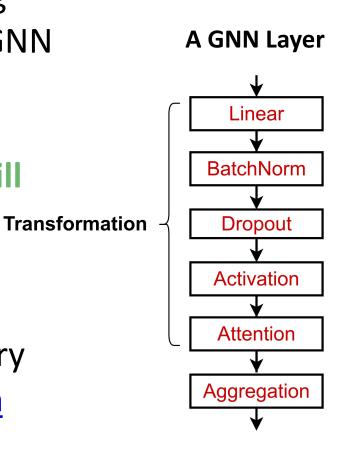


y = ax

Х

GNN Layer in Practice

- Summary: Modern deep learning modules can be included into a GNN layer for better performance
- Designing novel GNN layers is still an active research frontier! Tr
- Suggested resources: You can explore diverse GNN designs or try out your own ideas in <u>GraphGym</u>



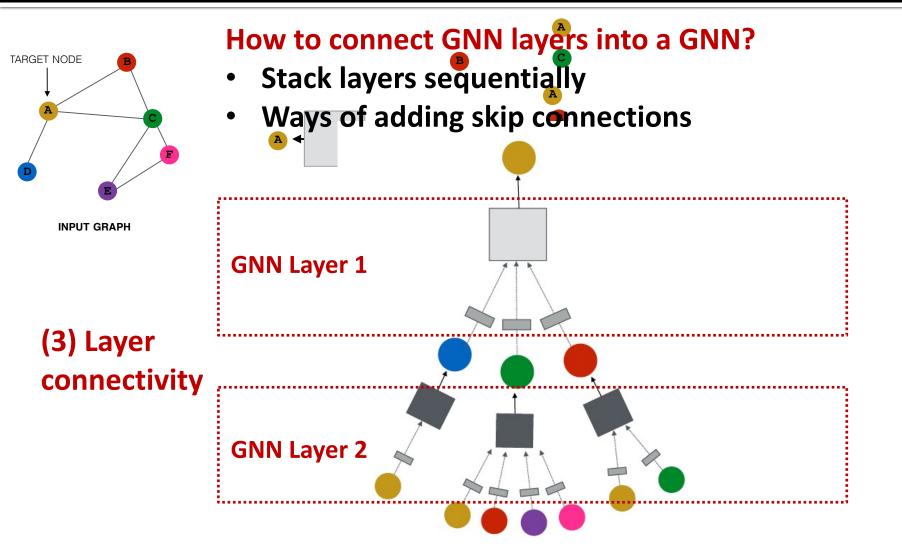
Stacking Layers of a GNN

Jiaxuan You, Stanford University

Adapted from Stanford CS 224W & CS 246



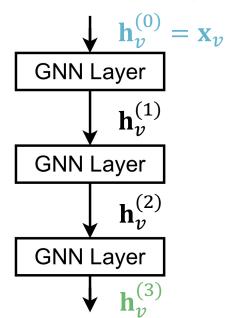
Stacking GNN Layers



Stacking GNN Layers

How to construct a Graph Neural Network?

- The standard way: Stack GNN layers sequentially
- Input: Initial raw node feature x_v
- **Output:** Node embeddings $\mathbf{h}_{v}^{(L)}$ after *L* GNN layers



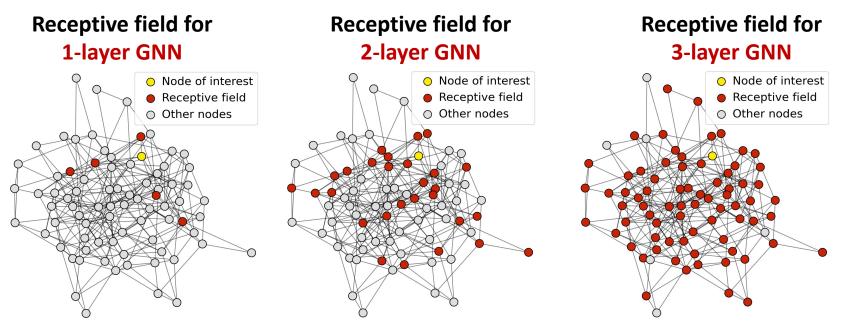
The Over-smoothing Problem

The Issue of stacking many GNN layers

- GNN suffers from the over-smoothing problem
- The over-smoothing problem: all the node embeddings converge to the same value
 - This is bad because we want to use node embeddings to differentiate nodes
- Why does the over-smoothing problem happen?

Receptive Field of a GNN

- Receptive field: the set of nodes that determine the embedding of a node of interest
 - In a K-layer GNN, each node has a receptive field of K-hop neighborhood



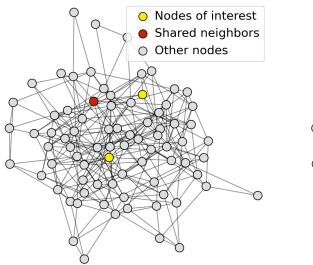
Receptive Field of a GNN

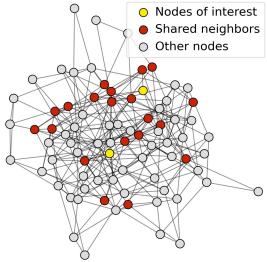
Receptive field overlap for two nodes
 The shared neighbors quickly grows when we increase the number of hops (num of GNN layers)

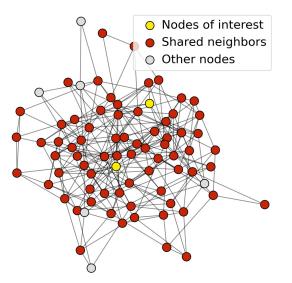
1-hop neighbor overlap Only 1 node

2-hop neighbor overlap About 20 nodes

3-hop neighbor overlap Almost all the nodes!







Receptive Field & Over-smoothing

- We can explain over-smoothing via the notion of receptive field
 - We knew the embedding of a node is determined by its receptive field
 - If two nodes have highly-overlapped receptive fields, then their embeddings are highly similar
 - Stack many GNN layers → nodes will have highlyoverlapped receptive fields → node embeddings will be highly similar → suffer from the oversmoothing problem

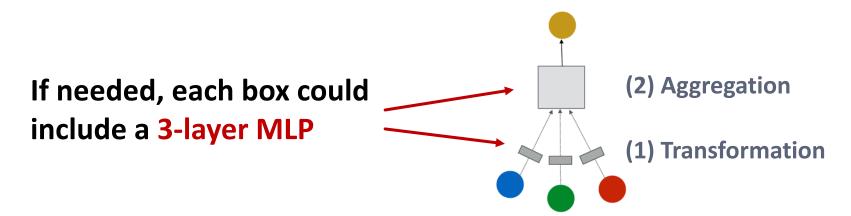
Next: how do we overcome over-smoothing problem?

Design GNN Layer Connectivity

- What do we learn from the over-smoothing problem?
 Lesson 1: Be cautious when adding GNN layers
 - Unlike neural networks in other domains (CNN for image classification), adding more GNN layers do not always help
 - Step 1: Analyze the necessary receptive field to solve your problem. E.g., by computing the diameter of the graph
 - Step 2: Set number of GNN layers L to be a bit more than the receptive field we like. Do not set L to be unnecessarily large!
- Question: How to enhance the expressive power of a GNN, if the number of GNN layers is small?

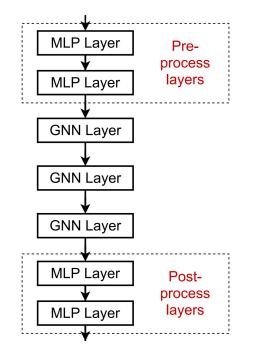
Expressive Power for Shallow GNNs

- How to make a shallow GNN more expressive?
- Solution 1: Increase the expressive power within each GNN layer
 - In our previous examples, each transformation or aggregation function only include one linear layer
 - We can make aggregation / transformation become a deep neural network!



Expressive Power for Shallow GNNs

- How to make a shallow GNN more expressive?
- Solution 2: Add layers that do not pass messages
 - A GNN does not necessarily only contain GNN layers
 - E.g., we can add MLP layers (applied to each node) before and after GNN layers, as pre-process layers and post-process layers



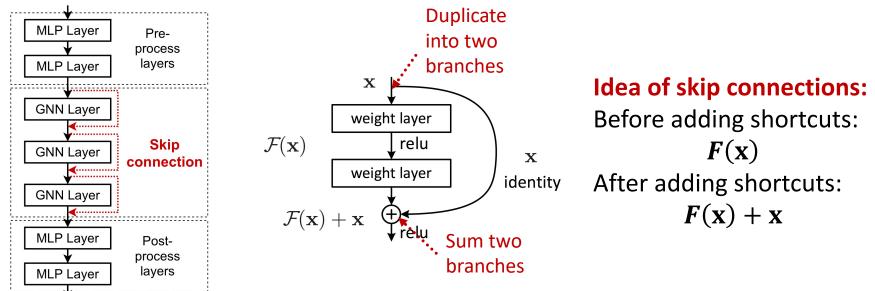
Pre-processing layers: Important when encoding node features is necessary.E.g., when nodes represent images/text

Post-processing layers: Important when reasoning / transformation over node embeddings are needed E.g., graph classification, knowledge graphs

In practice, adding these layers works great!

Design GNN Layer Connectivity

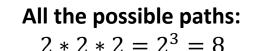
- What if my problem still requires many GNN layers?
 Lesson 2: Add skip connections in GNNs
 - Observation from over-smoothing: Node embeddings in earlier GNN layers can sometimes better differentiate nodes
 - Solution: We can increase the impact of earlier layers on the final node embeddings, by adding shortcuts in GNN

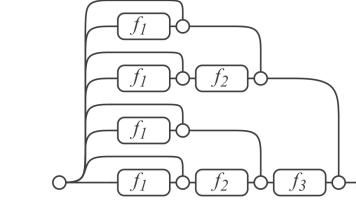


Idea of Skip Connections

Why do skip connections work?

- Intuition: Skip connections create a mixture of models
- N skip connections $\rightarrow 2^N$ possible paths
- Each path could have up to N modules
- We automatically get a mixture of shallow GNNs and deep GNNs





(b) Unraveled view of (a)

(a) Conventional 3-block residual network

Building block Skip connection

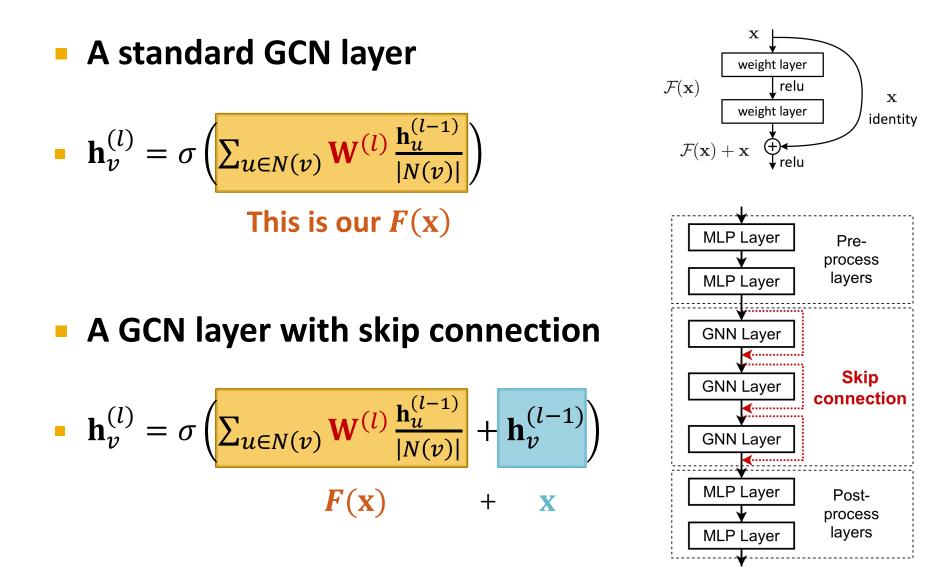
Residual module

Path 2: skip this module

Path 1: include this module

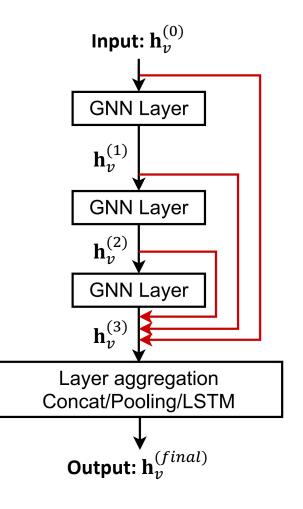
Veit et al. Residual Networks Behave Like Ensembles of Relatively Shallow Networks, ArXiv 2016

Example: GCN with Skip Connections



Other Options of Skip Connections

 Other options: Directly skip to the last layer
 The final layer directly aggregates from the all the node embeddings in the previous layers



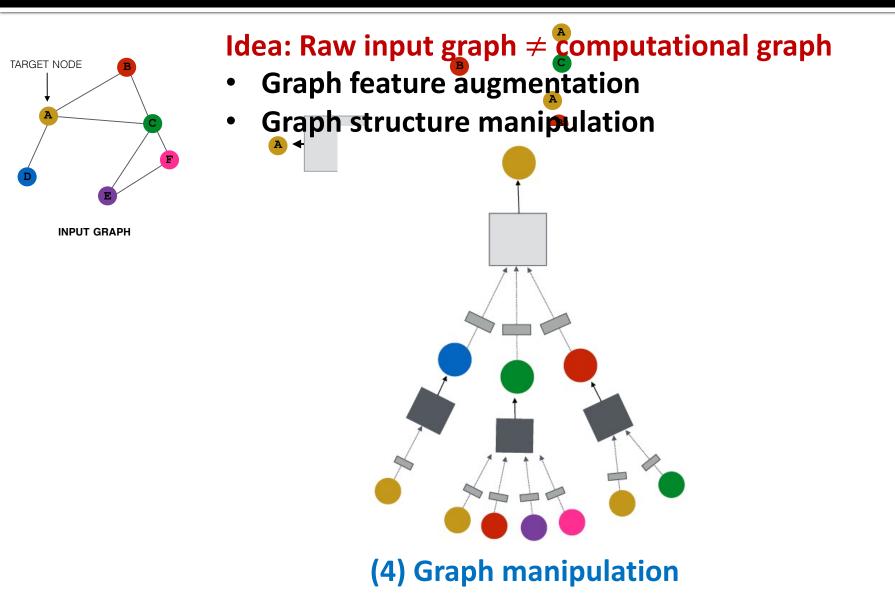
Graph Manipulation in GNNs

Jiaxuan You, Stanford University

Adapted from Stanford CS 224W & CS 246



General GNN Framework



Why Manipulate Graphs

Our assumption so far has been
Raw input graph = computational graph
Reasons for breaking this assumption

Feature level:

- The input graph lacks features → feature augmentation
- Structure level:
 - The graph is too sparse \rightarrow inefficient message passing
 - The graph is too dense \rightarrow message passing is too costly
 - The graph is too large → cannot fit the computational graph into a GPU
- It's just unlikely that the input graph happens to be the optimal computation graph for embeddings

Graph Manipulation Approaches

Graph Feature manipulation

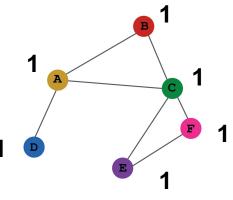
 The input graph lacks features -> feature augmentation

Graph Structure manipulation

- The graph is too dense -> Sample neighbors when doing message passing
- The graph is too large → Sample subgraphs to compute embeddings
 - Will cover later in lecture: Scaling up GNNs

Why do we need feature augmentation?

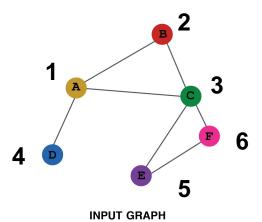
- (1) Input graph does not have node features
 - This is common when we only have the adj. matrix
- Standard approaches:
- a) Assign constant values to nodes



INPUT GRAPH

Why do we need feature augmentation?

- (1) Input graph does not have node features
 - This is common when we only have the adj. matrix
- Standard approaches:
- b) Assign unique IDs to nodes
 - These IDs are converted into one-hot vectors



One-hot vector for node with ID=5

$$[0, 0, 0, 0, 0, 1, 0]$$

Feature augmentation: constant vs. one-hot

	Constant node feature	One-hot node feature
Expressive power	Medium. All the nodes are identical, but GNN can still learn from the graph structure	High . Each node has a unique ID, so node-specific information can be stored
Inductive learning (Generalize to unseen nodes)	High . Simple to generalize to new nodes: we assign constant feature to them, then apply our GNN	Low. Cannot generalize to new nodes: new nodes introduce new IDs, GNN doesn't know how to embed unseen IDs
Computational cost	Low. Only 1 dimensional feature	High . $O(V)$ dimensional feature, cannot apply to large graphs
Use cases	Any graph, inductive settings (generalize to new nodes)	Small graph, transductive settings (no new nodes)

Why do we need feature augmentation?

- (2) Certain features can help GNN learning
- Other commonly used augmented features:
 - Node degree
 - PageRank

...

Clustering coefficient

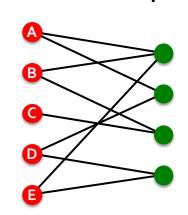
Any useful graph statistics can be used!

Add Virtual Nodes / Edges

- Motivation: Augment sparse graphs
- (1) Add virtual edges
 - Common approach: Connect 2-hop neighbors via virtual edges
 - Intuition: Instead of using adj. matrix A for GNN computation, use $A + A^2$



- Author-to-papers (they authored)
- 2-hop virtual edges make an author-author collaboration graph

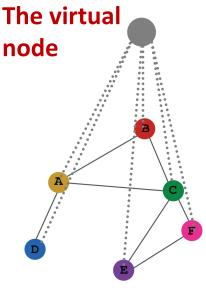


Papers

Authors

Add Virtual Nodes / Edges

- Motivation: Augment sparse graphs
 (2) Add virtual nodes
 - The virtual node will connect to all the nodes in the graph
 - Suppose in a sparse graph, two nodes have shortest path distance of 10
 - After adding the virtual node, all the nodes will have a distance of 2
 - Node A Virtual node Node B
 - Benefits: Greatly improves message passing in sparse graphs



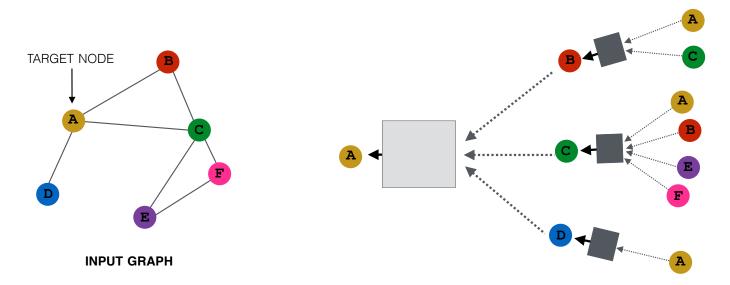
INPUT GRAPH

Hamilton et al. Inductive Representation Learning on Large Graphs, NeurIPS 2017

Node Neighborhood Sampling

Previously:

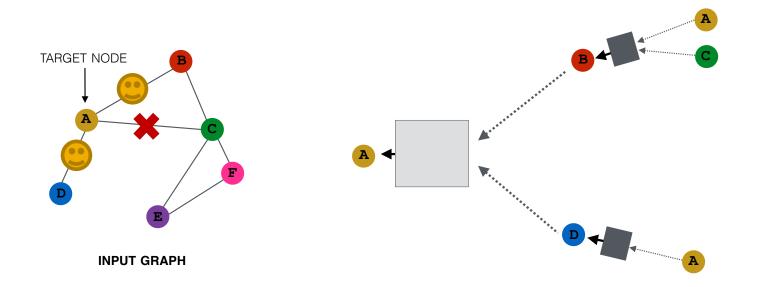
All the nodes are used for message passing



New idea: (Randomly) sample a node's neighborhood for message passing

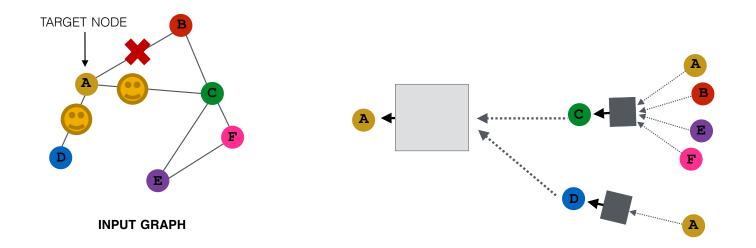
Neighborhood Sampling Example

- For example, we can randomly choose 2 neighbors to pass messages
 - Only nodes B and D will pass message to A



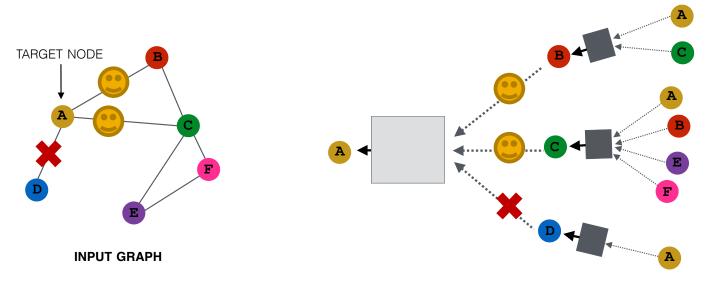
Neighborhood Sampling Example

- Next time when we compute the embeddings, we can sample different neighbors
 - Only nodes C and D will pass message to A



Neighborhood Sampling Example

- In expectation, we can get embeddings similar to the case where all the neighbors are used
 - Benefits: greatly reduce computational cost
 - And in practice it works great!



Summary of the Talk

Recap: A general perspective for GNNs

GNN Layer:

- Transformation + Aggregation
- Classic GNN layers: GCN, GraphSAGE, GAT

Layer connectivity:

- Deciding number of layers
- Skip connections

Graph Manipulation:

- Feature augmentation
- Structure manipulation

Resources: PyTorch Geometric + GraphGym

Layer connectivity Layer connectivity stage_dict = { 'stack': GNNStackStage, 'skipsum': GNNSkipStage,

'skipconcat': GNNSkipStage,

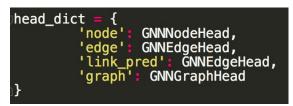
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GraphGym: Code Platform for GNN Design

Highly modularized pipeline for GNN research:

- Data loading, splitting
- GNN implementation
- Tasks: node/edge/graph
- Evaluation: accuracy, ROC AUC, .

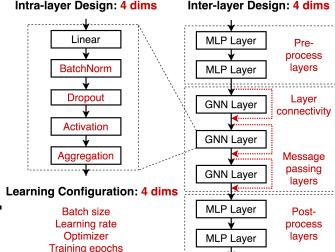
Prediction head for different tasks



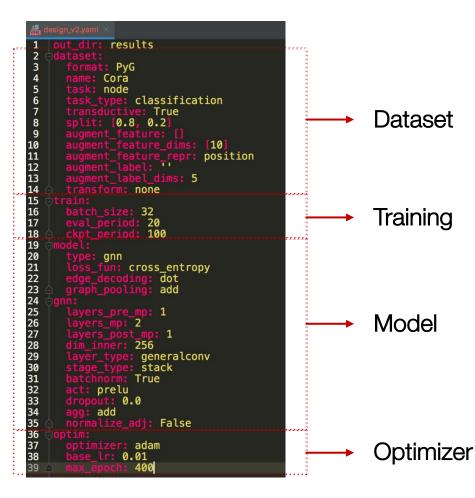
GNN layers

Jiaxuan You, Introduction to Graph Neural Networks

```
layer dict
     linear': Linear,
    'mlp': MLP,
               GCNConv,
    acnconv
                SAGEConv,
     sageconv
     gatconv
               GATConv,
        ineconv':
                  SplineConv,
     ginconv': GINConv.
                   GeneralConv,
     generalconv';
     generaledgeconv': GeneralE
    generalsampleedgeconv
```



GraphGym: Reproducible experiment management



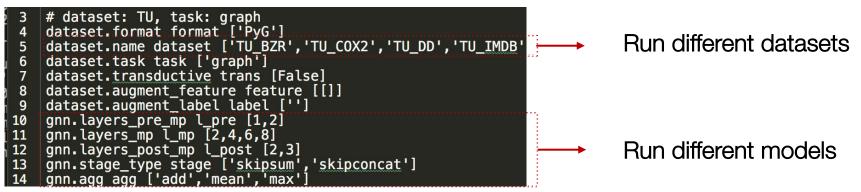
• One experiment: fully described by a configuration file

 Running an experiment is as simple as

python main.py --cfg design_v2.yaml --repeat 3

GraphGym: Scalable experiment management

A Grid of experimental settings



Launching thousands of GNNs in parallel



GraphGym: Scalable experiment management

Automatically generate experiment reports and figures

I_pre	l_mp	l_post	stage	agg	epoch	loss	loss_std	params	time_iter	time_iter_st	accuracy	accuracy_std
1	2		2 skipconcat	add	399	0.5678	0.0248	217256	0.1098	0.0075	0.886	0.0017
1	2		2 skipconcat	max	399	0.3754	0.0236	217256	0.0896	0.0026	0.9164	0.0017
1	2		2 skipconcat	mean	399	0.4885	0.0122	217256	0.0859	0.0046	0.9083	0.0011
1	2		2 skipsum	add	399	0.5624	0.022	295119	0.1121	0.0155	0.8853	0.0039
1	2		2 skipsum	max	399	0.3966	0.0054	295119	0.1049	0.003	0.9151	0.0025
1	2		2 skipsum	mean	399	0.4701	0.0118	295119	0.1027	0.0038	0.909	0.0028
1	2		3 skipconcat	add	399	0.5944	0.0231	199611	0.1138	0.0376	0.8844	0.0082
	-		2 11 1		200	0 4000	0.0000	100011	0.4.404	0.0500	0.0440	0.0000
Accuracy Ranking Accuracy Ranking Average	False Batch Nor	True	2- 1 0.0 0.3 3 - 2- 1 0.0 0.3 Drop	3- 2- 3-0.6	prelu relu prelu relu Activatic	2-		sum sum sum sum sum sum sum sum sum sum	4 6 4 6 sage passing h	8 3 2- 1 skipc	at skipsum star at skipsum star ver connectivit	sk
Ranking Averade	1			1- 3			0.001 0.01	0.1	adam sgo		200 40	0
Accuracy F Distribution				3-		2-	0.001 0.01		adam sg	3-2-		
	Pre-proce		Post-proce		Batch siz	ze	Learning rat		Optimizer	Tr	aining epochs	
8/24/21			liavuan	Vou Introduct	tion to Granh M	Joural Natwor	kc					66

Jiaxuan You, Introduction to Graph Neural Networks

Stanford Graph Learning Workshop

Stanford | ENGINEERING | Stanford Computer Forum

Stanford Data Science

- <u>https://snap.stanford.edu/graphlearning-workshop/</u>
 - Sept 16, 8am-5pm Pacific Time
 - Speakers: leaders from academia + industry
 - Will be live-streamed, free registration!
- New graph learning platform: Kumo
 - Pytorch Geometric + GraphGym + more!