Knowledge Based Obligations

RUC-ILLC Workshop on Deontic Logic

Eric Pacuit

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She was attacked only about 30m from her apartment’s door numerous times by a man (Winston Moseley) starting around 3:15 a.m.

The murder took place over a period of about thirty minutes, during which at least 38 alleged ”witnesses” failed to help the victim. (Note there is some dispute over what the witnesses actually saw and heard)
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**Issues:** obligations, group obligations, knowledge, group knowledge, default obligations, etc.
Why did the neighbors not act to help Kitty?

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An agent's obligations are often dependent on what the agent knows, and indeed one cannot reasonably be expected to respond to a problem if one is not aware of its existence.
Plan

✓ Introductory Example

2. Motivating Examples

3. Background: History Based Models

4. Knowledge Based Obligations.


5. Discussion
Some Related Work


(plus a lot of work on deontic logic and epistemic logic)
1. Uma is a physician whose neighbour is ill. Uma does not know and has not been informed. Uma has no obligation (as yet) to treat the neighbour.

2. Uma is a physician whose neighbour Sam is ill. The neighbour’s daughter Ann comes to Uma’s house and tells her. Now Uma does have an obligation to treat Sam, or perhaps call in an ambulance or a specialist.

3. Mary is a patient in St. Gibson’s hospital. Mary is having a heart attack. The caveat which applied in case 1. does not apply here. The hospital has an obligation to be aware of Mary’s condition at all times and to provide emergency treatment as appropriate.
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3. Mary is a patient in St. Gibson’s hospital. Mary is having a heart attack. The caveat which applied in case 1. does not apply here. The hospital has an obligation to be aware of Mary’s condition at all times and to provide emergency treatment as appropriate.
4. Uma has a patient with a certain condition C who is in the St. Gibson hospital mentioned above. There are two drugs $d$ and $d'$ which can be used for C, but $d$ has a better track record. Uma is about to inject the patient with $d$, but unknown to Uma, the patient is allergic to $d$ and for this patient $d'$ should be used. Nurse Rebecca is aware of the patient’s allergy and also that Uma is about to administer $d$. It is then Rebecca’s obligation to inform Uma and to suggest that drug $d'$ be used in this case.
Four Issues

1. The formal language and semantics must have machinery in which we can express statements of the form “after agent $i$ performs action $a$...”.

2. The formal language and semantics must have machinery in which we can express statements of the form “agent $i$ is obliged to perform action $a$”.

3. Certain actions are obligatory only in the presence of relevant information.

4. Certain obligations may disappear in the presence of relevant information: default obligations (example 4).
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Background: History Based Models


Background: History Based Models

The ‘Playground’

\[ t = 0 \]

\[ t = 1 \]

\[ t = 2 \]

\[ t = 3 \]
The ‘Playground’
Background: History Based Models

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The ‘Playground’: Notation
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- Given any set $X$, $X^*$ is the set of finite strings over $X$ and $X^\omega$ the set of infinite strings over $X$. Elements of $\Sigma^* \cup \Sigma^\omega$ will be called **histories**.
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- $\text{FinPre}(\mathcal{H}) = \{ H \mid \exists H' \in \mathcal{H} \text{ such that } H \preceq H' \}$ is the set of finite prefixes of the elements of $\mathcal{H}$.
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- $\epsilon$ is the empty string and $\text{FinPre}_{-\epsilon}(H) = \text{FinPre}(H) - \{\epsilon\}$. 
Definition
Let $\Sigma$ be any set of events. A set $\mathcal{H} \subseteq \Sigma^* \cup \Sigma^\omega$ is called a protocol provided $\text{FinPre}_{\epsilon}(\mathcal{H}) \subseteq \mathcal{H}$. 

Some assumptions:
1. If $\Sigma$ is assumed to be finite, then we say that $F$ is finitely branching.
2. If $\mathcal{H}$ is a rooted protocol, $F$ is a tree frame.
History-based Frames

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History-based Frames

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Definition
An ETL frame is a tuple $\langle \Sigma, \mathcal{H}, \{\sim_i\}_{i \in A} \rangle$ where $\Sigma$ is a (finite or infinite) set of events, $\mathcal{H}$ is a protocol, and for each $i \in A$, $\sim_i$ is an equivalence relation on the set of finite strings in $\mathcal{H}$. 
Background: History Based Models

History-based Frames

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Defining Uncertainty
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1. Histories or Runs?
   - Let $L$ be a set of local states.
   - A run $r \in \mathcal{R}$ is a function $r : \mathbb{N} \rightarrow L^{n+1}$
   - Agent $i$ cannot distinguish two points if it is in the same state in both: $(r, t) \sim_i (r', t')$ iff $r(t)_i = r'(t')_i$. 

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2. Local View Functions: Suppose that $\Sigma = \bigcup_{i \in \mathcal{A}} E_i$. For each $i \in \mathcal{A}$ define $\lambda_i : \text{FinPre}(\mathcal{H}) \rightarrow E_i^*$
   For each **finite** $H \in \mathcal{H}$,
   $$H \sim_i H' \text{ iff } \lambda_i(H) = \lambda_i(H')$$
Defining Uncertainty

Characterization Theorem(s) Each approach is “equivalent”.

Formal Languages

- $P\varphi$ (\(\varphi\) is true sometime in the past),
- $F\varphi$ (\(\varphi\) is true sometime in the future),
- $Y\varphi$ (\(\varphi\) is true at the previous moment),
- $N\varphi$ (\(\varphi\) is true at the next moment),
- $N_e\varphi$ (\(\varphi\) is true after event \(e\))
- $K_i\varphi$ (agent \(i\) knows \(\varphi\)) and
- $C_B\varphi$ (the group \(B \subseteq A\) commonly knows \(\varphi\)).
An ETL model is a structure $\langle H, \{\sim_i\}_{i \in A}, V \rangle$ where $\langle H, \{\sim_i\}_{i \in A} \rangle$ is an ETL frame and

$$V : At \rightarrow 2^{\text{finite}(H)}$$

is a valuation function.

Formulas are interpreted at pairs $H, t$:

$$H, t \models \varphi$$
Truth in a Model

- \( H, t \models P\varphi \) iff there exists \( t' \leq t \) such that \( H, t' \models \varphi \)
- \( H, t \models F\varphi \) iff there exists \( t' \geq t \) such that \( H, t' \models \varphi \)
- \( H, t \models N\varphi \) iff \( H, t + 1 \models \varphi \)
- \( H, t \models Y\varphi \) iff \( t > 1 \) and \( H, t - 1 \models \varphi \)
- \( H, t \models K_i\varphi \) iff for each \( H' \in \mathcal{H} \) and \( m \geq 0 \) if \( H_t \sim_i H'_m \) then \( H', m \models \varphi \)
- \( H, t \models C\varphi \) iff for each \( H' \in \mathcal{H} \) and \( m \geq 0 \) if \( H_t \sim_* H'_m \) then \( H', m \models \varphi \).

where \( \sim_* \) is the reflexive transitive closure of the union of the \( \sim_i \).
Background: History Based Models

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Background: History Based Models

Example 1 & 2
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\[ t = 0 \]

\[ t = 1 \]

\[ t = 2 \]

\[ t = 3 \]
Example 1 & 2

\[ \lambda_u(v) = \lambda_u(c) \]
Background: History Based Models

Example 1 & 2
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\[H, 1 \models \neg K_u S\]
Example 1 & 2

$t = 0$

$t = 1$

$t = 2$

$t = 3$

$\lambda_u(vm) = \lambda_u(cm)$
Learning from the Protocol
Background: History Based Models

Learning from the Protocol

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Background: History Based Models

Learning from the Protocol

\[ t = 0 \]

\[ t = 1 \]

\[ t = 2 \]

\[ t = 3 \]

\[ \neg KW \land \neg KR \]
Learning from the Protocol

$t = 0$

$t = 1$

$t = 2$

$t = 3$

$R \
W$

$R \n W$

$\neg R \n W$
Background: History Based Models

Learning from the Protocol

\( t = 0 \)

\( t = 1 \)

\( t = 2 \)

\( t = 3 \)

\( KW \land \neg KR \)
Background: History Based Models

Learning from the Protocol

$t = 0$

$t = 1$

$t = 2$

$t = 3$
Learning from the Protocol

\[ t = 0 \]

\[ t = 1 \]

\[ t = 2 \]

\[ !W \]

\[ t = 3 \]

\[ R \]

\[ \neg W \]
Background: History Based Models

Learning from the Protocol

\[ t = 0 \]

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Learning from the Protocol

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\[ KW \land \neg KR \]
\[KW \land KR\]
Parameters of the Models

1. Expressivity of the formal language. *Does the language include a common knowledge operator? A future operator? Both? Branching time or linear time?*
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3. Conditions on the reasoning abilities of the agents. Do the agents satisfy perfect recall? No miracles? Synchronization?
Background: History Based Models

Perfect Recall

t = 0

\[ e_2 \quad e_3 \quad e_4 \quad e_5 \]

t = 1

\[ e_1 \quad e_2 \quad e_3 \quad e_4 \quad e_5 \]

t = 2

\[ e_1 \quad e_3 \quad e_7 \quad e_6 \]

t = 3

\[ e_1 \quad e_3 \quad e_7 \quad e_6 \]
Perfect Recall

$t = 0$

$t = 1$

$t = 2$

$t = 3$

Background: History Based Models
Perfect Recall

$t = 0$

$t = 1$

$t = 2$

$t = 3$
Background: History Based Models

No Miracles

t = 0

\[ e_2 \quad e_4 \]

\[ e_1 \quad e_5 \quad e_2 \quad e_3 \]

\[ e_1 \quad e_3 \quad e_7 \quad e_6 \]

\[ e_1 \quad e_2 \quad e_7 \]

\[ e_1 \quad e_3 \]

\[ e_2 \quad e_4 \]

\[ e_2 \]
No Miracles

$t = 0$

$t = 1$

$t = 2$

$t = 3$
Theorem(s) Expressive language plus certain idealizations can lead to high complexity ($\Pi_1^1$-completeness).


See also,

Plan

✓ Introductory Example
✓ Motivating Examples
✓ Background: History Based Models

4. Knowledge Based Obligations.


5. Discussion
Actions

Assume a finite set, $\text{Act} \subseteq \Sigma$, of primitive actions.

Assume $\text{Act} = \bigcup_{i \in \mathcal{A}} C_i$ where $C_i \cap C_j \neq \emptyset$ for $i \neq j$. 
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Assume $\text{Act} = \bigcup_{i \in A} C_i$ where $C_i \cap C_j \neq \emptyset$ for $i \neq j$.

Given a finite global history $H$ and $a \in \text{Act}$,

$$a(H) = \{ H' \mid Ha \preceq H' \text{ and } H' \text{ a global history} \}$$
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\( H, t \models [a] \varphi \) iff \( H', t + 1 \models \varphi \) for each \( H' \in a(H) \)
Knowledge Based Obligations

Actions

\[ H, t \models [a]\varphi \text{ iff } H', t + 1 \models \varphi \text{ for each } H' \in a(H) \]
Knowledge Based Obligations

Actions

\[ H, t \models [a] \varphi \iff H', t+1 \models \varphi \text{ for each } H' \in a(H) \]

- When an action is performed, it is performed at the next moment of time.
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- When an action is performed, it is performed at the next moment of time.
- Only one agent can perform some action at any moment.
Actions

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- When an action is performed, it is performed at the next moment of time.
- Only one agent can perform some action at any moment.
- If no agents perform an action, then nature performs a ‘clock tick’.
Actions

\[ H, t \models [a] \varphi \iff H', t + 1 \models \varphi \text{ for each } H' \in a(H) \]

- When an action is performed, it is performed at the next moment of time.

- Only one agent can perform some action at any moment.

- If no agents perform an action, then nature performs a ‘clock tick’.

- Each agent knows \textit{when} it can perform an action.
  \[ (\langle a_i \rangle_T \rightarrow K_i \langle a_i \rangle_T) \]
Values: Informal Definition

All global histories will be presumed to have a value

Let \( \mathcal{G}(H) \) be the set of extensions of (finite history) \( H \) which have the highest possible value. (Assumptions are needed to make \( \mathcal{G}(H) \) well defined)
Values: Informal Definition

All global histories will be presumed to have a value

Let $\mathcal{G}(H)$ be the set of extensions of (finite history) $H$ which have the highest possible value. (Assumptions are needed to make $\mathcal{G}(H)$ well defined)

We will say that $a$ is good to be performed at $H$ if $\mathcal{G}(H) \subseteq a(H)$, i.e., there are no $H$-good histories which do not involve the performing of $a$. 
Values: Formal Definition

Let $\mathcal{H}$ be a set of global histories and $H \in \mathcal{H}$ a global history. For each $t \in \mathbb{N}$, let $\mathcal{F}(H_t) = \{ H' \in \mathcal{H} \mid H_t \preceq H' \}$.

A value function is a map $\text{val} : \inf(\mathcal{H}) \to \mathbb{R}$ such that

1. For all $t \in \mathbb{N}$, $\text{val}[\mathcal{F}(H_t)]$ is a closed and bounded subset of $\mathbb{R}$.
2. $\bigcap_{t \in \mathbb{N}} \text{val}[\mathcal{F}(H_t)] = \{ \text{val}(H) \}$

Define $\mathcal{G}(H_t) = \{ H' \mid H' \in \mathcal{F}(H_t) \text{ and } \text{val}(H') = \max(\text{val}[\mathcal{F}(H_t)]) \}$
Agent $i$ has a (knowledge based) obliged to perform action $a$ at global history $H$ and time $t$ iff $a$ is an action which $i$ (only) can perform, and $i$ knows that it is good to perform $a$.

For each $a \in \text{Act}$, let $G(a)$ be a formula:

$$H, t \models G(a) \text{ iff } G(H_t) \subseteq a(H_t)$$

Then we say that $i$ is obliged to perform action $a$ (at $H, t$) if $K_i(G(a))$ is true (at $H, t$).
Example 2

\[ \lambda_u(vm) = \lambda_u(cm) \]
Example 2

\[ t = 0 \]

\[ t = 1 \]

\[ t = 2 \]

\[ t = 3 \]

\[ v \]

\[ m \]

\[ c \]

\[ u \]

\[ m \]

\[ c \]

\[ \neg S \]

\[ S \]

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Example 2

\[ t = 0 \]

\[ t = 1 \]

\[ t = 2 \]

\[ t = 3 \]

\[ H, 2 \models K_u G(t) \]
Default Obligations (Sketch)

We introduce a modal operator $B_i$ which is intended to mean that “$i$ has a default belief that ….”

Assume a set of Grove spheres on the set $\mathcal{H}$: $\mathcal{S} = \{S_1, S_2, \ldots\}$ where for each $i \geq 1$, $S_i \subseteq S_{i+1} \subseteq \mathcal{H}$, and $\bigcup_{i=1}^{\infty} S_i = \mathcal{H}$.

Let $H$ be a finite global history, $\mathcal{D}(H)$ is the set of the most plausible histories which extend $H$.

$\mathcal{D}_i(h)$: the set of histories that $i$ considers plausible.
Default Obligations (Sketch)

\[ H, t \models B_i\phi \text{ iff for all } H', H'_t \in D_i(\lambda_i(H_t)), \ H', t \models \phi \]

\( K_i\phi \) semantically entails \( B_i\phi \).

\( B_i \) is a \textbf{KD45} operator.

\( B_iG(a) \): "agent \( i \) has a \textit{default knowledge based obligation} to perform \( a \)"
Knowledge Based Obligations

Our Language

- $G(a)$: “$a$ is a non-informational obligation”.
- $\langle a \rangle \top$: “action $a$ can be performed”.
- $K_i \langle a_i \rangle \top$: “agent $i$ knows that she can perform action $a_i$”.
- $K_i G(a_i)$: “$i$ has a (knowledge based) obligation to perform $a_i$”.
- $B_i \phi$: “agent $i$ (justifiably) believes $\phi$”.
- $B_i G(a_i)$: “agent $i$ has a default obligation to perform $a_i$”.

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Plan

✓ Introductory Example

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✓ Background: History Based Models

✓ Knowledge Based Obligations.


5. Discussion
Recall that Ann has the (knowledge based) obligation to tell Jill about her father’s illness \((K_a G(m))\).
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Clearly, Ann will not be under any obligation to tell Jill that her father is ill, if Ann justifiably believes that Jill would not treat her father even if she knew of his illness.
Recall that Ann has the (knowledge based) obligation to tell Jill about her father’s illness ($K_a G(m)$).

Clearly, Ann will not be under any obligation to tell Jill that her father is ill, if Ann justifiably believes that Jill would not treat her father even if she knew of his illness.

Thus, to carry out a deduction we will need to assume

$$K_j(K_u \text{sick } \leftrightarrow \Box \text{treat})$$
A similar assumption is needed to derive that Jill has an obligation to treat Sam.
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Obviously, if Jill has a good reason to believe that Ann always lies about her father being ill, then she is under no obligation to treat Sam.
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In other words, we need to assume

$$K_j(msg \leftrightarrow \text{sick})$$
Common Knowledge of Ethicality

These formulas can all be derived for one common assumption which we call *Common Knowledge of Ethicality*. 
Discussion

Common Knowledge of Ethicality

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1. The agents must (commonly) know the protocol.
2. The agents are all of the same “type” (social utility maximizers)
What is a Protocol?

Given the full tree $T$ of events, a **protocol** is *any* subtree of $T$. 
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- A protocol is the set of histories of an extensive game consistent with a strategy profile.
Defining a Protocol

1. What formal language should we use to define the protocol?
2. What models do we have in mind?

Given a formula $\phi$, two ways to think about defining a protocol:

- Set of histories: the set of histories $P$ in the full event tree $T$ such that $h \in P$ iff $h | = \phi$
- Set of models: the set $\text{Mod}(\phi)$ (the set of models of $\phi$)
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Knowledge of the Protocol
Discussion

Knowledge of the Protocol

\[ A \]

\[ E \]

\[ L \]

\[ L \]

\[ A \]

\[ E \]

\[ L \]

\[ R \]

\[ R \]

\[ \sim A \]
Two types of uncertainty?

Given two finite histories $h$ and $h'$,

$h \sim_i h'$ means given the events $i$ has observed, $h$ and $h'$ are indistinguishable
Two types of uncertainty?

Given two maximal histories $H$ and $H'$, 

*agent i may be uncertain which of the two will be the final outcome.*
Thank You!