

Logic, Interaction and Collective Agency

Lecture 5

ESSLLI'10, Copenhagen

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Correlations in UTI

Correlations arise from the **Team Authority** requirement:

$$\text{If } t_i = k \text{ then } \alpha_i = \alpha_i^k(s)$$

| $\lambda_E(I_E)(I_O)$ | M_O | H_O |
|-----------------------|-------|-------|
| •, M , MM | | |
| •, M , MH | | |
| ⋮ | | |
| •, H , MM | | |
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|-----------------------|-------|-------|-----------------------|-------|-------|
| \bullet, M, MM | | 0 | \bullet, M, MM | | |
| \bullet, M, MH | | 0 | \bullet, M, MH | | |
| \vdots | | | \vdots | | |
| \bullet, H, MM | 0 | | \bullet, H, MM | | |
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| \bullet, M, MH | | 0 | \bullet, M, MH | | |
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| \bullet, M, MH | | 0 | \bullet, M, MH | 0 | |
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|-----------------------|-------|-------|-----------------------|-------|-------|
| \bullet, M, MM | | 0 | \bullet, M, MM | | 0 |
| \bullet, M, MH | | 0 | \bullet, M, MH | 0 | |
| \vdots | | | \vdots | | |
| \bullet, H, MM | 0 | | \bullet, H, MM | | 0 |
| \bullet, H, MH | 0 | | \bullet, H, MH | 0 | |
| \vdots | | | \vdots | | |

Plan

1. Prior Beliefs and the Common Prior Assumption.
2. Correlations: strategies, beliefs, quantitatively, qualitatively.
3. General Discussion on Commitments and Intentions.
4. Wrap up.

Prior Beliefs and the Common Prior Assumption

Prior and posterior beliefs

- ▶ Different stages of information disclosure:

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 2. *Ex interim*: the players receive a **piece of (private) information**. They know more.

Prior and posterior beliefs

- ▶ Different stages of information disclosure:
 1. *Ex ante*: *A priori*. No information. Possibly background beliefs about who you are playing against (background beliefs about the “context of interaction”).
Prior beliefs.
 2. *Ex interim*: the players receive a piece of (private) information. They know more.
Posterior beliefs.

- ▶ We encountered priors many time already:
 - In type spaces, in UTIs, in Bayesian games...

► We encountered priors many time already:

- In **type spaces**, in UTIs, in Bayesian games...

$$\lambda_i(t_i)(s, t) = \frac{\Omega((s, t) \& t_i)}{\Omega(t_i)}$$

- ▶ We encountered priors many time already:
 - In type spaces, in UTIs, in Bayesian games...
 - We mostly used **common** priors:
 - ▶ $\Omega_i = \Omega_j$ for all $i, j \in \mathcal{A}$.
 - ▶ $\preceq_i = \preceq_j$ for all $i, j \in \mathcal{A}$ (in epistemic-plausibility models)

Common priors

- ▶ \nRightarrow same posteriors!

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Common priors

- ▶ \nrightarrow same posteriors! For the simple reason that agents can receive **different private information**.
- ▶ Olivier and I play card together.



Common priors

- ▶ \nrightarrow same posteriors! For the simple reason that agents can receive **different private information**.
- ▶ Olivier and I play card together. Before the cards are dealt, our common prior belief that the other end up with a Joker is $0.37 = 2/54$.
- ▶ We get 5 card each (and don't show them to each other). I end up with the 2 Jokers.
 - My posterior belief that Olivier has a Joker is 0.
 - Olivier's posterior belief that I have a Joker is $0.4 = 2/49$.



Common Prior Assumption

- ▶ Differences in posterior beliefs should be seen as coming from different information, not from different priors.

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“Harsanyi doctrine” [Aumann, 1976].

R. Aumann. *Agreeing to Disagree*. *Annals of Statistics*, Vol.4, No.6, 1976.

J.C. Harsanyi. *Games with incomplete informations played by bayesian players*. *Management Science* 14:159182, 320334, 486502, 1967-68.

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“Harsanyi doctrine” [Aumann, 1976].

- ▶ CPA not an innocuous assumption!

▶ Explanation

R. Aumann. *Agreeing to Disagree*. *Annals of Statistics*, Vol.4, No.6, 1976.

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Why CPA?

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- ▶ A controversial question!
 - See Morris (1995) for a thorough discussion.

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 - See Morris (1995) for a thorough discussion. One **methodological** observation:
 - ▶ Better to explain differences in posterior on the basis of **identifiable differences** in information or **plausible errors** in information processing.

Why CPA?

- ▶ A controversial question!
 - See Morris (1995) for a thorough discussion. One **methodological** observation:
 - ▶ Better to explain differences in posterior on the basis of **identifiable differences** in information or **plausible errors** in information processing.
 - ▶ Resorting on differences in priors often appears **ad hoc** (the resulting theory is “too permissive”).

S. Morris. *The Common Prior Assumption in Economic Theory*. *Economics and Philosophy*, 11(2): pgs. 227- 253, 1995.

Common Prior and Shared Perspective

- ▶ Common prior is already a form of **shared background** within a group, although a **relatively weak one**.
 - RCBR characterizes non-dominance; but RCBR + (non-correlated) CP characterizes Nash equilibrium.

Common Prior and Shared Perspective

- ▶ Common prior is already a form of **shared background** within a group, although a **relatively weak one**.
 - RCBR characterizes non-dominance; but RCBR + (non-correlated) CP characterizes Nash equilibrium.
- ▶ But the connection between group members can be strengthened if the priors are **correlated**.

Correlations

Correlated Strategies

| | | |
|---|------|------|
| | H | M |
| H | 3, 3 | 0, 4 |
| M | 4, 0 | 1, 1 |

Correlated Strategies

| | | |
|---|------|------|
| | H | M |
| H | 3, 3 | 0, 4 |
| M | 4, 0 | 1, 1 |

We each can make our choice dependent on a flip of the coin:

Correlated Strategies

| | | |
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| | H | M |
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We each can make our choice dependent on a flip of the coin:

- ▶ A mixed strategy is a random variable assigning actions to the outcome of a coin toss.
- ▶ The coins are *independent*: (H, H) is never rational.

Correlated Strategies

| | | |
|---|------|------|
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Suppose we base our choices on the *same coin*:

Correlated Strategies

| | | |
|---|------|------|
| | H | M |
| H | 3, 3 | 0, 4 |
| M | 4, 0 | 1, 1 |

Suppose we base our choices on the *same coin*:

- ▶ $\rho_E = (\text{Head} \Rightarrow H, \text{Tail} \Rightarrow M)$
- ▶ $\rho_O = (\text{Head} \Rightarrow H, \text{Tail} \Rightarrow M)$

Our choice are **correlated**, i.e. **not independent**

(In general, the players need not observe the *same* random event.)

Correlated Strategies

| | H | M |
|---|------|------|
| H | 3, 3 | 0, 4 |
| M | 4, 0 | 1, 1 |

| | H | M |
|---|-------|-------|
| H | $1/2$ | 0 |
| M | 0 | $1/2$ |

Convenient representation of correlated strategies in terms of the distribution of the external signal.

Correlated Equilibrium

Let \mathbb{G} be a game in strategic form and W a (measurable) set of **outside signals** and $\Omega(S)$ a probability distribution over S .

- ▶ A **correlated strategy** for player i is a function $\rho_i : W \rightarrow S_i$ assigning to each signal an action in S_i .
- ▶ The **expected value** for player i of the profile ρ :

$$EV_i(\rho) = \sum_{w \in W} \Omega(w) v_i(\rho(w))$$

- ▶ A **correlated equilibrium** is a profile ρ_i of correlated strategies such that, for all i :

$$\rho_i \in \operatorname{argmax}_{\rho'_i} (\rho'_i, \rho_{-i})$$

R.J. Aumann. *Correlated equilibrium as an expression of bayesian rationality*. *Econometrica*, 55(1-18), 1987.

Nash equilibrium \subsetneq Correlated Equilibrium

| | H | M |
|---|------|------|
| H | 3, 3 | 0, 0 |
| M | 0, 0 | 1, 1 |

Nash equilibrium \subsetneq Correlated Equilibrium

▶

| | H | M |
|---|------|------|
| H | 6, 6 | 2, 7 |
| M | 7, 2 | 0, 0 |

Nash equilibrium \subsetneq Correlated Equilibrium

▶

| | H | M |
|---|------|------|
| H | 6, 6 | 2, 7 |
| M | 7, 2 | 0, 0 |

- ▶ Three Nash equilibria: (M, H), (H, M) and $((2/3 M, 1/3 H), (2/3 M, 1/3 H))$.

Nash equilibrium \subsetneq Correlated Equilibrium

▶

| | H | M |
|---|------|------|
| H | 6, 6 | 2, 7 |
| M | 7, 2 | 0, 0 |

| | H | M |
|---|------------|----------|
| H | Blue Card | Red card |
| M | Green Card | |

- ▶ Now suppose we make our choice dependent on drawing one of three cards.

Nash equilibrium \subsetneq Correlated Equilibrium

▶

| | H | M | | H | M |
|---|------|------|---|-----|-----|
| H | 6, 6 | 2, 7 | H | 1/3 | 1/3 |
| M | 7, 2 | 0, 0 | M | 1/3 | |

- ▶ Now suppose we make our choice dependent on drawing one of three cards.

Nash equilibrium \subsetneq Correlated Equilibrium

▶

| | H | M | | H | M |
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- ▶ Now suppose we make our choice dependent on drawing one of three cards.
- ▶ This is a **correlated equilibrium**, with **expected value of 5** for each of us.

Nash equilibrium \subsetneq Correlated Equilibrium

▶

| | H | M | | H | M |
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- ▶ Now suppose we make our choice dependent on drawing one of three cards.
- ▶ This is a **correlated equilibrium**, with **expected value of 5** for each of us.
- ▶ No Nash equilibrium can give you that.

Nash equilibrium \subsetneq Correlated Equilibrium

| | | | | | |
|---|------|------|---|-----|-----|
| | H | M | | H | M |
| H | 6, 6 | 2, 7 | H | 1/3 | 1/3 |
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- ▶ Now suppose we make our choice dependent on drawing one of three cards.
- ▶ This is a **correlated equilibrium**, with **expected value of 5** for each of us.
- ▶ **No mixing** can give you the probability distribution ρ on $\Pi_i S_i$.

Nash equilibrium \subsetneq Correlated Equilibrium

| | | | | | |
|---|------|------|---|-----|-----|
| | H | M | | H | M |
| H | 6, 6 | 2, 7 | H | 1/3 | 1/3 |
| M | 7, 2 | 0, 0 | M | 1/3 | |

- ▶ Now suppose we make our choice dependent on drawing one of three cards.
- ▶ This is a **correlated equilibrium**, with **expected value of 5** for each of us.
- ▶ The probability distribution ρ on $\prod_i S_i$ **cannot be factorized** in independent probability distributions on S_i 's.

Correlations in Belief

- ▶ Small step from correlated strategies to correlated priors.

| | H | M |
|---|------|------|
| H | 3, 3 | 0, 0 |
| M | 0, 0 | 1, 1 |

Correlations in Belief

- ▶ Small step from correlated strategies to correlated priors.

| | | | | | |
|---|------|------|----------|-----|-----|
| | H | M | Ω | M | H |
| H | 3, 3 | 0, 0 | H | 1/2 | 0 |
| M | 0, 0 | 1, 1 | M | 0 | 1/2 |

- $\mathcal{T} = \{t_O, t_E\}$.

Correlations in Belief

- ▶ Small step from correlated strategies to correlated priors.

| | | | | | |
|---|------|------|----------|-----|-----|
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| H | 3, 3 | 0, 0 | H | 1/2 | 0 |
| M | 0, 0 | 1, 1 | M | 0 | 1/2 |

- $\mathcal{T} = \{t_O, t_E\}$.
- This example hides some conceptual complications...

▶ Explanation

Beliefs about correlations

| X | L | R | | Y | L | R |
|----------|---------|---------|----------|----------|---------|---------|
| U | 1, 1, 3 | 1, 0, 3 | | U | 1, 1, 2 | 0, 0, 0 |
| D | 0, 1, 0 | 0, 0, 0 | | D | 0, 0, 0 | 1, 1, 2 |
| | | | Z | | L | R |
| | | | U | | 1, 1, 0 | 1, 0, 0 |
| | | | D | | 0, 1, 3 | 0, 0, 3 |

- ▶ Three players: Ann, Bob and Charlie.

A. Brandenburger and A. Friedenberg. *Intrinsic Correlations in Games*. J.E.T., vol. 141, 2008.

Beliefs about correlations

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|----------|---------|---------|
| U | 1, 1, 2 | 0, 0, 0 |
| D | 0, 0, 0 | 1, 1, 2 |

| Z | L | R |
|----------|---------|---------|
| U | 1, 1, 0 | 1, 0, 0 |
| D | 0, 1, 3 | 0, 0, 3 |

- ▶ Ann chooses the **column**.

Beliefs about correlations

| X | L | R |
|----------|---------|---------|
| U | 1, 1, 3 | 1, 0, 3 |
| D | 0, 1, 0 | 0, 0, 0 |

| Y | L | R |
|----------|---------|---------|
| U | 1, 1, 2 | 0, 0, 0 |
| D | 0, 0, 0 | 1, 1, 2 |

| Z | L | R |
|----------|---------|---------|
| U | 1, 1, 0 | 1, 0, 0 |
| D | 0, 1, 3 | 0, 0, 3 |

- ▶ Ann chooses the **column**.

Beliefs about correlations

| X | L | R |
|----------|----------|----------|
| U | 1, 1, 3 | 1, 0, 3 |
| D | 0, 1, 0 | 0, 0, 0 |

| Y | L | R |
|----------|----------|----------|
| U | 1, 1, 2 | 0, 0, 0 |
| D | 0, 0, 0 | 1, 1, 2 |

| Z | L | R |
|----------|----------|----------|
| U | 1, 1, 0 | 1, 0, 0 |
| D | 0, 1, 3 | 0, 0, 3 |

- **Bob** chooses the **row**.

Beliefs about correlations

| | | | | | |
|----------|----------|---------|----------|---------|---------|
| X | L | R | Y | L | R |
| U | 1, 1, 3 | 1, 0, 3 | U | 1, 1, 2 | 0, 0, 0 |
| D | 0, 1, 0 | 0, 0, 0 | D | 0, 0, 0 | 1, 1, 2 |
| | Z | L | R | | |
| | U | 1, 1, 0 | 1, 0, 0 | | |
| | D | 0, 1, 3 | 0, 0, 3 | | |

- ▶ **Bob** chooses the **row**.

Beliefs about correlations

| X | L | R |
|----------|---------|---------|
| U | 1, 1, 3 | 1, 0, 3 |
| D | 0, 1, 0 | 0, 0, 0 |

| Y | L | R |
|----------|---------|---------|
| U | 1, 1, 2 | 0, 0, 0 |
| D | 0, 0, 0 | 1, 1, 2 |

| Z | L | R |
|----------|---------|---------|
| U | 1, 1, 0 | 1, 0, 0 |
| D | 0, 1, 3 | 0, 0, 3 |

- ▶ Charlie chooses the **matrix**.

Beliefs about correlations

| X | L | R |
|----------|---------|---------|
| U | 1, 1, 3 | 1, 0, 3 |
| D | 0, 1, 0 | 0, 0, 0 |

| Y | L | R |
|----------|---------|---------|
| U | 1, 1, 2 | 0, 0, 0 |
| D | 0, 0, 0 | 1, 1, 2 |

| Z | L | R |
|----------|---------|---------|
| U | 1, 1, 0 | 1, 0, 0 |
| D | 0, 1, 3 | 0, 0, 3 |

- ▶ Charlie chooses the **matrix**.

Beliefs about correlations

| X | L | R |
|----------|---------|---------|
| U | 1, 1, 3 | 1, 0, 3 |
| D | 0, 1, 0 | 0, 0, 0 |

| Y | L | R |
|----------|---------|---------|
| U | 1, 1, 2 | 0, 0, 0 |
| D | 0, 0, 0 | 1, 1, 2 |

| Z | L | R |
|----------|---------|---------|
| U | 1, 1, 0 | 1, 0, 0 |
| D | 0, 1, 3 | 0, 0, 3 |

- ▶ Charlie chooses the **matrix**.

Beliefs about correlations

| X | L | R |
|----------|---------|---------|
| U | 1, 1, 3 | 1, 0, 3 |
| D | 0, 1, 0 | 0, 0, 0 |

| Y | L | R |
|----------|---------|---------|
| U | 1, 1, 2 | 0, 0, 0 |
| D | 0, 0, 0 | 1, 1, 2 |

| Z | L | R |
|----------|---------|---------|
| U | 1, 1, 0 | 1, 0, 0 |
| D | 0, 1, 3 | 0, 0, 3 |

- Strategy **Y** is **rationalizable** for Charlie.

Beliefs about correlations

| X | L | R |
|----------|---------|---------|
| U | 1, 1, 3 | 1, 0, 3 |
| D | 0, 1, 0 | 0, 0, 0 |

| Y | L | R |
|----------|---------|---------|
| U | 1, 1, 2 | 0, 0, 0 |
| D | 0, 0, 0 | 1, 1, 2 |

| Z | L | R |
|----------|---------|---------|
| U | 1, 1, 0 | 1, 0, 0 |
| D | 0, 1, 3 | 0, 0, 3 |

- ▶ Strategy **Y** is **rationalizable** for Charlie.
- ▶ Take: $\lambda(t_C)(U, L) = 1/2$, $\lambda(t_C)(D, R) = 1/2$.

Beliefs about correlations

| X | L | R | | Y | L | R |
|----------|---------|---------|----------|----------|---------|---------|
| U | 1, 1, 3 | 1, 0, 3 | | U | 1, 1, 2 | 0, 0, 0 |
| D | 0, 1, 0 | 0, 0, 0 | | D | 0, 0, 0 | 1, 1, 2 |
| | | | Z | L | R | |
| | | | U | 1, 1, 0 | 1, 0, 0 | |
| | | | D | 0, 1, 3 | 0, 0, 3 | |

- ▶ Strategy **Y** is **rationalizable** for Charlie.
- ▶ Take: $\lambda(t_C)(U, L) = 1/2$, $\lambda(t_C)(D, R) = 1/2$. Then:

$$EV_C(X, t_C) = 1/2(3) + 1/2(0) = 1.5$$

Beliefs about correlations

| X | L | R | | Y | L | R |
|----------|---------|---------|----------|----------|---------|---------|
| U | 1, 1, 3 | 1, 0, 3 | | U | 1, 1, 2 | 0, 0, 0 |
| D | 0, 1, 0 | 0, 0, 0 | | D | 0, 0, 0 | 1, 1, 2 |
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Beliefs about correlations

| X | L | R | Y | L | R |
|----------|---------|---------|----------|---------|---------|
| U | 1, 1, 3 | 1, 0, 3 | U | 1, 1, 2 | 0, 0, 0 |
| D | 0, 1, 0 | 0, 0, 0 | D | 0, 0, 0 | 1, 1, 2 |

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| U | 1, 1, 3 | 1, 0, 3 | | U | 1, 1, 2 | 0, 0, 0 |
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- ▶ Strategy **Y** is **rationalizable** for Charlie.
- ▶ Take: $\lambda(t_C)(U, L) = 1/2$, $\lambda(t_C)(D, R) = 1/2$.
- ▶ **Y** is not rationalizable if Charlie doesn't believe that Ann and Bob correlate their choices. (Left as an exercise).

The Importance of Correlations

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 - Team Interactions generate correlations.
- ▶ Question: where do (beliefs in) correlations come from?
Answer: A player can think that other players' strategy choices are correlated, because he thinks what they believe about the game is correlated.

Correlations through types

| X | L | R |
|----------|---------|---------|
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- ▶ Instead: make strategy choices conditional on types/beliefs/hierarchies.

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- Instead: make strategy choices conditional on types/beliefs/hierarchies.

- Suppose $T_A = \{t_A, u_A\}$ and $T_B = \{t_B, u_B\}$ and:

$$\lambda(t_C)(t_A, t_B, U, L) = 1/2 = \lambda(t_C)(u_A, u_B, D, R)$$

Correlations through types

| t_C | UL | UR | DL | DR |
|-----------|-----|----|----|-----|
| $t_A t_B$ | 1/2 | 0 | 0 | 0 |
| $t_A u_B$ | 0 | 0 | 0 | 0 |
| $u_A t_B$ | 0 | 0 | 0 | 0 |
| $u_A u_B$ | 0 | 0 | 0 | 1/2 |

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- ▶ We get the same result: Charlie is rational at state

$$(t_C, \bullet), (s_A, s_B, Y)$$

Correlations through types

- ▶ Strategy choices conditional on signals.
 - $\rho_A = \{Head \Rightarrow U, Tail \Rightarrow D\}$, $\rho_B = \{Head \Rightarrow L, Tail \Rightarrow R\}$.
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- ▶ How much explanatory power?

Correlations through types

- ▶ **CI**: For any type t_i for agent i , the choices (s_j, s_k, \dots) for the other agents ($i \neq j, \neq k, \dots$) are independent (i.e. non-correlated), conditional on the type of the other players.

A. Brandenburger and A. Friedenberg. *Intrinsic Correlations in Games*. J.E.T., vol. 141, 2008.

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A. Brandenburger and A. Friedenberg. *Intrinsic Correlations in Games*. J.E.T., vol. 141, 2008.

Two routes to explain correlations

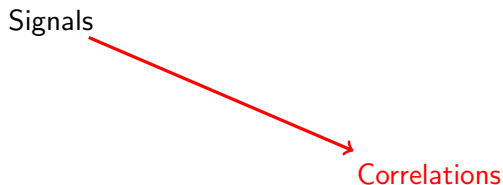
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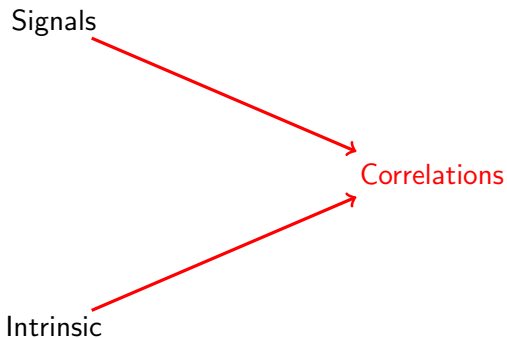
R.J. Aumann. *Correlated equilibrium as an expression of bayesian rationality.* *Econometrica*, 55(1-18), 1987.

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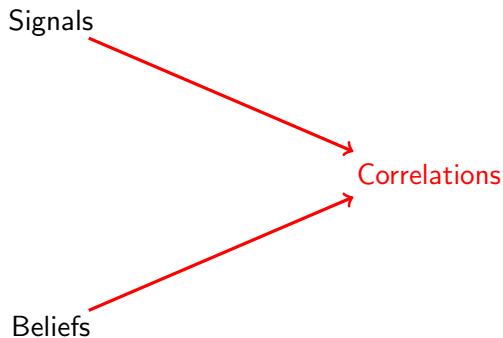
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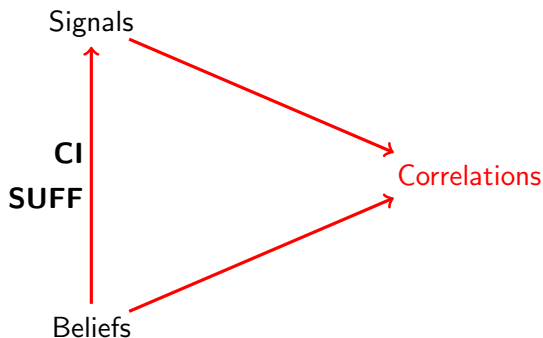
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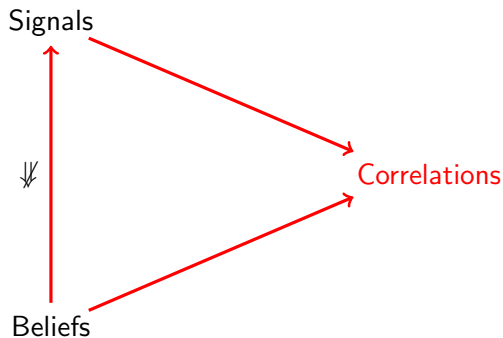
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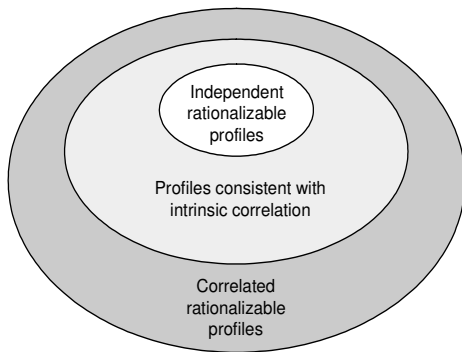


Figure 2.2

A. Brandenburger and A. Friedenberg. *Intrinsic Correlations in Games*. J.E.T., vol. 141, 2008.

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The kind of correlations that arise in *UTI* are akin to correlations induced by types. **Questions:**

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More general questions:

- What does it means to say that a team is rational at a given state?
- And what about **correlations in non-probabilistic models of beliefs?**

Additional requirements on team work.

What is a team?

1. Group identification.
 - Information about who's in and who's out.
 - Reasoning as group members.
 - Shared goal.
 - ▶ Group preference / utilities.
2. Shared commitments.
 - Shared intentions.
 - Sanctions for lapsing?
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Motivational attitudes which:

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M. Bratman. *Intention, Plans, Practical Reason*. Harvard UP, 1987.

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Commitments and Intentions

Key Philosophical Work:

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Intentions and Teamwork:

M. Gilbert. *On Social Facts*. Princeton UP, 1989.

J. Searle. *The Construction of Social Reality*. Free Press, 1995.

M. Bratman. *Faces of Intentions*. Cambridge UP, 1999.

R. Tuomela. *The Philosophy of Sociality*. Oxford UP, 2010.

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Formally - but mostly single-agent:

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P.R. Cohen and HJ Levesque. *Intention is Choice with Commitment*. *Artificial Intelligence*, 42(3), 1990.

F Gul, W Pesendorfer. *Temptation and Self-Control*. *Econometrica*, 2001.

⇒ See also (T. Icard, EP and Y. Shoham 2010).

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Formally - Multi-agent:

B. Dunin-Keplicz and R. Verbrugge. *Teamwork in Multi-Agent Systems*. Wiley, 2010.

⇒ See also (Lorini and Herzig:

<http://www.irit.fr/~Andreas.Herzig/Esslli09course/>)

Teamwork as **Shared cooperative activity**:



Teamwork as Shared cooperative activity:

A The Intention part:

1. Me:

1.1 I intend that we J.

1.2 I intend that we J in accordance with and because of meshing subplans of (1.1) and (2.1).

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3. Additional requirements:

3.1 The intentions in (1) and in (2) are not coerced by the other participant.

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B: The epistemic part:

1. It is common knowledge between us that (A).

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Conclusions: Teams in Interaction

1. Group identification.
 - Information about who's in and who's out.
 - Reasoning as group members.
 - Shared goal.
 - ▶ Group preference / utilities.
2. Shared commitments.
 - Shared intentions.
 - Sanctions for lapsing?
 - Shared praise[blame] for success[failure]?
3. Common knowledge (beliefs?) of the above?

Thanks and Merci (and Bedankt)

CPA not an innocuous assumption!

- ▶ Agreement Theorem:

*If two people have the **common priors**, and their posteriors for an event A are **common knowledge**, then these posterior have to be **the same**.*

CPA not an innocuous assumption!

- ▶ Agreement Theorem:

The Annals of Statistics
1976, Vol. 4, No. 6, 1236-1239

AGREEING TO DISAGREE¹

BY ROBERT J. AUMANN

Stanford University and the Hebrew University of Jerusalem

Two people, 1 and 2, are said to have *common knowledge* of an event E if both know it, 1 knows that 2 knows it, 2 knows that 1 knows it, 1 knows that 2 knows that 1 knows it, and so on.

THEOREM. *If two people have the same priors, and their posteriors for an event A are common knowledge, then these posteriors are equal.*

If two people have the same priors, and their posteriors for a given event A are common knowledge, then these posteriors must be equal. This is so even though they may base their posteriors on quite different information. In brief, people with the same priors *cannot agree to disagree*.

We publish this observation with some diffidence, since once one has the appropriate framework, it is mathematically trivial. Intuitively, though, it is not

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▶ [Back to Lecture](#)

An digression on CI and SUFF

Suppose a being in whose power to predict your choices you have enormous confidence. [...]

R. Nozick. *Newcomb's Problem and Two Principles of Choice*. 1969.

An digression on CI and SUFF

Suppose a being in whose power to predict your choices you have enormous confidence. [...] There are two boxes, (*A*) and (*B*). (*B*1) contains 1000 [euros]. (*B*2) contains either 1 million [euros] (*M*), or nothing.

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1. taking what is in both boxes
2. taking only what is in the second box.

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Furthermore, [the following is common knowledge]:

- ▶ If the being predicts you will take what is in both boxes, he does not put the 1 M in the second box.
- ▶ If the being predicts you will take only what is in the second box, he does put the 1M in the second box.

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The situation is as follows. First the being makes its prediction. Then it puts the 1M in the second box, or does not, depending upon what it has predicted. Then you make your choice. What do you do?

An digression on CI and SUFF

| | $A = 1M$ | $A = 0$ |
|---------|------------|---------|
| 1 Box | 1M | 0 |
| 2 Boxes | $1M + 100$ | 100 |



An digression on CI and SUFF

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- ▶ No uncertainty on types here.

▶ Back to Lecture

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- ▶ No uncertainty on types here.
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▶ Back to Lecture

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- ▶ No uncertainty on types here.
- ▶ SUFF is trivially satisfied (always for two agents).
- ▶ CI is violated.

▶ Back to Lecture