Logic, Interaction and Collective Agency

Lecture 3

ESSLLI'10, Copenhagen

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Plan for Today

1. Intro: group/team preferences, frames and identification.
2. Unreliable Team Interaction (I).
3. A short overview of Variable Frame Theory.
4. Unreliable Team Interaction (II).

Team preferences and team reasoning.

The Main Question(s)

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Hard Work	3, 3	0, 0
Minimal Work	0, 0	1, 1

- When there is scope for cooperation, what does it mean to say that the agents are rational?
  - ✓ Analytical question.

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- As such the question is under-specified.
  - One needs to specify the context of interaction (or of the game). This includes:
    - Information of the agents about all relevant aspects of interaction.
    - Additional group- or team-related aspects of the game.

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What is a team?

1. Group identification.
    - Information about who's in and who's out.
    - Reasoning as group members.
    - Shared goal.
        - Group preference / utilities.
2. Shared commitments.
    - Shared intentions.
    - Sanctions for lapsing?
    - Shared praise[blame] for success[failure]?
3. Common knowledge (beliefs?) of the above?

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Group- or team- preferences.

Groups or teams may have their own objectives/goals/"preferences":

C. List and P. Pettit. *Group Agency*. Forthcoming..

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Group preferences *vs* shared preferences.

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- Preferable for the team {Eric, Olivier} (?).

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  - "We're in the same boat."
  - Part of what it means to be a team member seems to adopt the team objectives, or at least to reason on the basis of them.

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N. Gold. *Teamwork*. Palgrave MacMillan, 2005.

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  - Primitives?
  - One recurring requirement: Paretian in the member's preferences. I.e. If a profile is Pareto-optimal then it is also most preferred for the team.

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Claim: Acting as a group member is different than:

- Individual action based on individual preferences.
- Individual action based on group preference.

(Unreliable) Team Interaction and Team Reasoning.
Bacharach (1999, 2006), Sugden (200X)

Step 1: Adding teams, conservatively

Definition

A game in strategic form TI is a tuple $\langle \mathcal{A}, , S_i, v_i \rangle$ such that :

- $\mathcal{A}$ is a finite set of agents.
- S_i is a finite set of *actions* or *strategies* for i.
- $v_i : \Pi_{i \in \mathcal{A}} S_i \longrightarrow \mathbb{R}$ is an *utility function* that assigns to every strategy profile $\sigma \in \Pi_{i \in \mathcal{A}} S_i$ the utility of that profile for agent i.

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1. Team interactions are generalization of games in strategic form:
    - Given a set of agent $\mathcal{A} = \{1, 2, ..., i\}$, the team interaction such that $M = \{\{1\}, \{2\}, ..., \{i\}\}$ is a game in strategic form.
2. Only individuals take action, but sometime they act for a team. How:
    - For any team $k \in M$, call $\alpha^k \in \Pi_{i \in k} S_i$ a protocol for k, and write α for a protocol for all team $k \in M$. $\mathfrak{P} = \Pi_{k \in M} \Pi_{i \in K} S_i$ is the set of all protocols.

Step 2: Types and Uncertainty

Definition

A type space for a team interaction TI is a tuple:

$$\mathcal{T} = \langle S, \{T_i\}_{\in \mathcal{A}}, \Omega \rangle$$

- $T_i = \{k \in M : i \in k\}$ is a set of types for player i.
- S a set of signal, the uncertainty domain.

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- Ω is a probability distribution on the set of states.
  - A Common Prior.

Terminology and Remarks:

- A state is a tuple:

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At each state, each agent belong to one and only one team.

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- An protocol for team k in an UTI is an function which gives, for each member i of k and each signal and action in A_i.
- Conditioning gives the functions $\lambda_i : T_i \rightarrow \Delta(S \times T_{-i})$:

$$\lambda_i(t_i)(s, t) = \frac{\Omega((s, t) \cap t_i)}{\Omega(t_i)}$$

Ex Ante Expected Value and Equilibrium

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- Given an type space $\mathcal{T}$ with Team Authority, for a team interaction TI, the *ex ante* expected value of protocol α for team k:

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- The protocol α is a *ex ante* UTI-equilibrium iff, for all $k \in M$,

$$\alpha \in argmax_{\beta \in \mathfrak{P}}(EV^k(\beta^k, \alpha^{-k}))$$

UTI, an example

Teams and utilities:

UTI, an example

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- Teams (M). Either:
  - we decide alone: $I_O = \{Olivier\}, I_E = \{Eric\}$;
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  - we decide alone: $I_O = \{Olivier\}, I_E = \{Eric\}$;
  - or as a team $C = \{Olivier, Eric\}$.
- Utilities for the (non-singleton) team is the average individual payoffs.

UTI, an example

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Ω	I_E	C
I_O	$4/9$	$2/9$
C	$2/9$	$1/9$

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The protocol $\alpha = (M, M, HH)$ is a *ex-ante* equilibrium.

$$EV^C(\alpha) = \Omega(I_E, I_E)v^C(\alpha^{t_{Olivier}}_{Olivier}, \alpha^{t_{Eric}}_{Eric}) + \Omega(C, I_E)v^C(\alpha^{t_{Olivier}}_{Olivier}, \alpha^{t_{Eric}}_{Eric}) + \Omega(I_0, C)v^C(\alpha^{t_{Olivier}}_{Olivier}, \alpha^{t_{Eric}}_{Eric}) + \Omega(C, C)v^C(\alpha^{t_{Olivier}}_{Olivier}, \alpha^{t_{Eric}}_{Eric})$$

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The protocol $\alpha = (M, M, HH)$ is a *ex-ante* equilibrium.

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EV^C(\alpha) = \Omega(I_E, I_E)v^C(M, \alpha_{Eric}^{t_{Eric}}) + \\
\Omega(C, I_E)v^C(\alpha_{Olivier}^{t_{Olivier}}, \alpha_{Eric}^{t_{Eric}}) + \\
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- Team Authority:

$$\text{If } t_i = k \text{ then } \alpha_i = \alpha_i^k(s)$$

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$$EV^C(\alpha) = \Omega(I_E, I_E)1 + \Omega(C, I_E)2 + \Omega(I_0, C)2 + \Omega(C, C)3$$

UTI, an example

The protocol $\alpha = (M, M, HH)$ is a *ex-ante* equilibrium.

$$EV^C(\alpha) = (4/9)1 +$$

$$(2/9)2 +$$

$$(2/9)2 +$$

$$(1/9)3$$

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The protocol $\alpha = (M, M, HH)$ is a *ex-ante* equilibrium.

$$EV^C(\alpha) = 1.66$$
$$EV^C(L, L, ML) = 1.33$$
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The protocol $\alpha = (M, M, HH)$ is a *ex-ante* equilibrium.

- MM maximizes EV^C given L, L.
- For either Olivier or Eric, L is the only EV-maximizer.
  - A strategy S_i of an individual is strictly dominated in a game $\mathbb{G}$ iff it is strictly dominated in an TI extending $\mathbb{G}$ such that $\{i\} \in M$.
    $\Rightarrow$ A consequence of Team Authority.

Frames and Variable Frame Theory

A short digression.

Being part of a given team	$\approx$	seeing the interactive situation through a specific frame.

Framing effect

Logicophilia, a virulent virus, threatens 600 participants of ESSLLI'10.

[Adapted from Tversky and Kahneman (1981)]

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1. You must choose between two prevention programs, resulting in:
   - A: 200 participants will be saved for sure.
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The Experiment:	
A: 0 + 200 for sure.	**B:** (33% 600) + (66% 0).
⇒ 72 % of the participants choose A over B.	
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  To a lesser extend, this is also true of the epistemic formalism that we have been using:
  - "Believing" A and $\vdash A \leftrightarrow B$ imply "Believing" B.

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A': 600 - 400 for sure.	**B':** (33% 600) + (66% 0).
⇒ 78 % of the participants choose B' over A'.	

- Standard decision- and game theory are extensional (see. e.g. Schick '97):
  - Choosing A and $A \leftrightarrow B$ imply Choosing A.

  To a lesser extend, this is also true of the epistemic formalism that we have been using:
  - "Believing" A and $\vdash A \leftrightarrow B$ imply "Believing" B.
- Note: this is different from logical omniscience.

Framing effect

The Experiment:	
A: 0 + 200 for sure.	**B:** (33% 600) + (66% 0).
⇒ 72 % of the participants choose A over B.	
A': 600 - 400 for sure.	**B':** (33% 600) + (66% 0).
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  - Choosing A and $A \leftrightarrow B$ imply Choosing A.

  To a lesser extend, this is also true of the epistemic formalism that we have been using:
  - "Believing" A and $\vdash A \leftrightarrow B$ imply "Believing" B.
- Decision problems in the logicophilia case to be an intensional context.

Variable Frame Theory through an example.

	x_1	x_2	x_3	x_4
y_1	3, 3	3, 3	2, 2	2, 4
y_2	3, 3	3, 3	0, 2	0, 4
y_3	2, 2	2, 0	1, 1	1, 1
y_4	4, 2	4, 0	1, 1	1, 1

Variable Frame Theory through an example.

	x_1	x_2	x_3	x_4
y_1	3, 3	3, 3	2, 2	2, 4
y_2	3, 3	3, 3	0, 2	0, 4
y_3	2, 2	2, 0	1, 1	1, 1
y_4	4, 2	4, 0	1, 1	1, 1

- Set of nondescript actions.

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- Set of nondescript actions.
- Utility functions defined for each player on the nondescript action profiles.

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- Frames are predicates of actions profiles.

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y_4	4, 2	4, 0	1, 1	1, 1

- Set of nondescript actions.
- Utility functions defined for each player on the nondescript action profiles.
- Frames are predicates of actions profiles. E.g. $F_{Team} = \{H_T, M_T, \}$ with $H_T = \{x_1, y_1, \}$ and $M_T = \{x_2, y_2, \}$.

Variable Frame Theory through an example.

	x_1	x_2	x_3	x_4
y_1	3, 3	3, 3	2, 2	2, 4
y_2	3, 3	3, 3	0, 2	0, 4
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- Utility functions defined for each player on the nondescript action profiles.
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Variable Frame Theory through an example.

	x_1	x_2	x_3	x_4
y_1	3, 3	3, 3	2, 2	2, 4
y_2	3, 3	3, 3	0, 2	0, 4
y_3	2, 2	2, 0	1, 1	1, 1
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- Set of nondescript actions.
- Utility functions defined for each player on the nondescript action profiles.
- Frames are predicates of actions profiles.
- Each frame F occurs with a certain probability $v(F)$. E.g. If $v(F_{Team}) = 1$, i.e. common knowledge of team frame.

Variable Frame Theory through an example.

	x_1	x_2	x_3	x_4
y_1	3, 3	3, 3	2, 2	2, 4
y_2	3, 3	3, 3	0, 2	0, 4
y_3	2, 2	2, 0	1, 1	1, 1
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- Set of nondescript actions.
- Utility functions defined for each player on the nondescript action profiles.
- Frames are predicates of actions profiles.
- Each frame F occurs with a certain probability $v(F)$. E.g. If $v(F_{Ind}) = 1$, i.e. common knowledge of individual frame.

Variable Frame Theory through an example.

	x_1	x_2	x_3	x_4
y_1	3, 3	3, 3	2, 2	2, 4
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y_3	2, 2	2, 0	1, 1	1, 1
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- Set of nondescript actions.
- Utility functions defined for each player on the nondescript action profiles.
- Frames are predicates of actions profiles.
- Each frame F occurs with a certain probability $v(F)$.

[Bacharach, 2006] for formal details.

Sugden [2005]: team reasoning under CK of team membership.

Team reasoning and pro-group I-mode.

- Question: What is, if any, the difference between team or group agency and individual agency with group preferences?

Some terminology

- Acting as team member

$$\Rightarrow \left[\text{Adopting the team's preferences} + \text{Team reasoning} \right]$$

Some terminology

- Team agency / We-Mode

  $\Rightarrow \left[\text{Adopting the team's preferences} + \text{Team reasoning} \right]$

From [Tuolema 1995, Forthcoming]

Some terminology

- Team agency / We-Mode
  $\Rightarrow \left[\text{Adopting the team's preferences} + \text{Team reasoning} \right]$

- Pro-Group I mode $\Rightarrow \left[\text{Having the team's preferences} \right]$

From [Tuolema 1995, Forthcoming]

Some terminology

- Team agency / We-Mode
  $\Rightarrow \left[\text{Adopting the team's preferences} + \text{Team reasoning} \right]$
  - We write the paper together.
- Pro-Group I mode $\Rightarrow \left[\text{Having the team's preferences} \right]$
  - I write the paper with Eric.

From [Tuolema 1995, Forthcoming]

Some terminology

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  - We write the paper together.

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  - I write the paper with Eric.

- Question:What is the specific import of team reasoning?

From [Tuolema 1995, Forthcoming]

Some terminology

- Team agency / We-Mode

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  - We write the paper together.

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  - I write the paper with Eric.

- Question: Can we reduce UTI to Bayesian Games, i.e. uncertainty about the payoffs?

From [Tuolema 1995, Forthcoming]

Bayesian Games

Informally: structures to reasons about games with incomplete information, i.e. where there is uncertainty about the structure of the game.

See [Harsanyi, 67-68] and [Myerson, 1991] for a modern introduction.

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Bayesian Games

Informally: structures to reasons about games with incomplete information, i.e. where there is uncertainty about the structure of the game.

- Each players can be of certain types.
- Payoffs are dependent from strategy choice and types.
- Historical note: predates the use of types for imperfect and higher-order information! See [Brandenburger'10] and the talk today/tomorrow.

See [Harsanyi, 67-68] and [Myerson, 1991] for a modern introduction.

Bayesian Games

Formally:

Bayesian Games

Formally:

Definition

A Bayesian Game $\mathcal{B}$ is a tuple $\langle \mathcal{A}, S_i, \mathcal{T}_i, v_i, \lambda_i \rangle$ such that :

- $\mathcal{A}$ is a finite set of agents.
- S_i is a finite set of *actions*. for i. We write S for the set $\Pi_{i \in A} S_i$ of all action profiles.
- $\mathcal{T}_i$ is a finite set of *types* for i.
- A strategy $\sigma_i : T_i \longrightarrow A_i$ is a function assigning to each type of i an action in A_i.
- $v_i : (S \times \mathcal{T}_i) \longrightarrow \mathbb{R}$ is an utility function given that she is of type t_i.
- $\lambda_i : T_i \longrightarrow \Delta(\mathcal{T}_{-i})$.

Bayesian Games

Formally:

Definition

A Bayesian Game $\mathcal{B}$ is a tuple $\langle \mathcal{A}, S_i, \mathcal{T}_i, v_i, \lambda_i \rangle$ such that :

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- $v_i : (S \times \mathcal{T}_i) \longrightarrow \mathbb{R}$ is an utility function given that she is of type t_i.
- Ω is a common prior over $(\mathcal{T})$.

The ex ante expected value of profile σ for player i is defined as :

$$EV_i(\sigma) = \sum_t \Omega(t) v_i((\sigma_i(t_i), \sigma_{-i}(t_{-i})), t_i)$$

A Bayesian equilibrium is a strategy profiles σ such that, for all i,

$$\sigma_i \in argmax_{\sigma'_i}(EV_i(\sigma'_i, \sigma_{-i}))$$

The ex ante expected value of profile σ for player i is defined as :

$$EV_i(\sigma) = \sum_{t_i} \Omega(t_i) \left(\sum_{t_{-i}} \Omega(t_{-i}/t_i) v_i((\sigma_i(t_i), \sigma_{-i}(t_{-i})), t_i) \right)$$

A Bayesian equilibrium is a strategy profiles σ such that, for all i,

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The ex ante expected value of profile σ for player i is defined as :

$$EV_i(\sigma) = \sum_{t_i} \Omega(t_i)\left(EV_i(\sigma/t_i)\right)$$

A Bayesian equilibrium is a strategy profiles σ such that, for all i,

$$\sigma_i \in argmax_{\sigma'_i}(EV_i(\sigma'_i, \sigma_{-i}))$$

From UTIs to Bayesian Games

Let $\langle TI, \mathcal{T} \rangle$ be an unreliable team interaction with no external uncertainty. The Bayesian Game $\mathcal{B}_{UTI}$ based on $\langle TI, \mathcal{T} \rangle$ is defined as follow:

- $\mathcal{A}$ is the set of individuals in TI.
- S_i is the same as in TI.
- $\mathcal{T}_i$ the same as in $\mathcal{T}$, i.e. *types* for i:
  - When $t_i = k$ we say that i is a benefactor for $k \in M$.
- $v_i(s, t_i) = v^{t_i}(s)$.(Ignoring states of uncertainty for now).

Definition

Let α be a protocol in a given UTI, and σ a strategy profile in $\mathcal{B}_{UTI}$. Then σ agrees with α whenever, for all $t_i \in \mathcal{T}_i$:

$$\alpha_i^{t_i} = \sigma_i(t_i)$$

If α is an UTI equilibrium, then there is a Bayesian Equilibrium σ in $\mathcal{B}_{UTI}$ that agrees with α.

Proof.

Sketch:

1. If σ agrees with α, maximization of $EV_i((\sigma'_i, \sigma_{-i})/t_i)$ is equivalent to maximizing $EV^k((a_i, \alpha_{-i})/t_i)$ because:
   - For $s = \sigma(t_i, t_{-i})$; $v_i(s, t_i) = v^k(s)$ and;

If α is an UTI equilibrium, then there is a Bayesian Equilibrium σ in $\mathcal{B}_{UTI}$ that agrees with α.

Proof.

Sketch:

1. If σ agrees with α, maximization of $EV_i((\sigma'_i, \sigma_{-i})/t_i)$ is equivalent to maximizing $EV^k((a_i, \alpha_{-i})/t_i)$ because:
   - For $s = \sigma(t_i, t_{-i})$; $v_i(s, t_i) = v^k(s)$ and;
   - Strategy-wise, for all j, $\alpha_j^{t_j} = \sigma_j(t_j)$.

If α is an UTI equilibrium, then there is a Bayesian Equilibrium σ in $\mathcal{B}_{UTI}$ that agrees with α.

Proof.

Sketch:

1. If σ agrees with α, maximization of $EV_i((\sigma'_i, \sigma_{-i})/t_i)$ is equivalent to maximizing $EV^k((a_i, \alpha_{-i})/t_i)$ because:
   - For $s = \sigma(t_i, t_{-i})$; $v_i(s, t_i) = v^k(s)$ and;
   - Strategy-wise, for all j, $\alpha_j^{t_j} = \sigma_j(t_j)$.
2. If α is an UTI-equilibrium, then for all i, α_i^k maximizes $EV^k((\alpha_i^{t_i}, \alpha_{-i})/t_i)$.

□

If α is an UTI equilibrium, then there is a Bayesian Equilibrium σ in $\mathcal{B}_{UTI}$ that agrees with α.

UTI-equilibria $\subseteq$ Bayesian equilibria in $\mathcal{B}_{UTI}$.

If α is an UTI equilibrium, then there is a Bayesian Equilibrium σ in $\mathcal{B}_{UTI}$ that agrees with α.

UTI-equilibria $\subsetneq$ Bayesian equilibria in $\mathcal{B}_{UTI}$.

UTI-equilibria $\subsetneq$ Bayesian equilibria

UTI-equilibria $\subsetneq$ Bayesian equilibria

	Hard Work	Minimal Work
Hard Work	3, 3 (3)	0, 4 (2)
Minimal Work	4, 0 (2)	1, 1 (1)

UTI-equilibria $\subsetneq$ Bayesian equilibria

	Hard Work	Minimal Work
Hard Work	3, 3 (3)	0, 4 (2)
Minimal Work	4, 0 (2)	1, 1 (1)

- Preliminary observation:
  - Let w be the probability that $t_i = C$ for either player in a type space $\mathcal{T}$ for this TI. If (M, M, HH) is an UTI-equilibrium then $w \geq 1/3$.

See [Bacharach, 1999] for details.

UTI-equilibria $\subsetneq$ Bayesian equilibria

	Hard Work	Minimal Work
Hard Work	3, 3 (3)	0, 4 (2)
Minimal Work	4, 0 (2)	1, 1 (1)

UTI-equilibria $\subsetneq$ Bayesian equilibria

	Hard Work	Minimal Work
Hard Work	3, 3 (3)	0, 4 (2)
Minimal Work	4, 0 (2)	1, 1 (1)

- Let T be a type space for this game such that $w = 1/6$.

UTI-equilibria $\subsetneq$ Bayesian equilibria

	Hard Work	Minimal Work
Hard Work	3, 3 (3)	0, 4 (2)
Minimal Work	4, 0 (2)	1, 1 (1)

- Let T be a type space for this game such that $w = 1/6$. The strategy profile σ which agrees with (M, M, HH) is an equilibria in the Bayesian Game for this UTI.

UTI-equilibria $\subsetneq$ Bayesian equilibria

The Bayesian Game:

I_0, I_E (0.69)	H	M
H	3, 3	0, 4
M	4, 0	1, 1

I_0, C_E (0.14)	H	M
H	3, 3	0, 2
M	4, 2	1, 1

C_0, I_E (0.14)	H	M
H	3, 3	2, 4
M	2, 0	1, 1

C_0, C_E (0.03)	H	M
H	3, 3	2, 2
M	2, 2	1, 1

UTI-equilibria $\subsetneq$ Bayesian equilibria

The Bayesian Game:

I_0, I_E (0.69)	H	M
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C_0, I_E (0.14)	H	M
H	3, 3	2, 4
M	2, 0	1, 1

C_0, C_E (0.03)	H	M
H	3, 3	2, 2
M	2, 2	1, 1

$$EV_E((M,H),(M,H)) = \sum_t \Omega(t) v_E((M,H)(t_E),(M,H)(t_O)), t_E)$$

UTI-equilibria $\subsetneq$ Bayesian equilibria

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I_0, I_E (0.69)	H	M
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C_0, C_E (0.03)	H	M
H	3, 3	2, 2
M	2, 2	1, 1

$$EV_E((M,H),(M,H)) =$$
$$\Omega(I_O, I_E)v_E((M,M), I_E) \; + \; \Omega(I_O, C_E)v_E((M,H), C_E) \; +$$
$$\Omega(C_O, I_E)v_E((H,M), I_E) \; + \; \Omega(C_O, C_E)v_E((H,H), C_E)$$

UTI-equilibria $\subsetneq$ Bayesian equilibria

The Bayesian Game:

I_0, I_E (0.69)	H	M
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C_0, C_E (0.03)	H	M
H	3, 3	2, 2
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$$EV_E((M,H),(M,H)) =$$
$$\Omega(I_O, I_E) v_E((M,M), I_E) \ + \ \Omega(I_O, C_E) v_E((M,H), C_E) \ +$$
$$\Omega(C_O, I_E) v_E((H,M), I_E) \ + \ \Omega(C_O, C_E) v_E((H,H), C_E)$$

UTI-equilibria $\subsetneq$ Bayesian equilibria

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I_0, I_E (0.69)	H	M
H	3, 3	0, 4
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M	2, 0	1, 1

C_0, C_E (0.03)	H	M
H	3, 3	2, 2
M	2, 2	1, 1

$$EV_E((M,H),(M,H)) =$$
$$\Omega(I_O, I_E)(1) \; + \; \Omega(I_O, C_E) v_E((M,H), C_E) \; +$$
$$\Omega(C_O, I_E) v_E((H,M), I_E) \; + \; \Omega(C_O, C_E) v_E((H,H), C_E)$$

UTI-equilibria $\subsetneq$ Bayesian equilibria

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H	3, 3	2, 4
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C_0, C_E (0.03)	H	M
H	3, 3	2, 2
M	2, 2	1, 1

$$EV_E((M,H),(M,H)) =$$

$$\Omega(I_O, I_E)(1) + \Omega(I_O, C_E)(2) +$$

$$\Omega(C_O, I_E) v_E((H,M), I_E) + \Omega(C_O, C_E) v_E((H,H), C_E)$$

UTI-equilibria $\subsetneq$ Bayesian equilibria

The Bayesian Game:

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C_0, C_E (0.03)	H	M
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$$EV_E((M,H),(M,H)) =$$
$$\Omega(I_O, I_E)(1) \ + \ \Omega(I_O, C_E)(2) +$$
$$\Omega(C_O, I_E)(4) \ + \ \Omega(C_O, C_E) v_E((H,H), C_E)$$

UTI-equilibria $\subsetneq$ Bayesian equilibria

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I_0, I_E (0.69)	H	M
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I_0, C_E (0.14)	H	M
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$$EV_E((M,H),(M,H)) = \\ \Omega(I_O, I_E)(1) + \Omega(I_O, C_E)(2) + \\ \Omega(C_O, I_E)(4) + \Omega(C_O, C_E)(3)$$

UTI-equilibria $\subsetneq$ Bayesian equilibria

The Bayesian Game:

I_0, I_E (0.69)	H	M
H	3, 3	0, 4
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C_0, I_E (0.14)	H	M
H	3, 3	2, 4
M	2, 0	1, 1

C_0, C_E (0.03)	H	M
H	3, 3	2, 2
M	2, 2	1, 1

$$EV_E((M,H),(M,H)) =$$

$$0.69(1) \ + \ 0.14(2) +$$

$$0.14(4) \ + \ 0.03(3)$$

UTI-equilibria $\subsetneq$ Bayesian equilibria

The Bayesian Game:

I_0, I_E (0.69)	H	M
H	3, 3	0, 4
M	4, 0	1, 1

I_0, C_E (0.14)	H	M
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M	4, 2	1, 1

C_0, I_E (0.14)	H	M
H	3, 3	2, 4
M	2, 0	1, 1

C_0, C_E (0.03)	H	M
H	3, 3	2, 2
M	2, 2	1, 1

$$EV_E((M, H), (M, H)) = 1.62$$

UTI-equilibria $\subsetneq$ Bayesian equilibria

The Bayesian Game:

I_0, I_E (0.69)	H	M
H	3, 3	0, 4
M	4, 0	1, 1

I_0, C_E (0.14)	H	M
H	3, 3	0, 2
M	4, 2	1, 1

C_0, I_E (0.14)	H	M
H	3, 3	2, 4
M	2, 0	1, 1

C_0, C_E (0.03)	H	M
H	3, 3	2, 2
M	2, 2	1, 1

$$EV_E((M,H),(M,H)) = 1.62$$

$$EV_E((H,H),(M,H)) = .75$$

$$EV_E((M,M),(M,H)) = 1.56$$

$$EV_E((H,M),(M,H)) = 1.3$$

Some Remarks

(L, L, HH) UTI-equ. only if $w \geq 1/3$.

$((M, H), (M, H))$ Bayesian equ. even for $1/6 \leq w < 1/3$.

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- What's going on in the zone $1/6 \leq w < 1/3$?

Some Remarks

(L, L, HH) UTI-equ. only if $w \geq 1/3$.

$((M, H), (M, H))$ Bayesian equ. even for $1/6 \leq w < 1/3$.

- What's going on in the zone $1/6 \leq w < 1/3$?
  - The profile $((M, H), (M, H))$ is a sub-optimal Bayesian equilibrium for the team if $1/6 \leq w < 1/3$.
    - ((L, L),(L, L)) is also an Bayesian equilibria, which as a better expected value for the team.

Some Remarks

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$((M, H), (M, H))$ Bayesian equ. even for $1/6 \leq w < 1/3$.

- What's going on in the zone $1/6 \leq w < 1/3$?
  - The profile $((M, H), (M, H))$ is a sub-optimal Bayesian equilibrium for the team if $1/6 \leq w < 1/3$.
    - ((L, L),(L, L)) is also an Bayesian equilibria, which as a better expected value for the team.
  - Individual benefactors (Pro-group I-mode decision makers) who don't team reason have no way to exclude this sub-optimal equilibrium.
  - Team Reasoning is the missing ingredient.

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$\Rightarrow$ UTI can be seen as games between teams.

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- *Ex interim* rationality in UTI?
  - Still open.

Coming up next

- Other modes of shared attitudes: correlations.