# Logic, Interaction and Collective Agency

Lecture 1

ESSLLI'10, Copenhagen

Eric Pacuit    Olivier Roy

TiLPS, Tilburg University    University of Groningen
ai.stanford.edu/~epacuit    philos.rug.nl/~olivier

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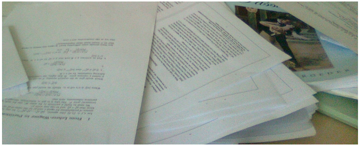
# Writing a paper together



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| | Hard Work | Minimal Work |
|---|---|---|
| Hard Work | 3, 3 | 0, 0 |
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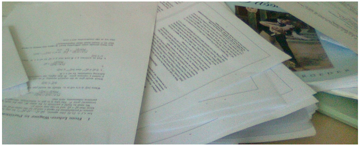

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Problem of Coordination.

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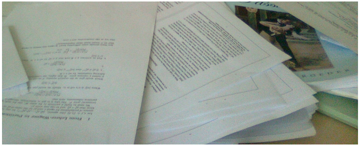

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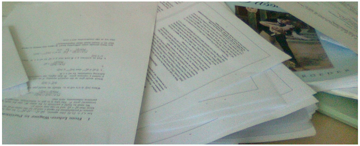
| Minimal Work | 4, 0 | 1, 1 |



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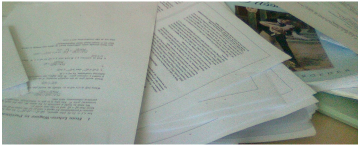

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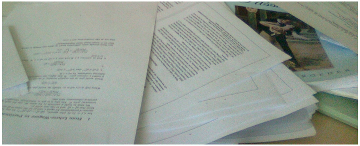
Intuitively, we solve these problem by working together.
This is the question of collective agency.



# Plan for the week

1. The problem of collective agency:

2. Group attitudes (I): a non-standard introduction.
3. Acting on team preferences, frames and team reasoning.
4. Group attitudes (II): Correlations.
5. Commitments, intentions and cooperative agency?

# Plan for the week

1. The problem of collective agency:
   - Individual and group agency.
   - Games.
   - Beliefs (Type Spaces) and rationality.
2. Group attitudes (I): a non-standard introduction.
3. Acting on team preferences, frames and team reasoning.
4. Group attitudes (II): Correlations.
5. Commitments, intentions and cooperative agency?

Individual vs collective agency

# Different contexts of agency

# Different contexts of agency

- Individual decision making and individual action against nature.
  - Ex: Gambling.

# Different contexts of agency

- Individual decision making and individual action against nature.
- Individual decision making in interaction.
  - Ex: Playing chess.

# Different contexts of agency

- Individual decision making and individual action against nature.
- Individual decision making in interaction.
- Collective decision making.
  - Ex: Carrying the piano.

# Different contexts of agency

- Individual decision making and individual action against nature.
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Interaction - Formal models

# Situations of Interaction

In this course we will mostly study situations of interaction in terms of Games in Strategic or Normal form.

# Situations of Interaction

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|---|---|---|
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# Situations of Interaction

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- $v_i : \Pi_{i \in \mathcal{A}} S_i \longrightarrow \mathbb{R}$ is an *utility function* that assigns to every strategy profile $\sigma \in \Pi_{i \in \mathcal{A}} S_i$ the utility valuation of that profile for agent $i$.

# Games of Interest

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| | Hard Work | Minimal Work |
|---|---|---|
| Hard Work | 1, 1 | 0, 0 |
| Minimal Work | 0, 0 | 1, 1 |

- Coordination Games.

# Games of Interest

| | Hard Work | Minimal Work |
|---|---|---|
| Hard Work | 2, 1 | 0, 0 |
| Minimal Work | 0, 0 | 1, 2 |

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- Prisoner's Dilemma.

# Games of Interest

| | Hard Work | Minimal Work |
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| Hard Work | 3, 3 | 0, 4 |
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- In general: games with scope for cooperation.

# The Main Question(s)

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|---|---|---|
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- When there is scope for cooperation, what will the agents do? If they are rational?
  - Descriptive question.

# The Main Question(s)

| | Hard Work | Minimal Work |
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- When there is scope for cooperation, what should the agents do? If they are rational?
  - Normative question.

# The Main Question(s)

| | Hard Work | Minimal Work |
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- When there is scope for cooperation, what does it mean to say that they are rational?
  - Analytical question.

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- When there is scope for cooperation, what does it mean to say that they are rational?
  - ✓ Analytical question.
- Our main focus in this course.

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  - One needs to specify the context of interaction (or of the game). This includes:
    - (possibly) some additional group- or team-related aspects of the game.
    - Information of the agents about all relevant aspects of the game.

# Information in games

# What does it *mean* to be *(perfectly) rational*?

| | $H$ | $M$ |
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| | $H$ | $M$ |
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| $H$ | 3,3 | 0,0 |
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Ann's best choice depends on what she *expects* Bob to do, and this depends on what she *thinks* Bob expects her to do, and so on...

# What does it *mean* to be *(perfectly) rational*?

| | $H$ | $M$ |
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| $H$ | 3,3 | 0,0 |
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Instrumental Rationality: maximize *given your current information* (Bayesian Decision Theory)

# Information in games situations

- Various states of information disclosure.

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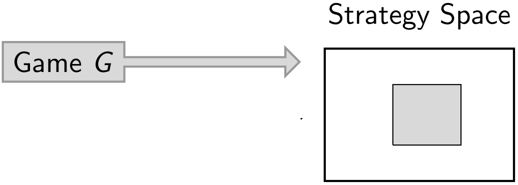
  - strategic information (what will the other players do?)
  - higher-order information (what are the other players thinking?)
- Varieties of informational attitudes
  - hard ("knowledge")
  - soft ("beliefs")

# Finding the "rational" choice

$\boxed{\text{Game } G}$

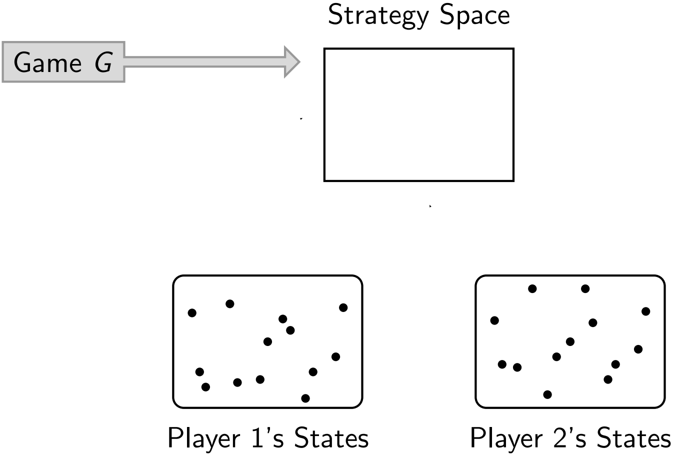
$G$: available actions, payoffs, structure of the decision problem

# Finding the “rational” choice



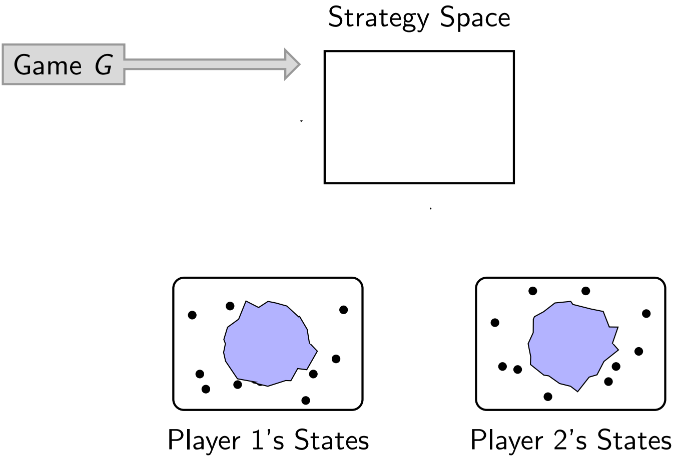

**solution concepts** are systematic descriptions of what players *do*

# Finding the "rational" choice



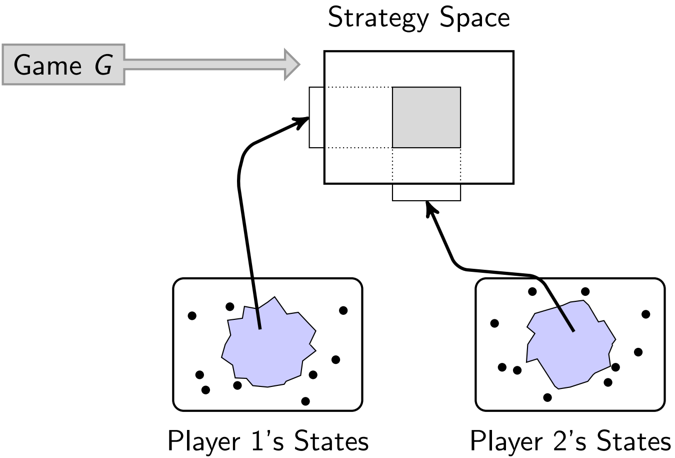

Consider possible *information states* of the players

# Finding the "rational" choice




Restrict to information states satisfying some rationality condition

# Finding the "rational" choice




Project onto the strategy space

## Time for some details...

Two general modeling strategies:

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1. Harsanyi type spaces: sorted structure with maps between players' "states"

J. Harsanyi. *Games with incomplete information played by "bayesian" players I-III. Management Science Theory* **14**: *159-182, 1967-68.*

# Time for some details...

Two general modeling strategies:

1. Harsanyi type spaces: sorted structure with maps between players' "states"

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2. Partition model: single set of *states* with partitions describing the players' (hard) information

R. Aumann. *Interactive Epistemology I & II*. International Journal of Game Theory (1999).

# Literature

See, for example,


P. Battigalli and G. Bonanno. *Recent results on belief, knowledge and the epistemic foundations of game theory*. Research in Economics (1999).

B. de Bruin. *Explaining Games*. Ph.D. Thesis, ILLC (2004).

A. Brandenburger. *The Power of Paradox: Some Recent Developments in Interactive Epistemology*. International Journal of Game Theory (2007).

# An Example

| | | Bob $H$ | Bob $M$ |
|---|---|---|---|
| Ann | $H$ | 3,3 | 0,0 |
| Ann | $M$ | 0,0 | 1,1 |

# An Example

| Ann \ Bob | $H$ | $M$ |
|---|---|---|
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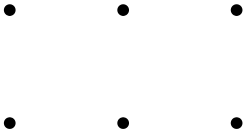
A set of information states



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Bob

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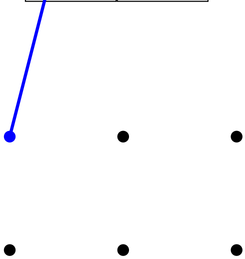

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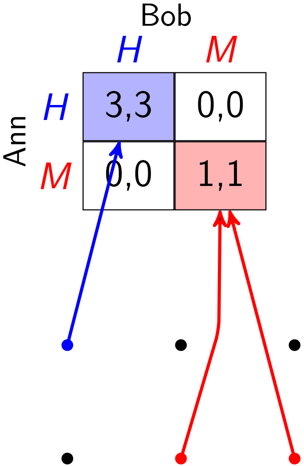


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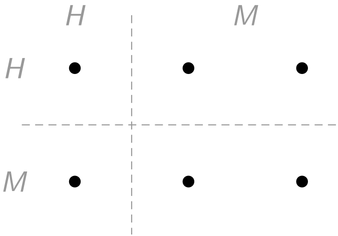


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|---|---|---|---|
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| | $H$ | $M$ | |
|---|---|---|---|
| $H$ | $\frac{1}{8}$ ● | $\frac{1}{16}$ ● | $\frac{1}{16}$ ● |
| $M$ | $\frac{1}{8}$ ● | $\frac{1}{8}$ ● | $\frac{1}{2}$ ● |

A common prior

# An Example

| | | Bob | |
|---|---|---|---|
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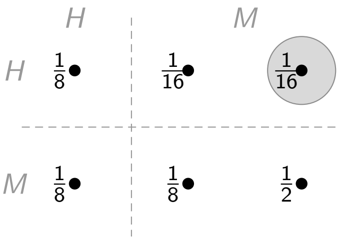


Suppose Ann chooses $H$ and Bob chooses $M$

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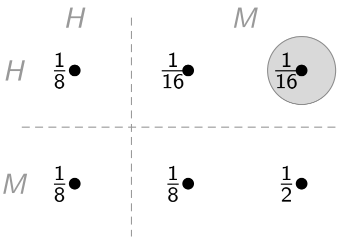


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*Are these choices rational?*

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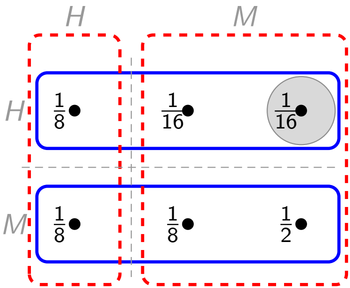


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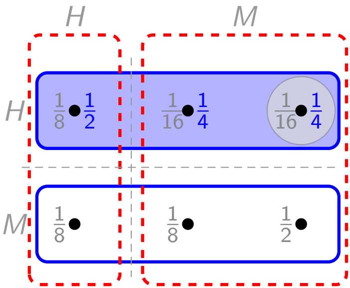


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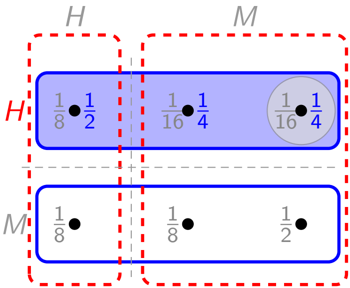


$$3 \cdot P_A(H) + 0 \cdot P_A(M) \geq 0 \cdot P_A(H) + 1 \cdot P_A(M)$$

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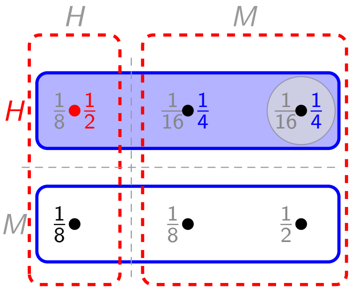


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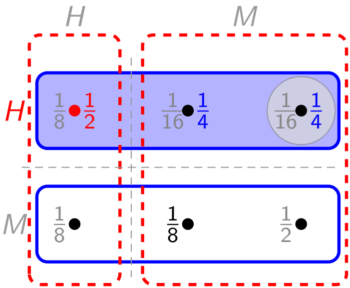


$$3 \cdot \frac{1}{2} + 0 \cdot P_A(M) \geq 0 \cdot \frac{1}{2} + 1 \cdot P_A(M)$$

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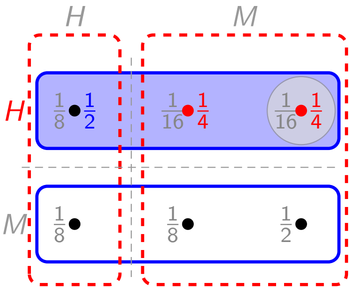


$$3 \cdot \frac{1}{2} + 0 \cdot \frac{1}{2} \geq 0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2}$$

# An Example

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|---|---|---|---|
| | | $H$ | $M$ |
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Suppose Ann chooses $H$ and Bob chooses $M$

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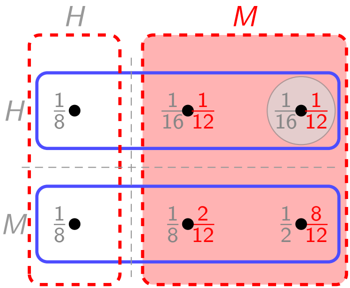


$$0 \cdot P_B(H) + 1 \cdot P_B(M) \geq 3 \cdot P_B(H) + 0 \cdot P_B(M)$$

# An Example

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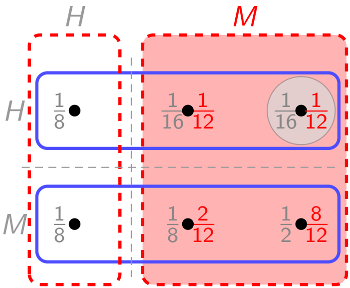


$$0 \cdot \frac{2}{12} + 1 \cdot \frac{10}{12} \geq 3 \cdot \frac{2}{12} + 0 \cdot \frac{10}{12}$$

# An Example

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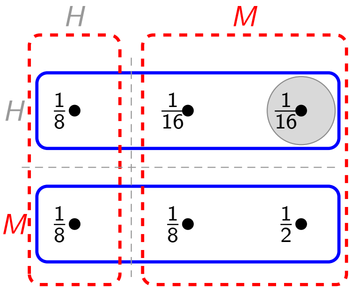


Suppose Ann chooses $H$ and Bob chooses $M$

*Are these choices rational?*
Yes.

# An Example

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|---|---|---|---|
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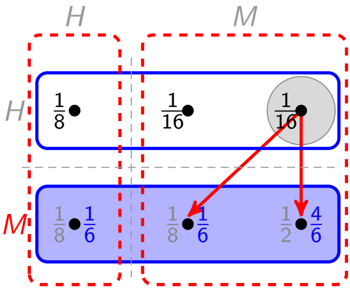



Suppose Ann chooses $H$ and Bob chooses $M$

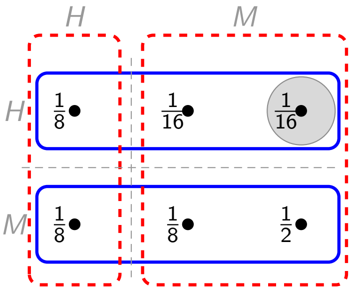
*Are these choices rational?*
Yes.

Bob (Ann) *knows* that Ann (Bob) is *rational*

$$0 \cdot \frac{1}{6} + 1 \cdot \frac{5}{6} \geq 3 \cdot \frac{1}{6} + 0 \cdot \frac{5}{6}$$

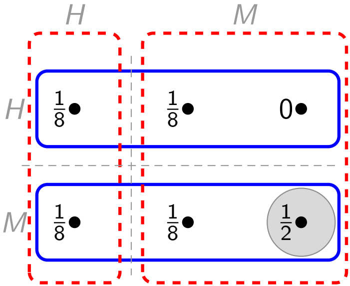
# Two Issues

| Ann \ Bob | $H$ | $M$ |
|---|---|---|
| $H$ | 3,3 | 0,0 |
| $M$ | 0,0 | 1,1 |



# Two Issues

| | | Bob $H$ | Bob $M$ |
|---|---|---|---|
| Ann | $H$ | 3,3 | 0,0 |
| | $M$ | 0,0 | 1,1 |




1. Zero probability $\neq$ "impossible"

# Two Issues

| | Bob: *H* | Bob: *M* |
|---|---|---|
| Ann: *H* | 3,3 | 0,0 |
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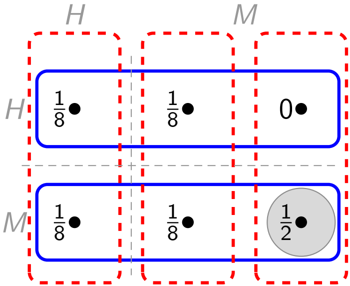


1. Zero probability $\neq$ "impossible"
2. Different "types" of players can make the same choice

## Two Issues

| Ann \ Bob | $H$ | $M$ |
|---|---|---|
| $H$ | 3,3 | 0,0 |
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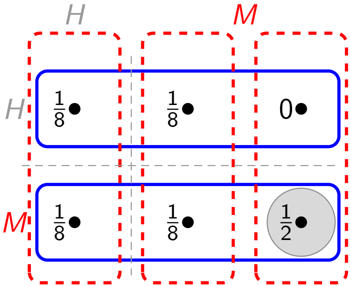
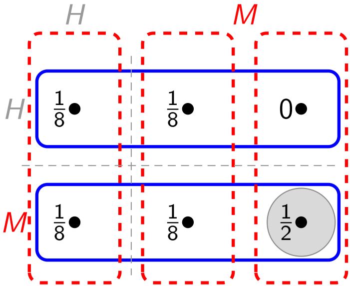



1. Zero probability $\neq$ "impossible"
2. Different "types" of players can make the same choice

- Are Ann and Bob rational?

# Two Issues

| | | Bob | |
|---|---|---|---|
| | | $H$ | $M$ |
| Ann | $H$ | 3,3 | 0,0 |
| | $M$ | 0,0 | 1,1 |




1. Zero probability $\neq$ "impossible"
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- Are Ann and Bob rational? Yes.

# Two Issues

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| $H$ | 3,3 | 0,0 |
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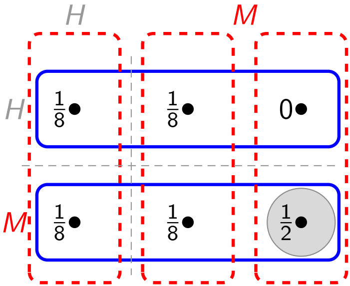


1. Zero probability $\neq$ "impossible"
2. Different "types" of players can make the same choice

- Are Ann and Bob rational? Yes.
- Do they *know* that each other is rational? No.

# Two Issues

| Ann \ Bob | $H$ | $M$ |
|---|---|---|
| $H$ | 3,3 | 0,0 |
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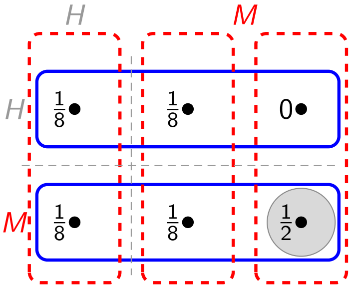


1. Zero probability $\neq$ "impossible"
2. Different "types" of players can make the same choice

- Are Ann and Bob rational? Yes.
- Do they *know* that each other is rational? No.
  ($though\ Pr_{Bob}(Irrat(Ann)) = 0$)

## Harsanyi Type Space

Based on the work of John Harsanyi on games with *incomplete information*, game theorists have developed an elegant formalism that makes precise talk about beliefs, knowledge and rationality:

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- Each **type** is assigned a joint probability over the space of types and actions

$$\lambda_i : T_i \to \Delta(T_{-i} \times S_{-i})$$

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Player $i$'s types

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The set of all probability distributions

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$$\lambda_i : T_i \to \Delta(\underbrace{T_{-i}}_{\text{The other players' types}} \times S_{-i})$$

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The other players' choices

# Returning to the Example: A Game Model

Ann (rows) / Bob (columns)

| | $H$ | $M$ |
|---|---|---|
| $H$ | 3,3 | 0,0 |
| $M$ | 0,0 | 1,1 |

$t_A$

| | $H$ | $M$ |
|---|---|---|
| $t_B$ | 0 | 0.5 |
| $u_B$ | 0.2 | 0.3 |

$t_B$

| | $H$ | $M$ |
|---|---|---|
| $t_A$ | 0 | 1 |

$u_B$

| | $H$ | $M$ |
|---|---|---|
| $t_A$ | 0.4 | 0.6 |

# Returning to the Example: A Game Model

|  | Bob: $H$ | Bob: $M$ |
|---|---|---|
| Ann: $H$ | 3,3 | 0,0 |
| Ann: $M$ | 0,0 | 1,1 |

- One type for Ann ($t_A$) and two types for Bob ($t_B, u_B$)

|  | $H$ | $M$ |
|---|---|---|
| $t_B$ | 0 | 0.5 |
| $u_B$ | 0.2 | 0.3 |

$t_A$

|  | $H$ | $M$ |
|---|---|---|
| $t_A$ | 0 | 1 |

$t_B$

|  | $H$ | $M$ |
|---|---|---|
| $t_A$ | 0.4 | 0.6 |

$u_B$

# Returning to the Example: A Game Model

| | | Bob: $H$ | Bob: $M$ |
|---|---|---|---|
| Ann | $H$ | 3,3 | 0,0 |
| Ann | $M$ | 0,0 | 1,1 |

- One type for Ann ($t_A$) and two types for Bob ($t_B, u_B$)
- A **state** is a tuple of choices and types: $(M, M, t_A, t_B)$

$t_A$:

| | $H$ | $M$ |
|---|---|---|
| $t_B$ | 0 | 0.5 |
| $u_B$ | 0.2 | 0.3 |

$t_B$:

| | $H$ | $M$ |
|---|---|---|
| $t_A$ | 0 | 1 |

$u_B$:

| | $H$ | $M$ |
|---|---|---|
| $t_A$ | 0.4 | 0.6 |

# Returning to the Example: A Game Model

| Ann \ Bob | $H$ | $M$ |
|---|---|---|
| $H$ | 3,3 | 0,0 |
| $M$ | 0,0 | 1,1 |

- One type for Ann ($t_A$) and two types for Bob ($t_B, u_B$)
- A **state** is a tuple of choices and types: $(M, t_A, M, u_B)$
- Calculate **expected utility** in the usual way...

$t_A$

| | $H$ | $M$ |
|---|---|---|
| $t_B$ | 0 | 0.5 |
| $u_B$ | 0.2 | 0.3 |

$t_B$

| | $H$ | $M$ |
|---|---|---|
| $t_A$ | 0 | 1 |

$u_B$

| | $H$ | $M$ |
|---|---|---|
| $t_A$ | 0.4 | 0.6 |

# Returning to the Example: A Game Model

| Ann \ Bob | $H$ | $M$ |
| --- | --- | --- |
| $H$ | 3,3 | 0,0 |
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$t_A$

| | $H$ | $M$ |
| --- | --- | --- |
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$t_B$

| | $H$ | $M$ |
| --- | --- | --- |
| $t_A$ | 0 | 1 |

$u_B$

| | $H$ | $M$ |
| --- | --- | --- |
| $t_A$ | 0.4 | 0.6 |

# Returning to the Example: A Game Model

| | | Bob $H$ | Bob $M$ |
|---|---|---|---|
| Ann | $H$ | 3,3 | 0,0 |
| Ann | $M$ | 0,0 | 1,1 |

- $M$ is **rational** for Ann ($t_A$)
  $0 \cdot 0.2 + 1 \cdot 0.8 \geq 3 \cdot 0.2 + 0 \cdot 0.8$

$t_A$:

| | $H$ | $M$ |
|---|---|---|
| $t_B$ | 0 | 0.5 |
| $u_B$ | 0.2 | 0.3 |

$t_B$:

| | $H$ | $M$ |
|---|---|---|
| $t_A$ | 0 | 1 |

$u_B$:

| | $H$ | $M$ |
|---|---|---|
| $t_A$ | 0.4 | 0.6 |

# Returning to the Example: A Game Model

| | | Bob $H$ | Bob $M$ |
|---|---|---|---|
| Ann | $H$ | 3,3 | 0,0 |
| Ann | $M$ | 0,0 | 1,1 |

- $M$ is **rational** for Ann ($t_A$)
  $0 \cdot 0.2 + 1 \cdot 0.8 \geq 3 \cdot 0.2 + 0 \cdot 0.8$
- $M$ is **rational** for Bob ($t_B$)
  $0 \cdot 0 + 1 \cdot 1 \geq 3 \cdot 0 + 0 \cdot 1$

$t_A$

| | $H$ | $M$ |
|---|---|---|
| $t_B$ | 0 | 0.5 |
| $u_B$ | 0.2 | 0.3 |

$t_B$

| | $H$ | $M$ |
|---|---|---|
| $t_A$ | 0 | 1 |

$u_B$

| | $H$ | $M$ |
|---|---|---|
| $t_A$ | 0.4 | 0.6 |

# Returning to the Example: A Game Model

Bob

| Ann | $H$ | $M$ |
|---|---|---|
| $H$ | 3,3 | 0,0 |
| $M$ | 0,0 | 1,1 |

- $M$ is **rational** for Ann ($t_A$)
  $0 \cdot 0.2 + 1 \cdot 0.8 \geq 3 \cdot 0.2 + 0 \cdot 0.8$
- $M$ is **rational** for Bob ($t_B$)
  $0 \cdot 0 + 1 \cdot 1 \geq 3 \cdot 0 + 0 \cdot 1$
- Ann thinks Bob may be irrational

$t_A$

| | $H$ | $M$ |
|---|---|---|
| $t_B$ | 0 | 0.5 |
| $u_B$ | 0.2 | 0.3 |

$t_B$

| | $H$ | $M$ |
|---|---|---|
| $t_A$ | 0 | 1 |

$u_B$

| | $H$ | $M$ |
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# Returning to the Example: A Game Model

| | | Bob $H$ | Bob $M$ |
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  $0 \cdot 0.2 + 1 \cdot 0.8 \geq 3 \cdot 0.2 + 0 \cdot 0.8$
- $M$ is **rational** for Bob ($t_B$)
  $0 \cdot 0 + 1 \cdot 1 \geq 3 \cdot 0 + 0 \cdot 1$
- Ann thinks Bob may be irrational
  $P_A(Irrat[B]) = 0.3$, $P_A(Rat[B]) = 0.7$

$t_A$:

| | $H$ | $M$ |
|---|---|---|
| $t_B$ | 0 | 0.5 |
| $u_B$ | 0.2 | 0.3 |

$t_B$:

| | $H$ | $M$ |
|---|---|---|
| $t_A$ | 0 | 1 |

$u_B$:

| | $H$ | $M$ |
|---|---|---|
| $t_A$ | 0.4 | 0.6 |

**Notation**:

- Suppose $s \in S = S_1 \times \cdots \times S_n$ (pure strategy profiles) and $t \in T_1 \times \cdots \times T_n$ (set of types).
- Let $s_{-i} \in S_1 \times \cdots \times S_{i-1} \times S_{i+1} \times \cdots \times S_n$ be the other agents' choices (similarly for types) and $s_i \in S_i$ agent $i$'s choice.
- Write $(s_i, s_{-i})$ for $s$.
- For $t_i \in T_i$, let $p_{t_i} = \lambda(t_i) \in \Delta(S_{-i} \times T_{-i})$

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Agent $i$'s expected value at state $(s, t)$ is:

$$EV_i(s,t) = \sum_{t'_{-i}} \sum_{\sigma'_{-i}} p_{t_i}(s'_{-i}, t'_{-i}) u_i(s_i, s'_{-i})$$

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Sum over all possible types and choices

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$t_i$'s 1st-order beliefs

**Notation**:

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Agent $i$'s utility

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$$EV_i(s,t) = \sum_{t'_{-i}} \sum_{\sigma'_{-i}} p_{t_i}(s'_{-i}, t'_{-i}) u_i(s_i, s'_{-i})$$

Agent $i$ is rational at state $(s, t)$ whenever:

$$s_i \in \text{argmax}_{s' \in S_i}(EV_i(s[s_i \mapsto s'_i], t))$$

## Beyond Game Models: Bayesian Games

The components of a **Bayesian game** for a set of agents $\mathcal{A}$:

- $S = S_1 \times \cdots \times S_n$ where $S_i$ is the set of actions for player $i$;
- $T = T_1 \times \cdots \times T_n$ where $T_i$ is the set of types for player $i$;
- $u = (u_1, \ldots, u_n)$ where $u_i : A \times T_i \to \Re$ is a utility function.
- $p : T \to [0, 1]$ is a common prior over types; and
- $\alpha_i : T_i \to A_i$ is a pure strategy function
- Let $s_i$ be a mixed strategy and $s_i(a_i|t_i)$ denote the probability agent $i$ plays $a_i$ given that $i$'s type is $t_i$.
- *Ex post expected value*:
  $EU_i(s, t) = \sum_{a \in A} \prod_{j \in \mathcal{A}} s_j(a_j|t_j) u_i(a, t)$
- *Ex interim expected value*:
  $EU_i(s, t_i) = \sum_{t_{-i} \in T_{-i}} p(t_{-i}|t_i) EU_i(s, (t_i, t_{-i}))$
- *Ex ante expected value*:
  $EU_i(s) = \sum_{t_i \in T_i} p(t_i) EU_i(s, t_i)$

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- $u = (u_1, \ldots, u_n)$ where $u_i : A \times T_i \to \mathfrak{R}$ is a utility function.
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# Beyond Game Models: Bayesian Games

The components of a **Bayesian game** for a set of agents $\mathcal{A}$:

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# Rationalizability

**Definition**

Strategy $s_i'$ of agent $i$ is rationalizable if there exists a state $(s, t)$ in a type structure $\mathcal{T}$ such that $s_i = s_i'$ and $s_i'$ is rational at $(s, t)$.

# Rationalizability

| | Hard Work | Minimal Work |
|---|---|---|
| Hard Work | 3, 3 | 0, 0 |
| Minimal Work | 0, 0 | 1, 1 |

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Observations: "*Methodological Individualism*"

1. Coordination on Pareto sub-optimal outcome is rationalizable.

# Rationalizability

| | Hard Work | Minimal Work |
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Observations: "*Methodological Individualism*"

1. Coordination on Pareto sub-optimal outcome is rationalizable.
2. The cooperative outcome (HW,HW) in the PD is not rationalizable. (Why?)

## Teamwork once again

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Intuitively, Yes.

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*But then more machinery is needed...*

# What is a team?

Any group?

# What is a team?

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- Surely not. But interesting phenomena at this level already.
  - $\Rightarrow$ Coalitional powers (c.f. Pauly 2002).

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Then a group with:

i A certain (hierarchical) structure?

ii Whose members identify with the group (c.f. Gold 2005)?

  - Information about who's in and who's out.
  - Reasoning and acting as group members.

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   - Shared intentions.
   - Sanctions for lapsing?
   - Shared praise[blame] for success[failure]?

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Note: None of these are necessary conditions!

# Recap

Acting as a team involve:

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- Team-reasoning (Agency Transformation).

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Later this week.

# Tomorrow

- Building the common perspective: (a non-standard introduction to) common knowledge, and common modes of reasoning.