Models of Strategic Reasoning
Lecture 3

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Lecture 1: Introduction, Motivation and Background

Lecture 2: The Dynamics of Rational Deliberation

Lecture 3: Reasoning to a Solution: Common Modes of Reasoning in Games

Lecture 4: Reasoning to a Model: Iterated Belief Change as Deliberation

Lecture 5: Reasoning in Specific Games: Experimental Results

General comments

- Extensive games, imprecise probabilities, other notions of stability, weaken common knowledge assumptions,...
- Generalizing the basic model
- Why assume deliberators are in a "information feedback situation"?
- Deliberation in decision theory.

J. McKenzie Alexander. *Local interactions and the dynamics of rational deliberation*. Philosophical Studies 147 (1), 2010.

Consider a social network $\langle N, E \rangle$ (connected graph)

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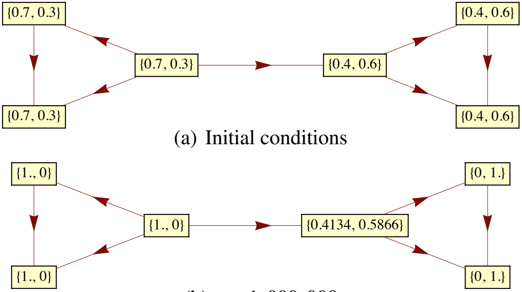
$\mathbf{p}'_{\mathbf{a},\mathbf{b}}(\mathbf{t}+\mathbf{1})$ is represents the incremental refinement of player a's state of indecision given his knowledge about player b's state of indecision (at time $t+1$).

Pool this information to form your new probabilities:

$$\mathbf{p}_i(t+1) = \sum_{j=1}^{k} w_{i,i_j} \mathbf{p}'_{i,i_j}(t+1)$$

		Billy	
		Boxing	Ballet
Maggie	Boxing	$(2,1)$	$(0,0)$
	Ballet	$(0,0)$	$(1,2)$

Fig. 7 The game of Battle of the Sexes.



(a) Initial conditions

(b) $t = 1{,}000{,}000$

Fig. 8 Battle of the Sexes played by Nash deliberators ($k = 25$) on two cycles connected by a bridge edge (values rounded to the nearest 10^{-4}).

The value of information

Why is it better to make a "more informed" decision?
Suppose that you can either choose know, or perform a costless experiment and make the decision later. What should you do?

I. J. Good. *On the principle of total evidence*. British Journal for the Philosophy of Science, 17, pgs. 319 - 321, 1967.

"Never decide today what you might postpone until tomorrow in order to learn something new"

Choose between n acts $A_1, \ldots, A_n$ or perform a cost-free experiment E with possible results $\{e_k\}$, then decide.

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Then,

$$U(\text{Choose now}) = \max_j \sum_i p(K_i)U(A_j \mathbin{\&} K_i)$$

$$= \max_j \sum_k \sum_i p(K_i)p(e_k \mid K_i)U(A_j \mathbin{\&} K_i)$$

The value of an informed decision conditional on e:

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Compare $\max_j \sum_k \sum_i p(K_i) p(e_k \mid K_i) U(A_j \,\&\, K_i)$ and $\sum_k \max_j \sum_i p(e_k \mid K_i) p(K_i) U(A_j \,\&\, K_i)$

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$\sum_k \max_j g(k,j)$ is greater than or equal to $\max_j \sum_k g(k,j)$, so the second is greater than or equal to the first.

The Cost of Thinking

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L. J. Savage. *Difficulties in the theory of personal probability*. Philosophy of Science, 34(4), pgs. 305 - 310, 1967.

What are the players deliberating/reasoning *about*?

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Their preferences?

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Their preferences? The model?

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Their preferences? The model? The other players? What to *do*?

▸ Conclusion

I. Douven. *Decision theory and the rationality of further deliberation*. Economics and Philosophy, 18, pgs. 303 - 328, 2002.

Deliberation in Decision Theory

"deliberation crowds out prediction"

F. Schick. *Self-Knowledge, Uncertainty and Choice*. The British Journal for the Philosophy of Science, 30:3, pgs. 235 - 252, 1979.

I. Levi. *Feasibility*. in *Knowledge, belief and strategic interaction*, C. Bicchieri and M. L. D. Chiara (eds.), pgs. 1 - 20, 1992.

W. Rabinowicz. *Does Practical deliberation Crowd Out Self-Prediction?*. Erkenntnis, 57, 91-122, 2002.

Conclusion

Meno's Paradox

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Levi's Argument

1. If you have access to self-knowledge and logical omniscience to apply the principles of rational choice to determine which options are admissible, then the principles of rational choice are vacuous for the purposes of deciding what to do.
2. If you do not have access to self-knowledge and logical omniscience in this sense, then the principles of rational choice are inapplicable for the purposes of deciding what do.

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If all the previous conditions are satisfied, then no inadmissible option is feasible from the deliberating agent's point of view when deciding what to do: $C(A) = A$.

"Though this result is not contradictory, it implies the vacuousness of principles of rational choice for the purpose of deciding what to do...If they are useless for this purpose, then by the argument of the previous section, they are useless for passing judgement on the rationality of choice as well." (L, pg. 10)

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(Earlier argument: "If X is merely giving advice, it is pointless to advise Sam to do something X is sure Sam will not do...The point I mean to belabor is that passing judgement on the rationality of Sam's choices has little merit unless it gives advice to how one should choose in predicaments similar to Sam's in relevant aspects")

Weak Thesis: In a situation of choice, the DM does not assign extreme probabilities to options among which his choice is being made.

Strong Thesis: In a situation of choice, the DM does not assign any probabilities to options among which his choice is being made.

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"...the probability assignment to A may still be available to the subject in his purely doxastic capacity but not in his capacity of an agent or practical deliberator. The agent *qua* agent must abstain from assessing the probability of his options." (Rabinowicz, pg. 3)

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- Deliberation as a *feedback* process: change in inclinations causes a change in probabilities assigned to various options, which in turn may change my inclinations towards particular options....

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- Implications for game theory (*common knowledge of rationality* implies, in particular, that agents satisfy *Smugness*).

Skip

Game Plan

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✓ The Dynamics of Rational Deliberation

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Eg., Iterated removal of weakly/strictly dominated strategies

1. iterative procedures narrow down or assist in the search for a equilibria

   successive stages of strategy deletion may correspond to different levels of belief (in a lexicographic probability system)

2. iterative procedures represent a rational deliberation process

   successive stages of a strategy deletion can be interpreted as tracking successive steps of reasoning that players can perform

Aumann "versus" Lewis on Common Knowledge

Aumann *defines* common knowledge to be the infinite conjunction of iterations of "everyone knows that" operators.

Lewis offers an analysis of how common knowledge is *achieved*

R. Cubitt and R. Sugden. *Common Knowledge, Salience and Convention: A Reconstruction of David Lewis' Game Theory*. Economics and Philosophy, 19, pgs. 175 - 210, 2003.

The Fixed-Point Definition

Separating the fixed-point/iteration definition of common knowledge/belief:

J. Barwise. *Three views of Common Knowledge*. TARK (1987).

J. van Benthem and D. Saraenac. *The Geometry of Knowledge*. Aspects of Universal Logic (2004).

A. Heifetz. *Iterative and Fixed Point Common Belief*. Journal of Philosophical Logic (1999).

Reason to Believe

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- Anyone who accept the rules of arithmetic has a reason to believe $618 \times 377 = 232,986$, but most of us do not hold have firm beliefs about this.

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- Anyone who accept the rules of arithmetic has a reason to believe $618 \times 377 = 232,986$, but most of us do not hold have firm beliefs about this.
- Definition: $R_i(\varphi)$ means φ is true within some logic of reasoning that is *endorsed* by (that is, accepted as a normative standard by) person i...φ must be either regarded as *self-evident* or derivable by rules of inference (deductive or inductive)

A indicates to i that φ

A is a "state of affairs"

$A\ ind_i\ \varphi$: i's reason to believe that A holds *provides* i's reason for believing that φ is true.

(A1) For all i, for all A, for all φ: $[R_i(A\ holds) \wedge (A\ ind_i\ \varphi)] \Rightarrow R_i(\varphi)$

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- $[(A\ \mathit{ind}_i\ \varphi) \wedge (A\ \mathit{ind}_i \psi)] \Rightarrow A\ \mathit{ind}_i(\varphi \wedge \psi)$
- $[(A\ \mathit{ind}_i[A'\ \mathit{holds}]) \wedge (A'\ \mathit{ind}_i x)] \Rightarrow A\ \mathit{ind}_i \varphi$

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- $[(A \; ind_i[A' \; holds]) \wedge (A' \; ind_i x)] \Rightarrow A \; ind_i \varphi$
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- $[(A\ ind_i\ R_j[A'\ holds]) \wedge R_i(A'\ ind_j \varphi)] \Rightarrow A\ ind_i R_j(\varphi)$

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Reflexive Common Indicator

- A *holds* $\Rightarrow R_i(A$ *holds*$)$
- A *ind*$_i$ $R_j(A$ *holds*$)$
- A *ind*$_i$ φ
- $(A$ *ind*$_i$ $\psi) \Rightarrow R_i[A$ *ind*$_j$ $\psi]$

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Theorem. (Lewis) For all states of affairs A, for all propositions φ, and for all groups G: if A holds, and if A is a reflexive common indicator in G that φ, then $R^G(\varphi)$ is true.

Lewis and Aumann

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Example. Suppose there is an agent $i \not\in G$ that is *authoritative* for each member of G.

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Example

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What should they do?

R. Sugden. *The Logic of Team Reasoning*. Philosophical Explorations (6)3, pgs. 165 - 181 (2003).

Example

		B	
		l	r
A	l		
	r		

Example

			B	
			l	r
A	l		10,10	0,0
	r		0,0	11,11

A: What should I do?

Example

		B l	B r
A	l	10,10	0,0
A	r	0,0	11,11

A: What should I do? r if the probability of B choosing r is $> \frac{10}{21}$ and l if the probability of B choosing l is $> \frac{11}{21}$

(symmetric reasoning for B)

Example

		B: l	B: r
A	l	10,10	0,0
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A: What should *we* do?

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A: What should *we* do? **Team Reasoning**: an escape from the infinite regress? why should this "mode of reasoning" be endorsed?

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$R_i(\varphi_i)$ vs. $R_j(\varphi_i)$: Suppose i reliable takes a bus every Monday. The other commuters may all make the inductive inference that i will take the bus next Monday (M_i). In fact, we may assume that this is a *common mode of reasoning*, so everyone reliably makes the inference that i will catch the bus next Monday. So, $R_j(M_i)$, $R_iR_j(M_i)$, but i should still be *free* to choose whether he wants to take the bus on Monday, so $\neg R_i(M_i)$ and $\neg R_j(R_i(M_i))$, etc.

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Awareness of Common Reason: for all $i \in G$ and all propositions φ,

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Common Attribution of Common Reason: for all $i \in G$, for all propositions φ for which i is not the subject

$$inf(R^G) : \varphi \rightarrow R_i(\varphi)$$

Team Maximising

$inf(R_i) : R^N[opt(v, N, s^N)]$,
R^N[each $i \in N$ endorses team maximising with respect to N and v],
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i acts on reasons if for all s_i, $R_i[ought(i, s_i)] \Rightarrow choice(i, s_i)$

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$opt(v, N, s^N)$: s^N is maximal for the group N w.r.t. v

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Recursive definition: *i*'s endorsement of the rule depends on *i* having a reason to believe everyone else endorses the rule...

Team modes of reasoning, group identification, frames and team preferences, ...

Reasoning Based Expected Utility Procedure

R. Cubitt and R. Sugden. *The reasoning-based expected utility procedure*. Games and Economic Behavior, 2010.

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Example: RBEU (reasoning based expected utility):

- accumulate strategies that maximize expected utility for every possibly probability distribution
- delete strategies that do not maximize probability against any probability distribution
- accumulated strategies must receive positive probability, deleted strategies must receive zero probability

RBEU: Example

	L	R
U	1,1	1,1
M_1	0,0	1,0
M_2	2,0	0,0
B	0,2	0,0

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RBEU Example 2

	l	r
u	1,1	0,0
d	0,0	0,0

RBEU Example 2

	l	r
u	1,1	0,0
d	0,0	0,0

$$S^+ = \{u, l\}$$
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RBEU Example 2

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RBEU Example 3

	L	R
u	1,1	1,0
d	1,0	0,1

RBEU Example 3

	L	R
u	1,1	1,0
d	1,0	0,1

$S^+ = \{u\}$
$S^- = \emptyset$

RBEU Example 3

	L	R
u	1,1	1,0
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R. Cubitt and R. Sugden. *Common reasoning in games: A Lewisian analysis of common knowledge of rationality*. Discussion paper, 2011.

Today: foundational issues (value of information, deliberation in decision theory), Lewisian common knowledge, common modes of reasoning

Tomorrow: Dynamic logic perspective on games.