Epistemic Game Theory

Lecture 4

ESSLLI'12, Opole

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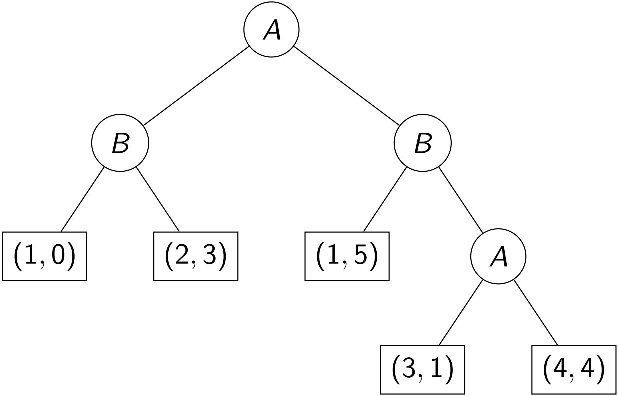
Plan for the week

1. **Monday** Basic Concepts.
2. **Tuesday** Epistemics.
3. **Wednesday** Fundamentals of Epistemic Game Theory.
4. **Thursday** Trees, Puzzles and Paradoxes.
   - Strict Dominance in the Tree: Common knowledge of Rationality and backward induction.
   - Weak dominance and admissibility in the matrix.
5. **Friday** More Paradoxes, Extensions and New Directions.

Backwards Induction

Invented by Zermelo, Backwards Induction is an iterative algorithm for “solving” and extensive game.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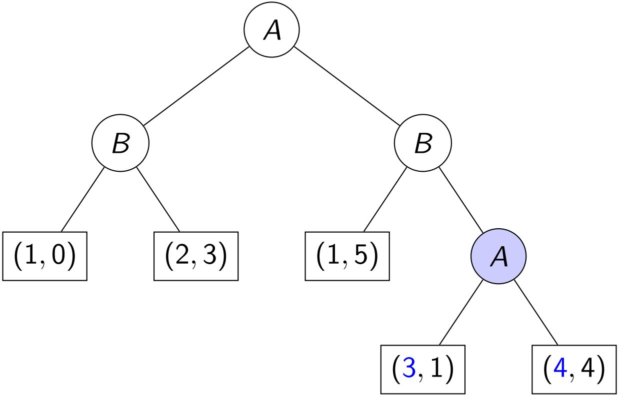
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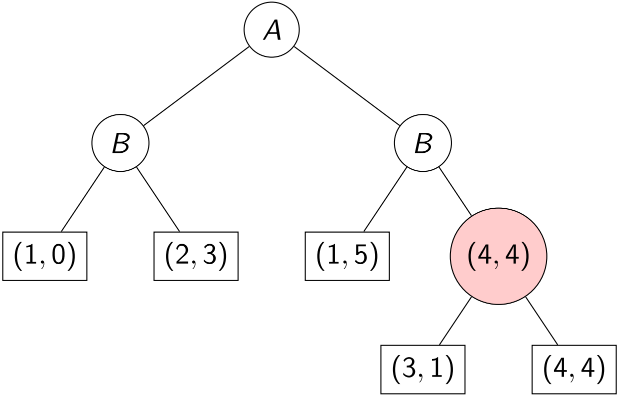
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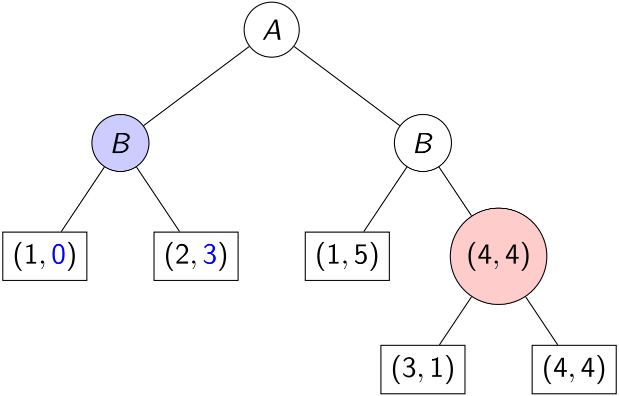
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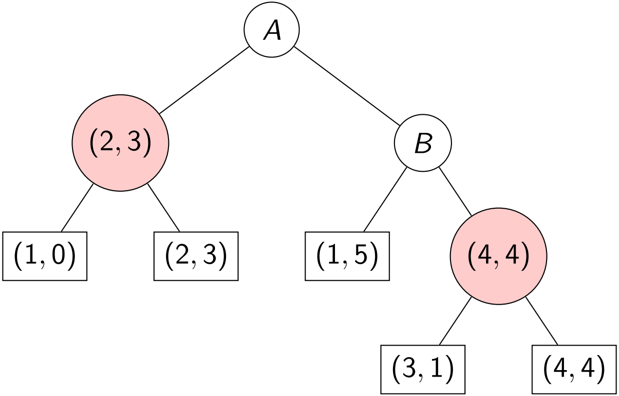
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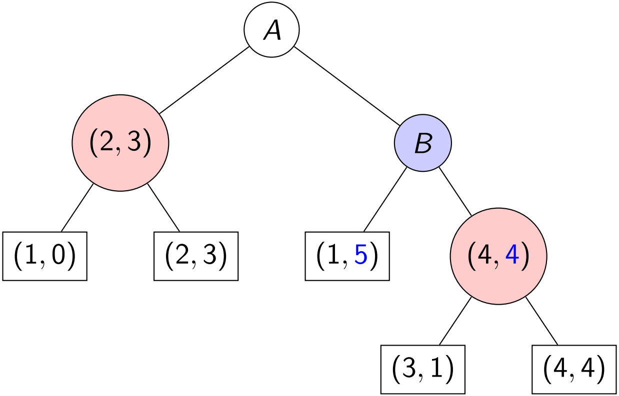
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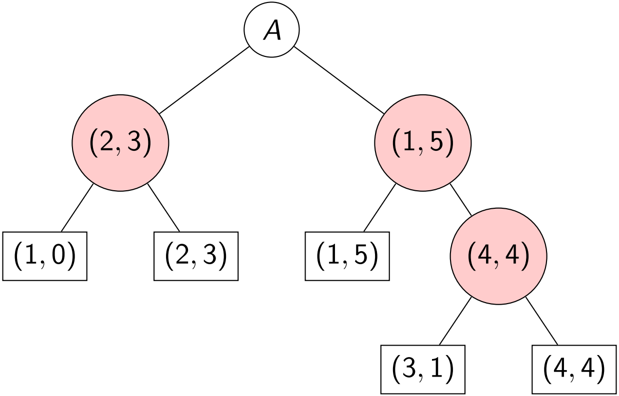
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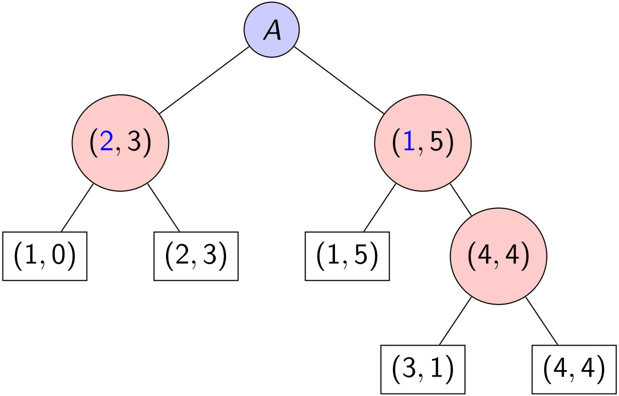
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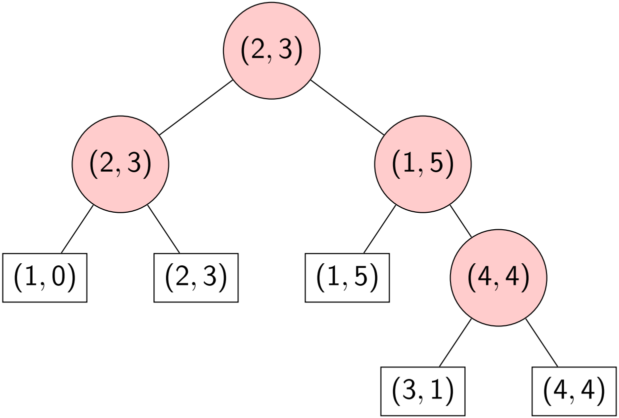
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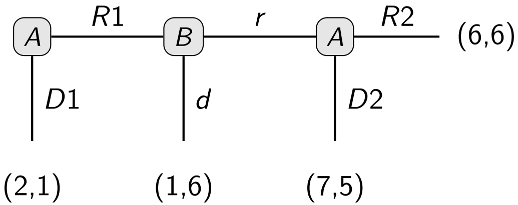

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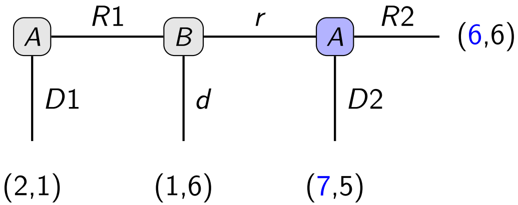

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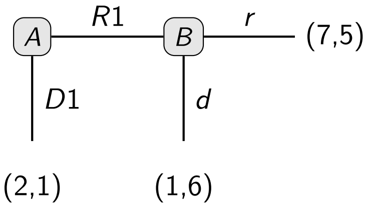

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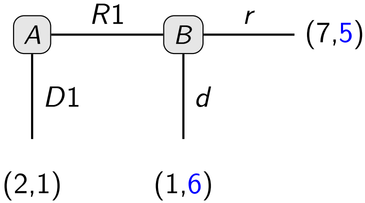

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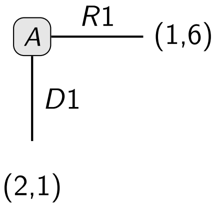

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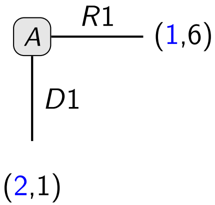

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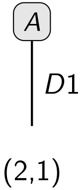

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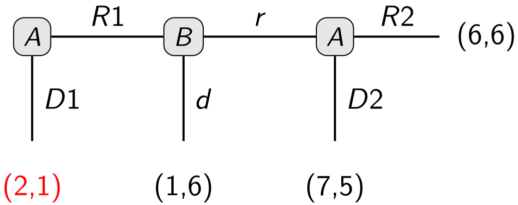

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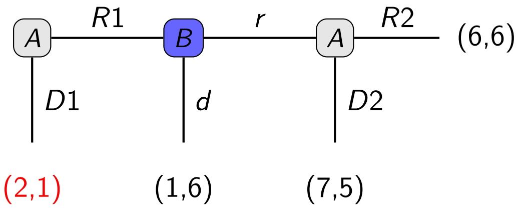

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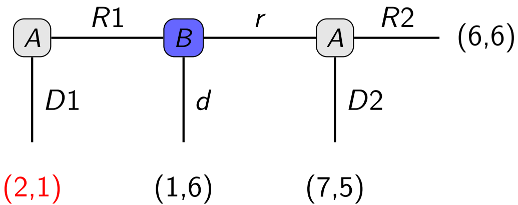

What should Bob thinks of Ann?

- Either she doesn't believe that he is rational and that he believes that she would choose $R2$.
- Or Ann made a "mistake" (= irrational move) at the first turn.

Either way, rationality is not "common knowledge".

R. Aumann. *Backwards induction and common knowledge of rationality*. Games and Economic Behavior, 8, pgs. 6 - 19, 1995.

R. Stalnaker. *Knowledge, belief and counterfactual reasoning in games*. Economics and Philosophy, 12, pgs. 133 - 163, 1996.

J. Halpern. *Substantive Rationality and Backward Induction*. Games and Economic Behavior, 37, pp. 425-435, 1998.

Models of Extensive Games

Let Γ be a *non-degenerate* extensive game with perfect information. Let Γ_i be the set of nodes controlled by player i.

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(A1) If $w \sim_i w'$ then $\sigma_i(w) = \sigma_i(w')$.

Rationality

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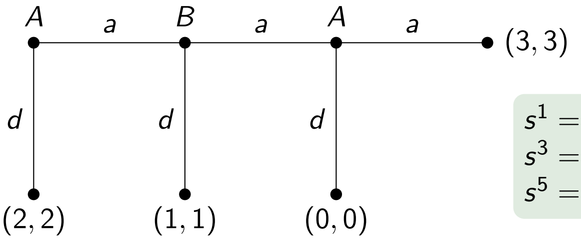
i **is rational at** v **in** w provided for all strategies $s_i \neq \sigma_i(w)$, $h_i^v(\sigma(w')) \geq h_i^v((\sigma_{-i}(w'), s_i))$ for some $w' \in [w]_i$.

Substantive Rationality

i is **substantively rational** in state w if i is rational at a vertex v in w of every vertex in $v \in \Gamma_i$



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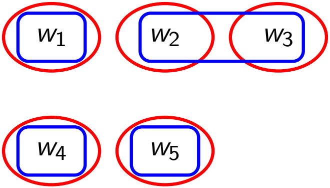

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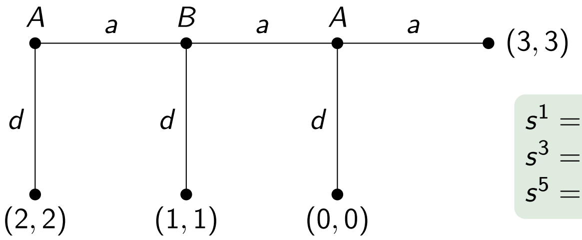
(F1) v is reached in $f(w, v)$ (i.e., v is on the path determined by $\sigma(f(w, v))$)

(F2) If v is reached in w, then $f(w, v) = w$

(F3) $\sigma(f(w, v))$ and $\sigma(w)$ agree on the subtree of Γ below v



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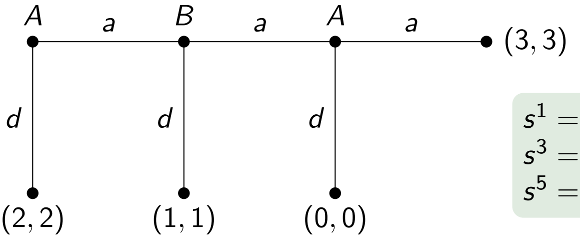
- $W = \{w_1, w_2, w_3, w_4, w_5\}$ with $\sigma(w_i) = s^i$
- $[w_i]_A = \{w_i\}$ for $i = 1, 2, 3, 4, 5$
- $[w_i]_B = \{w_i\}$ for $i = 1, 4, 5$ and $[w_2]_B = [w_3]_B = \{w_2, w_3\}$



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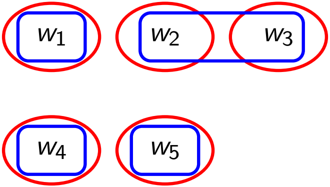

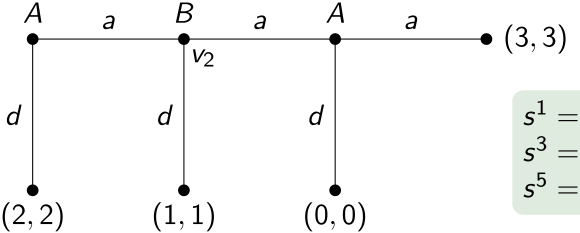

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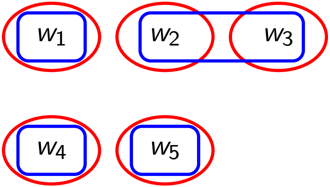
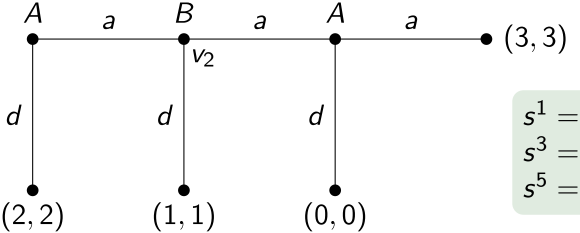


It is **common knowledge** at w_1 that if vertex v_2 were reached, Bob would play down.



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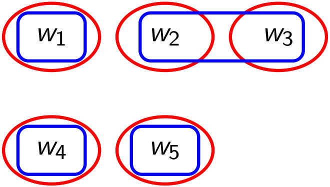
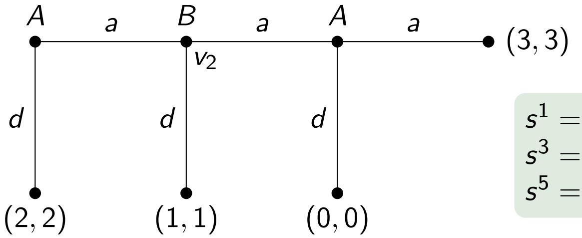


Bob is not rational at v_2 in w_1



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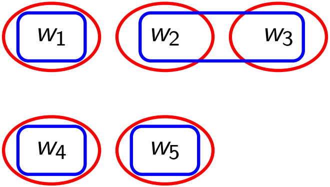
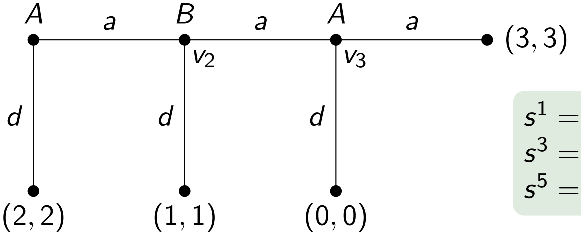


Bob is rational at v_2 in w_2



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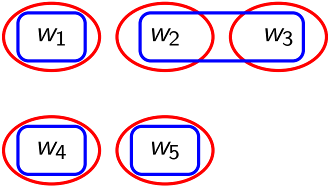

Note that $f(w_1, v_2) = w_2$ and $f(w_1, v_3) = w_4$, so there is common knowledge of S-rationality at w_1.

Aumann's Theorem: If Γ is a non-degenerate game of perfect information, then in all models of Γ, we have $C(A - Rat) \subseteq BI$

Stalnaker's Theorem: There exists a non-degenerate game Γ of perfect information and an extended model of Γ in which the selection function satisfies F1-F3 such that $C(S - Rat) \not\subseteq BI$.

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Revising beliefs during play:

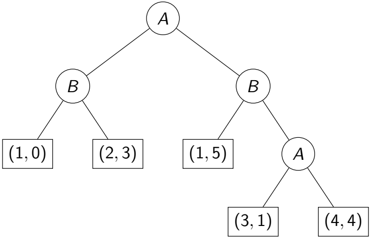
"Although it is common knowledge that Ann would play across if v_3 were reached, if Ann were to play across at v_1, Bob would consider it possible that Ann would play down at v_3"

F4. For all players i and vertices v, if $w' \in [f(w,v)]_i$ then there exists a state $w'' \in [w]_i$ such that $\sigma(w')$ and $\sigma(w'')$ agree on the subtree of Γ below v.

Theorem (Halpern). If Γ is a non-degenerate game of perfect information, then for every extended model of Γ in which the selection function satisfies F1-F4, we have $C(S-Rat) \subseteq BI$. Moreover, there is an extend model of Γ in which the selection function satisfies F1-F4.

J. Halpern. *Substantive Rationality and Backward Induction*. Games and Economic Behavior, 37, pp. 425-435, 1998.

Proof of Halpern's Theorem



- Suppose $w \in C(S - Rat)$. We show by induction on k that for all w' reachable from w by a finite path along the union of the relations $\sim_i$, if v is at most k moves away from a leaf, then $\sigma_i(w)$ is i's backward induction move at w'.

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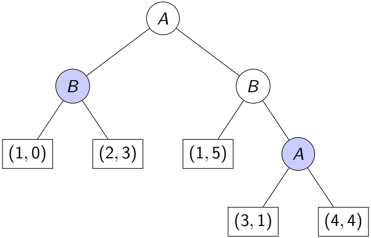

- Base case: we are at most 1 move away from a leaf. Suppose $w \in C(S - Rat)$. Take any w' reachable from w.

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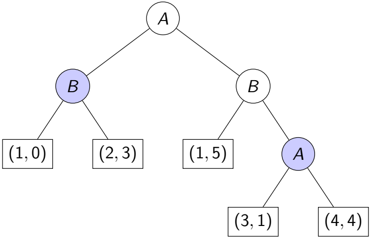

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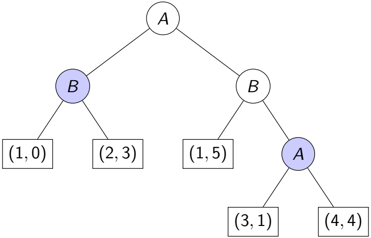

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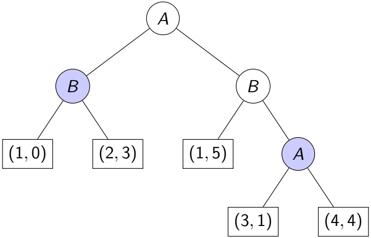

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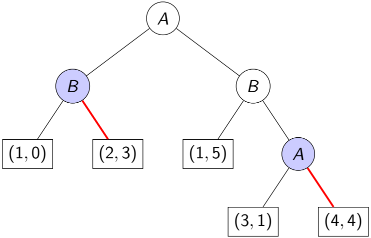

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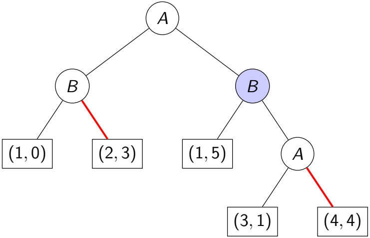

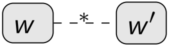

- Suppose $w \in C(S - Rat)$. Take any w' reachable from w. Assume, towards contradiction, that $\sigma(w)_i(v) = a$ is not the BI move for player i.

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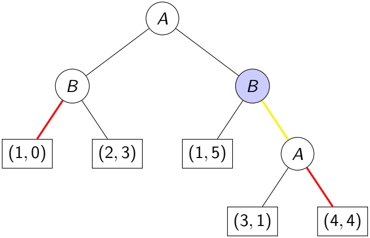
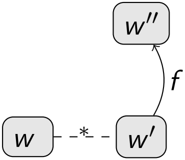
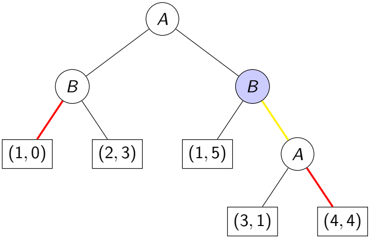
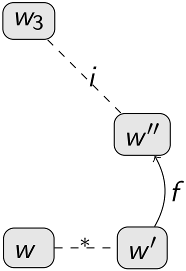




- Induction step. Suppose $w \in C(S - Rat)$. Take any w' reachable from w. Assume, towards contradiction, that $\sigma(w)_i(v) = a$ is not the BI move for player i. Since w is also in $C(S - Rat)$, we know by definition i must be rational at $w'' = f(w', v)$. But then, by F3 and our IH, all players play according to the BI solution after v at w''.

Proof of Halpern's Theorem



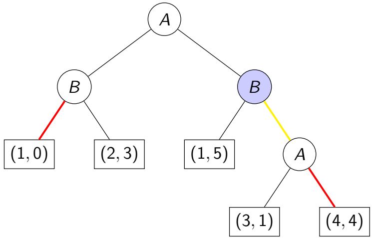


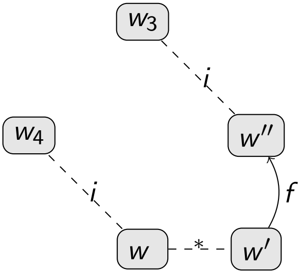
- i's rationality at w'' means, in particular, that there is a $w_3 \in [w'']_i$ such that

$$h_i^v(\sigma_i(w''), \sigma_{-i}(w_3)) \geq h_i^v((bi_i, \sigma_{-i}(w_3)))$$

for bi_i i's backward induction strategy.

Proof of Halpern's Theorem





- But then by F4 there must exists $w_4 \in [w]_i$ such that $\sigma(w_4)$ $\sigma(w_3)$ at the same in the sub-tree starting at v.

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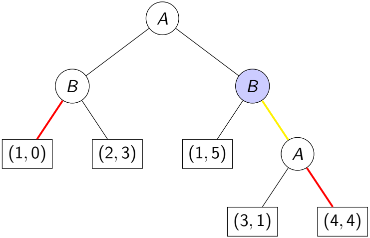

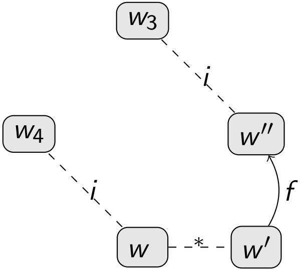

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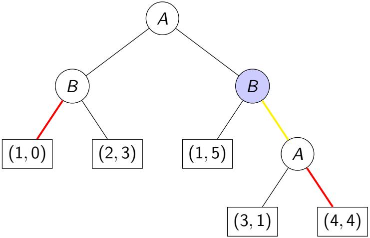

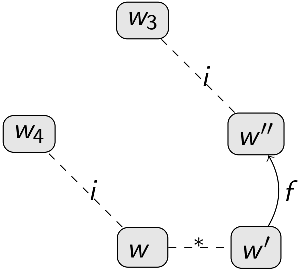

- But then by F4 there must exists $w_4 \in [w]_i$ such that $\sigma(w_4)$ $\sigma(w_3)$ at the same in the sub-tree starting at v. Since w_4 is reachable from w, in that state all players play according to the backward induction after v, and so this is also true of w_3. But then since the game is non-degenerate, playing something else than bi_i must make i strictly worst off at that state, a contradiction.

Some remarks

> *Aumann has proved that common knowledge of substantive rationality implies the backward induction solution in games of perfect information.*
>
> *Joseph Halpern*

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Aumann's theorem is a special case of Halpern's, where the converse of (F4) also holds. Beliefs (in fact, knowledge) are fixed.

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Stalnaker's theorem, as we saw, uses a more liberal belief revision policy.

Belief revision is key in extensive games. You might observe things you didn't expect, revise your beliefs on that, and make your decision for the next move.

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Are there, then, epistemic conditions using more liberal belief revision policies that still imply BI?

Yes. We just saw one... But by now dominant view on epistemic conditions for BI is:

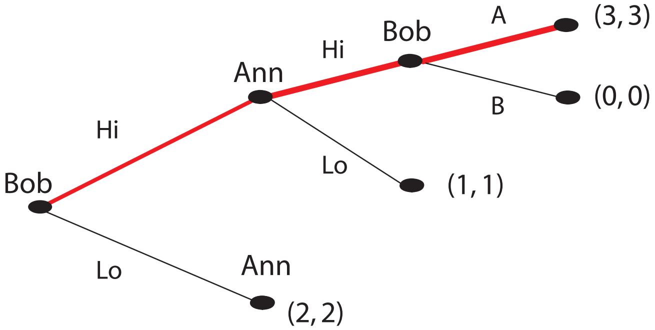
- Rationality and common strong belief in rationality implies BI.

Strong belief in rationality := a belief that you keep as long as you don't receive information that contradicts it.

Battigalli, P. and Siniscalchi, M. "Strong belief and forward induction reasoning". *Journal of Economic Theory*. 106(2), 2002.

Keep 'hoping' for rationality: a solution to the backward induction paradox. *Synthese*. 169(2), 2009.

From backward induction to weak dominance in the matrix



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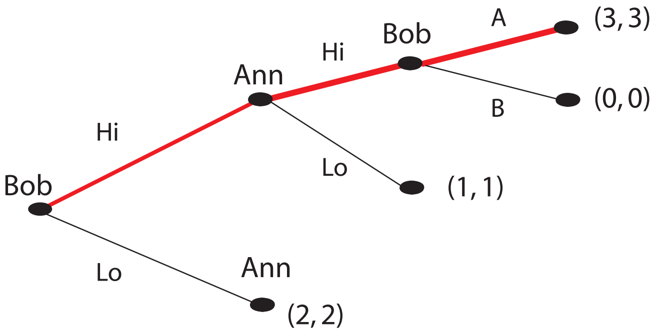
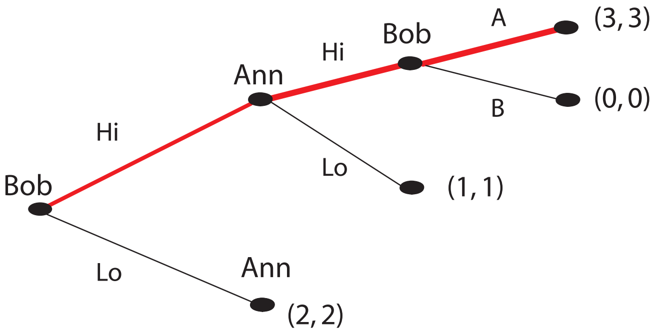


	Hi, A	Hi, B	Lo, A	Lo, B
Hi	3, 3	0, 0	2, 2	2, 2
Lo	1, 1	1, 1	2, 2	2, 2

From backward induction to weak dominance in the matrix



	Hi, A	Hi, B	Lo, A	Lo, B
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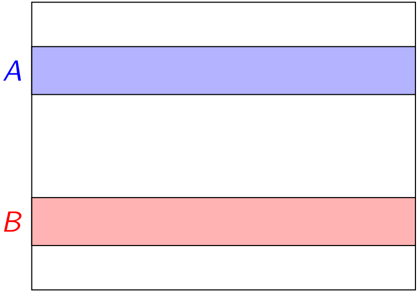
Weak Dominance



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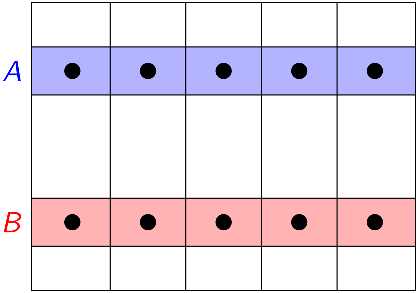

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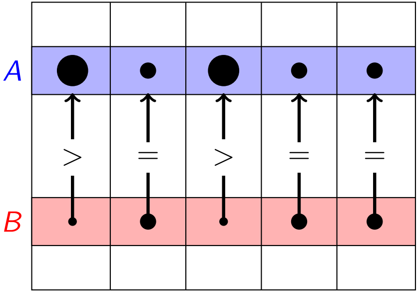

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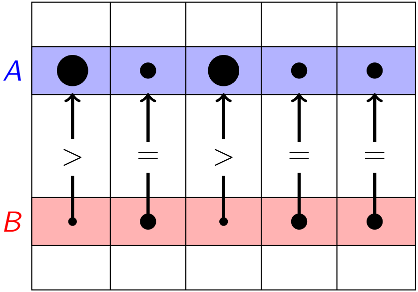

- All strictly dominated strategies are weakly dominated.

Weak Dominance

Suppose that $G = \langle N, \{S_i\}_{i\in N}, \{u_i\}_{i\in N}\rangle$ is a strategic game. A strategy $s_i \in S_i$ is weakly dominated (possibly by a mixed strategy) with respect to $X \subseteq S_{-i}$ iff there is **no full support probability measure** $p \in \Delta^{>0}(X)$ such that s_i is a best response with respect to p.

Strategic Reasoning and Admissibility

	L	R
U	1,1	0,1
D	0,2	1,0

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Strategic Reasoning and Admissibility

	L	R
U	1,1	0,1
D	0,2	1,0

Suppose rationality incorporates *admissibility* (or *cautiousness*).

1. Both Row and Column should use a *full-support* probability measure
2. But, if Row thinks that Column is **rational** then should she not assign probability 1 to L?

Strategic Reasoning and Admissibility

"The argument for deletion of a weakly dominated strategy for player i is that he contemplates the possibility that every strategy combination of his rivals occurs with positive probability. However, this hypothesis clashes with the logic of iterated deletion, which assumes, precisely, that eliminated strategies are not expected to occur."

Mas-Colell, Whinston and Green. *Introduction to Microeconomics*. 1995.

Strategic Reasoning and Admissibility

The condition that the players incorporate admissibility *into their rationality calculations seems to conflict with the condition that the players think the other players are rational (there is a tension between admissibility and strategic reasoning)*

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Does assuming that it is commonly known that players play only admissible strategies lead to a process of iterated removal of weakly dominated strategies?

Strategic Reasoning and Admissibility

The condition that the players incorporate admissibility *into their rationality calculations seems to conflict with the condition that the players think the other players are rational (there is a tension between admissibility and strategic reasoning)*

Does assuming that it is commonly known that players play only admissible strategies lead to a process of iterated removal of weakly dominated strategies? No!

L. Samuelson. *Dominated Strategies and Common Knowledge*. Games and Economic Behavior (1992).

Iterated Admissibility

		Bob L	Bob R
Ann	T	1,1	1,0
Ann	B	1,0	0,1

Iterated Admissibility

		Bob L	Bob R
Ann	T	1,1	1,0
Ann	B	1,0	0,1

T weakly dominates B

Iterated Admissibility

		Bob	
		L	R
Ann	T	1,1	1,0
	B	1,0	0,1

Then L strictly dominates R.

Iterated Admissibility

		Bob	
		L	R
Ann	T	1,1	1,0
	B	1,0	0,1

The IA set

Iterated Admissibility

		Bob	
		L	R
Ann	T	1,1	1,0
	B	1,0	0,1

But, now what is the reason for not playing B?

Iterated Admissibility

		Bob L	Bob R
Ann	T	1,1	1,0
Ann	B	1,0	0,1

Theorem (Samuelson). There is no model of this game satisfying common knowledge of rationality (where "rationality" incorporates admissibility)

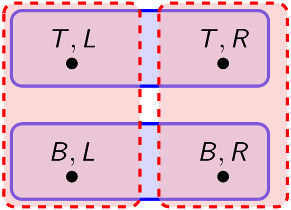
Common Knowledge of Admissibility

		Bob L	Bob R
Ann	T	1,1	1,0
Ann	B	1,0	0,1

There is no model of this game with *common knowledge* of admissibility.

Common Knowledge of Admissibility

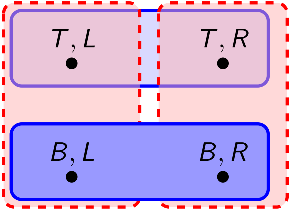
		Bob	
		L	R
Ann	T	1,1	1,0
	B	1,0	0,1



The "full" model of the game

Common Knowledge of Admissibility

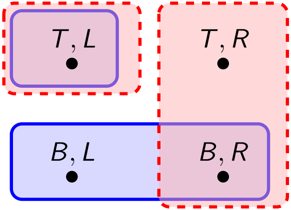
Ann \ Bob	L	R
T	1,1	1,0
B	1,0	0,1



The "full" model of the game: *B is not admissible given Ann's information*

Common Knowledge of Admissibility

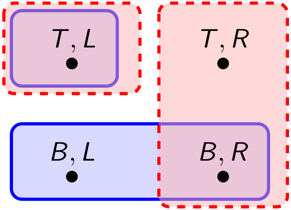
		Bob	
		L	R
Ann	T	1,1	1,0
	B	1,0	0,1



What is wrong with this model?

Common Knowledge of Admissibility

		Bob L	R
Ann	T	1,1	1,0
	B	1,0	0,1

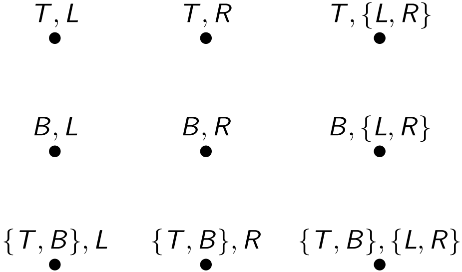


Privacy of Tie-Breaking/No Extraneous Beliefs: If a strategy is *rational* for an opponent, then it cannot be "ruled out".

Common Knowledge of Admissibility

Bob

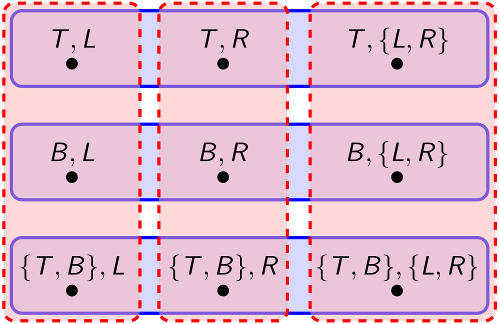
Ann		L	R
	T	1,1	1,0
	B	1,0	0,1



Moving to choice *sets*.

Common Knowledge of Admissibility

Ann \ Bob	L	R
T	1,1	1,0
B	1,0	0,1



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T	1,1	1,0
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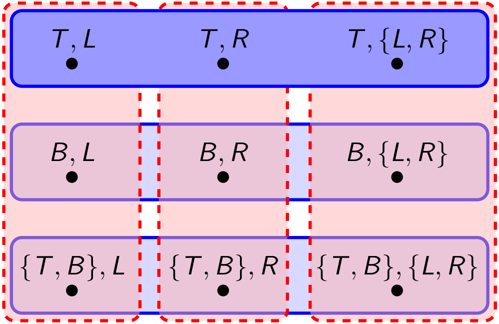
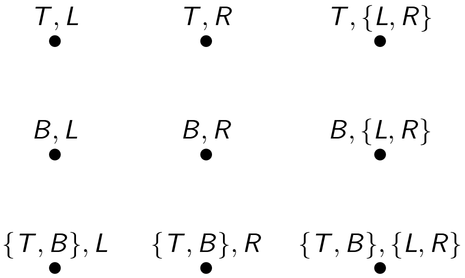


Ann thinks: Bob *has a reason to play L* OR Bob *has a reason to play R* OR Bob *has not yet settled on a choice*

Common Knowledge of Admissibility

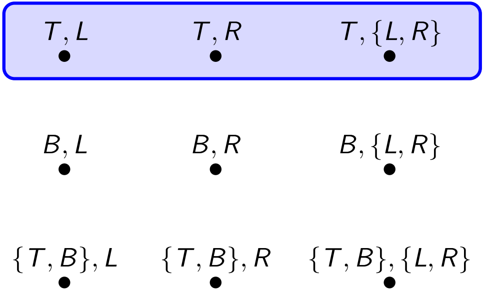
		Bob: L	Bob: R
Ann	T	1,1	1,0
	B	1,0	0,1



Still there is no model with common knowledge that players have *admissibility-based reasons*

Common Knowledge of Admissibility

Ann \ Bob	L	R
T	1,1	1,0
B	1,0	0,1

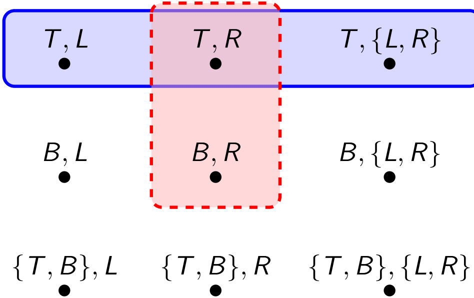


there is a reason to play T provided Ann considers it possible that Bob might play R (actually three cases to consider here)

Common Knowledge of Admissibility

Bob

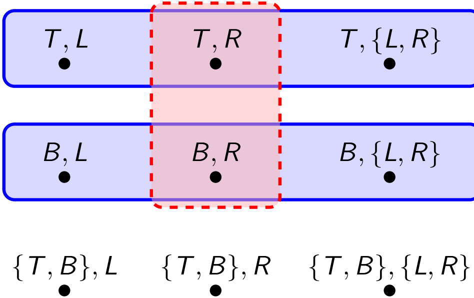
Ann		L	R
	T	1,1	1,0
	B	1,0	0,1



But there is a reason to play R provided it is possible that Ann has a reason to play B

Common Knowledge of Admissibility

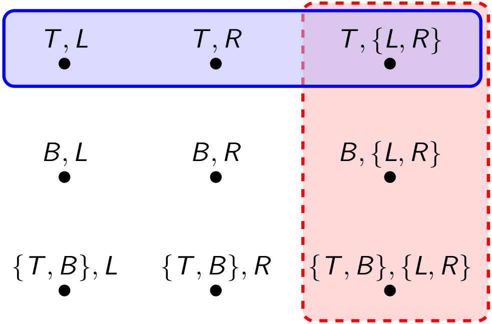
		Bob	
		L	R
Ann	T	1,1	1,0
	B	1,0	0,1



But, there is no reason to play B if there is a reason for Bob to play R.

Common Knowledge of Admissibility

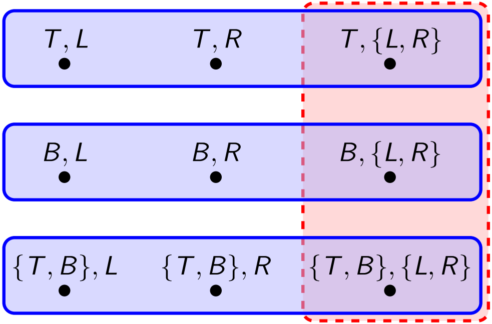
Ann \ Bob	L	R
T	1,1	1,0
B	1,0	0,1



R can be ruled out unless there is a possibility that B will be played.

Common Knowledge of Admissibility

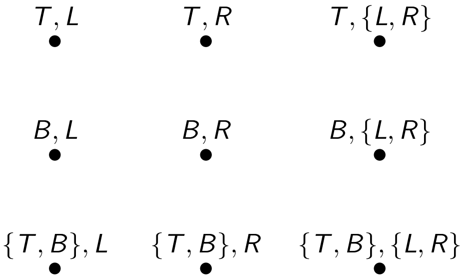
		Bob	
		L	R
Ann	T	1,1	1,0
	B	1,0	0,1



there is no reason to play B if R is a *possible play for Bob*.

Common Knowledge of Admissibility

Ann \ Bob	L	R
T	1,1	1,0
B	1,0	0,1



We can check all the possibilities and see we cannot find a model...

Both Including and Excluding a Strategy

One solution is to assume that players consider some strategies *infinitely more likely than other strategies*.

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Lexicographic Probability System: a sequence of probability distributions each infinitely more likely than the next.

	1 L	[1] R
U	1,1	0,1
D	0,2	1,0

A. Brandenburger, A. Friedenberg, H. J. Keisler. *Admissibility in Games*. Econometrica (2008).

LPS: $(\mu_0, \mu_1, \ldots, \mu_{n-1})$ (each μ_i is a probability measure with disjoint supports)

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(s_i, t_i) is **rational** provided (i) s_i lexicographically maximizes i's expected payoff under the LPS associated with t_i, **and** (ii) the LPS associated with t_i has full support.

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A player **assumes** E provided she considers E infinitely more likely than *not*-E.

The key notion is **rationality and common assumption of rationality** (RCAR).

But, there's more...

"Under admissibility, Ann considers everything possible. But this is only a decision-theoretic statement. Ann is in a game, so we imagine she asks herself: "What about Bob? What does he consider possible?" If Ann truly considers everything possible, then it seems she should, in particular, allow for the possibility that Bob does not! Alternatively put, it seems that a full analysis of the admissibility requirement should include the idea that other players do not conform to the requirement." (pg. 313)

A. Brandenburger, A. Friedenberg, H. J. Keisler. *Admissibility in Games*. Econometrica (2008).

Irrationality

		2		
		l	c	r
1	t	4,0	4,1	0,1
	m	0,0	0,1	4,1
	m	3,0	2,1	2,1

Irrationality

		2		
		l	c	r
1	t	4,0	4,1	0,1
	m	0,0	0,1	4,1
	m	3,0	2,1	2,1

- The IA set

Irrationality

		2		
		l	c	r
1	t	4,0	4,1	0,1
	m	0,0	0,1	4,1
	m	3,0	2,1	2,1

- All (L, b_i) are irrational, (C, b_i), (R, b_i) are rational if b_i has full support, irrational otherwise
- D is optimal then either $\mu(C) = \mu(R) = \frac{1}{2}$ or μ assigns positive probability to both L and R.

Irrationality

		2		
		l	c	r
1	t	4,0	4,1	0,1
	m	0,0	0,1	4,1
	m	3,0	2,1	2,1

- Fix a rational (D, a) where a assumes that Bob is rational. $(a \mapsto (\mu_0, \ldots, \mu_{n-1})$
- Let μ_i be the first measure assigning nonzero probability to $\{L\} \times T_B$ ($i \neq 0$ since a assumes Bob is rational).

Irrationality

		2		
		l	c	r
1	t	4,0	4,1	0,1
	m	0,0	0,1	4,1
	m	3,0	2,1	2,1

- Let μ_i be the first measure assigning nonzero probability to $\{L\} \times T_B$ $(i \neq 0)$.
- for each μ_k with $k < i$: (i) μ_k assigns probability $\frac{1}{2}$ to $\{C\} \times T_B$ and $\frac{1}{2}$ to $\{R\} \times T_B$; and (ii) U, M, D are each optimal under μ_k.

Irrationality

		2		
		l	c	r
1	t	4,0	4,1	0,1
	m	0,0	0,1	4,1
	m	3,0	2,1	2,1

- for each μ_k with $k < i$: (i) μ_k assigns probability $\frac{1}{2}$ to $\{C\} \times T_B$ and $\frac{1}{2}$ to $\{R\} \times T_B$; and (ii) U, M, D are each optimal under μ_k.
- D must be optimal under μ_i and soμ_i assigns positive probability to both $\{L\} \times T_B$ and $\{R\} \times T_B$.

Irrationality

		2		
		l	c	r
1	t	4,0	4,1	0,1
	m	0,0	0,1	4,1
	m	3,0	2,1	2,1

- D must be optimal under μ_i and soμ_i assigns positive probability to both $\{L\} \times T_B$ and $\{R\} \times T_B$.
- Rational strategy-type pairs are each infinitely more likely that irrational strategy-type pairs. Since, each point in $\{L\} \times T_B$ is irrational, μ_i must assign positive probability to irrational pairs in $\{R\} \times T_B$.

Irrationality

		2		
		l	c	r
1	t	4,0	4,1	0,1
	m	0,0	0,1	4,1
	m	3,0	2,1	2,1

- μ_i must assign positive probability to irrational pairs in $\{R\} \times T_B$.
- *This can only happen if there are types of Bob that do not consider everything possible.*