PHIL 308S: Voting Theory and Fair Division

Lecture 9

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What's Next?

- "Epistemic Democracy" (Condorcet Jury Theorem)
- Voting on issues, judgement aggregation, and Sen's Liberal Paradox
- Strategizing
- Distance-based rationalizations of voting methods
- Changing the basic framework: Approval Voting, Voting by "grading"

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Fair Division

Proceduralist Justifications

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"identifies a set of ideals with which any collective decision-making procedure ought to comply. [A] process of collective decision making would be more or less justifiable depending on the extent to which it satisfies them...What justifies a [collective] decision-making procedure is strictly a necessary property of the procedure—one entailed by the definition of the procedure alone."

J. Coleman and J. Ferejohn. *Democracy and social choice*. Ethics, 97(1): 6-25, 1986..

Epistemic Justifications

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"An epistemic interpretation of voting has three main elements: (1) an independent standard of correct decisions that is, an account of justice or of the common good that is independent of current consensus and the outcome of votes; (2) a cognitive account of voting that is, the view that voting expresses beliefs about what the correct policies are according to the independent standard, not personal preferences for policies; and (3) an account of decision making as a process of the adjustment of beliefs, adjustments that are undertaken in part in light of the evidence about the correct answer that is provided by the beliefs of others. (p. 34) "

J. Cohen. *An epistemic conception of democracy*. Ethics, 97(1): 26-38, 1986.

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Why should we buy the idea, though, that there really is such a thing as an objectively "best" choice? Aren't values relative, and isn't the point of voting to strike a balance between conflicting opinions, not to determine a correct one?"

H. P. Young. *Optimal Voting Rules*. The Journal of Economic Perspectives, 9:1, pgs. 51 - 64, 1995.

"...in many situations , differences of opinion arise from differences in values, not erroneous judgments. In this case it seems better to adopt the view that group choice is an exercise in finding a compromise between conflicting opinions." (Young, p. 60)

Suppose there are equally skilled individuals, each with a probability $p > 1/2$ of "choosing correctly".

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Let F be a decision method. $\pi(F, \mathbf{p})$ is the probability of getting the answer correct, given the skills of each individual $\mathbf{p}$.

Expert rule

$$\pi(F^e, \mathbf{p}) = p$$

Majority Rule

$$\pi(F^m, \mathbf{p})$$

Majority Rule

$$\pi(F^m, \mathbf{p}) = p^3$$

The probability everyone is correct is p^3

Majority Rule

$$\pi(F^m, \mathbf{p}) = p^3 + 3p^2(1-p)$$

The probability everyone is correct is p^3

The probability that 1 and 2 are correct: $p^2(1-p)$

The probability that 2 and 3 are correct: $p^2(1-p)$

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Theorem. Assume $p_1 \geq p_2 > p_3 > 1/2$, then the simple majority rule is preferred to the expert rule.

Consider 3 votes, each with a confidence level $p = 2/3$.

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The probability of at least m voters being correct is:

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$$= 3 * 4/27 + 1 * 8/27$$

$$= 20/27$$

Condorcet Jury Theorem

State of the world $\mathbf{x}$ takes values 0 and 1

R_i is the event that voter i votes correctly.

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Condorcet Jury Theorem. Suppose Independence and Competence. As the group size increases, the probability $Pr(M_n)$ that a majority votes correctly (i) increases and (ii) converges to one.

D. Austen-Smith and J. Banks. *Aggregation, Rationality and the Condorcet Jury Theorem*. The American Political Science Review, 90, 1, pgs. 34 - 45, 1996.

D. Estlund. *Opinion Leaders, Independence and Condorcet's Jury Theorem*. Theory and Decision, 36, pgs. 131 - 162, 1994.

F. Dietrich. *The premises of Condorcet's Jury Theorem are not simultaneously justified*. Episteme, 2008.

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Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

YYY	YYN	YNY	YNN	NYY	NYN	NNY	NNN
1	1	1	3	1	3	3	0

S. Brams, D. M. Kilgour, and W. Zwicker. *"The paradox of multiple elections"*. Social Choice and Welfare, 15(2): 211 - 236, 1998.

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Outcome by majority vote

Proposition 1: N (7 - 6)

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But there is no support for NNN!

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Anscombe's Paradox

	Issue 1	Issue 2	Issue 3
Voter 1	Yes	Yes	No
Voter 2	No	No	No
Voter 3	No	Yes	Yes
Voter 4	Yes	No	No
Voter 5	Yes	No	Yes
Majority	Yes	No	Yes

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Voters 4 & 5 support the outcome on a majority of issues

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Voters 1,2 & 3 do not support the outcome on a majority of issues

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Majority	Yes	No	Yes

Voters 4 & 5 support the outcome on a majority of issues
Voters 1,2 & 3 do not support the outcome on a majority of issues

A majority of voters do not support the majority outcome on a majority of issues.

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Judgement Aggregation

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Preference aggregations vs. *judgement aggregation*

- Judgements of preference, value judgements, beliefs

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- What should be done? What is the best alternative?

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- Judgements of preference, value judgements, beliefs
- What should be done? What is the best alternative?
- The Pareto conditions (see forthcoming work by W. Rabinowicz, S. Hartmann and S. Rafiee Rad)

Doctrinal Paradox

Suppose that three experts *independently* formed opinions about three propositions. For example,

1. p: "Carbon dioxide emissions are above the threshold x"
2. $p \rightarrow q$: "If carbon dioxide emissions are above the threshold x, then there will be global warming"
3. q: "There will be global warming"

Doctrinal Paradox

	p	$p \rightarrow q$	q
Expert 1			
Expert 2			
Expert 3			

Doctrinal Paradox

	p	$p \rightarrow q$	q
Expert 1	True	True	
Expert 2			
Expert 3			

Doctrinal Paradox

	p	$p \rightarrow q$	q
Expert 1	True	True	True
Expert 2			
Expert 3			

Doctrinal Paradox

	p	$p \to q$	q
Expert 1	True	True	True
Expert 2	True		False
Expert 3			

Doctrinal Paradox

	p	$p \rightarrow q$	q
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3			

Doctrinal Paradox

	p	$p \to q$	q
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Expert 2	True	False	False
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Doctrinal Paradox

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Expert 1	True	True	True
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Group			

Doctrinal Paradox

	p	$p \to q$	q
Expert 1	True	True	True
Expert 2	True	False	False
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Group	True		

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Expert 1	True	True	True
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The Logic of Group Decisions, II

(Kornhauser and Sager 1993)

p: a valid contract was in place

q: there was a breach of contract

r: the court is required to find the defendant liable.

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

The Logic of Group Decisions, II

(Kornhauser and Sager 1993)

Should we accept r?

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

The Logic of Group Decisions, II

(Kornhauser and Sager 1993)

Should we accept r? No, a simple majority votes no.

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

The Logic of Group Decisions, II

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Should we accept r? Yes, a majority votes yes for p and q and $(p \wedge q) \leftrightarrow r$ is a legal doctrine.

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

Many Variants!

See
`http://personal.lse.ac.uk/LIST/doctrinalparadox.htm`
for many generalizations!

Kornhauser and Sager. *Unpacking the court*. Yale Law Journal, 1986.

C. List and P. Pettit. *Aggregating Sets of Judgments: An Impossibility Result*. Economics and Philosophy 18: 89-110, 2002.

Sen's Liberal Paradox

Two members of a small society Lewd and Prude each have a personal copy of *Lady Chatterley's Lover*, consider

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l: Lewd reads the book;
p: Prude reads the book;
$l \to p$: If Lewd reads the book, then so does Prude.

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Lewd desires to read the book, and if he reads it, then so does Prude (Lewd enjoys the thought of Prude's moral outlook being corrupted)

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Prude desires to not read the book, and that Lewd not read it either, but in case Lewd does read the book, Prude wants to read the book to be informed about the dangerous material Lewd has read.

Sen's Liberal Paradox

	l	p	$l \rightarrow p$

Sen's Liberal Paradox

	l	p	$l \rightarrow p$
Lewd	True	True	True

Sen's Liberal Paradox

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Lewd	True	True	True
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1. Society assigns to each individual the liberal right to determine the collective desire on those propositions that concern only the individual's private sphere
   l is Lewd's case, p is Prude's case

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2. Unanimous desires of all individuals must be respected.

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2. Unanimous desires of all individuals must be respected.

So, society must be inconsistent!