PHIL 308S: Voting Theory and Fair Division

Lecture 7

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Arrow's Theorem

K. Arrow. *Social Choice & Individual Values*. 1951.

Also, see

J. Geanakoplos. *Three Brief Proofs of Arrow's Impossibility Theorem*. Economic Theory, **26**, 2005.

A. Taylor. *Social Choice and The Mathematics of Manipulation*. Cambridge University Press, 2005.

W. Gaertner. *A Primer in Social Choice Theory*. Oxford University Press, 2006.

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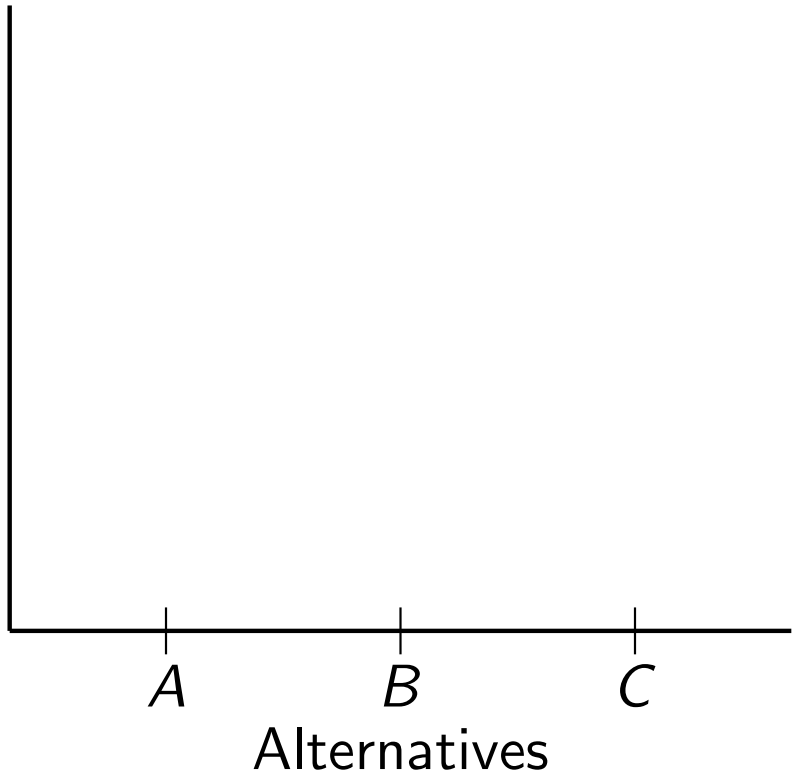
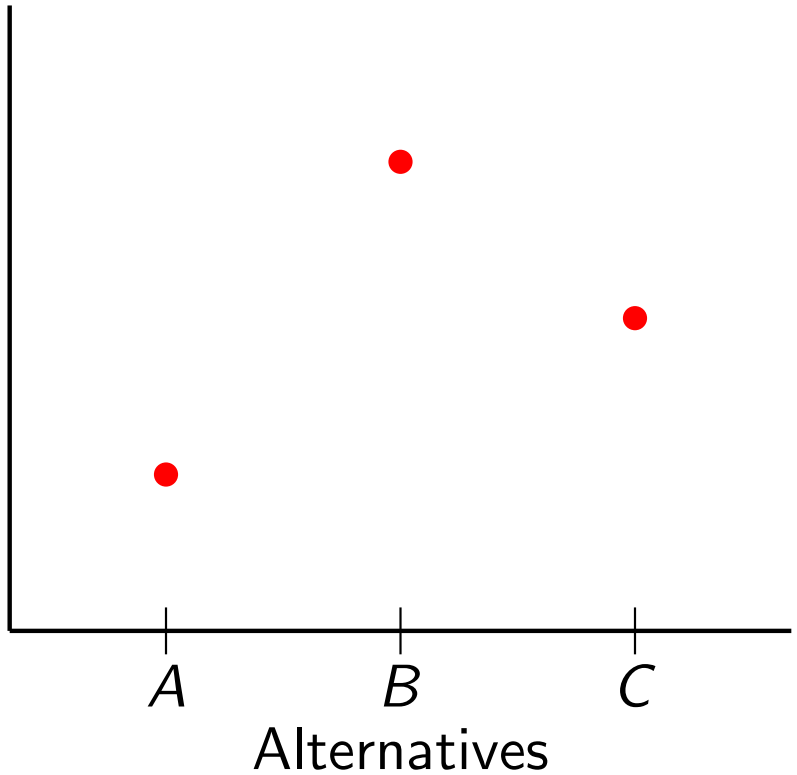
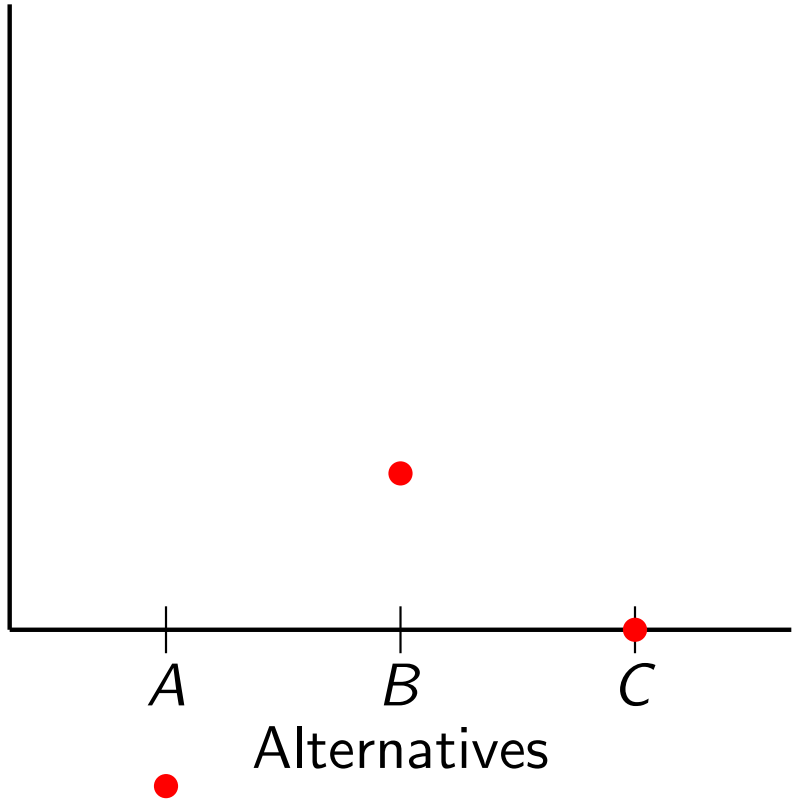
1	1	1
A	B	C
B	C	A
C	A	B


A
B
C
Alternatives



Plot B P_2 C P_2 A



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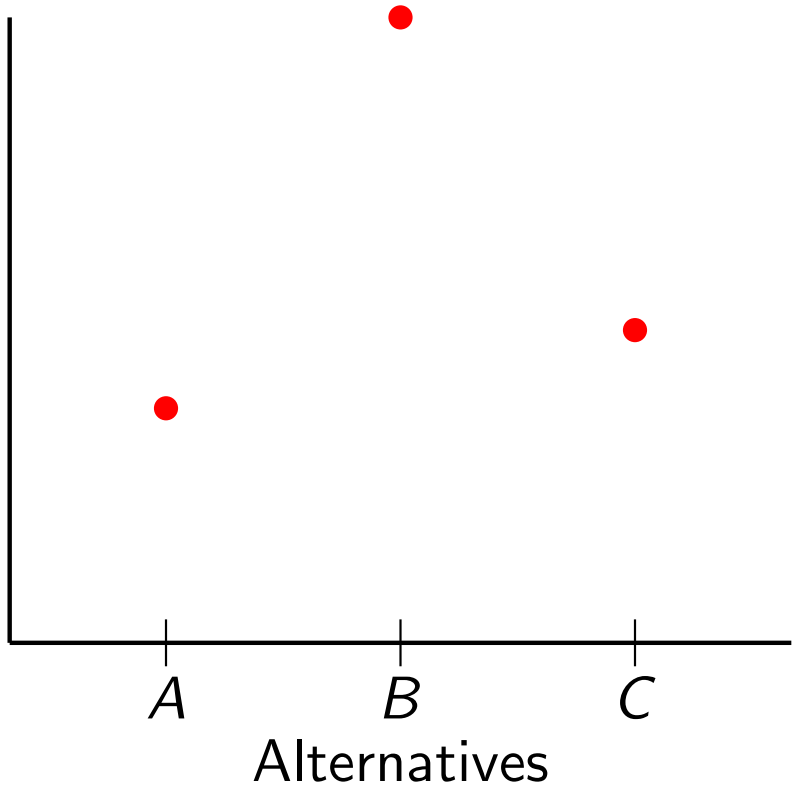

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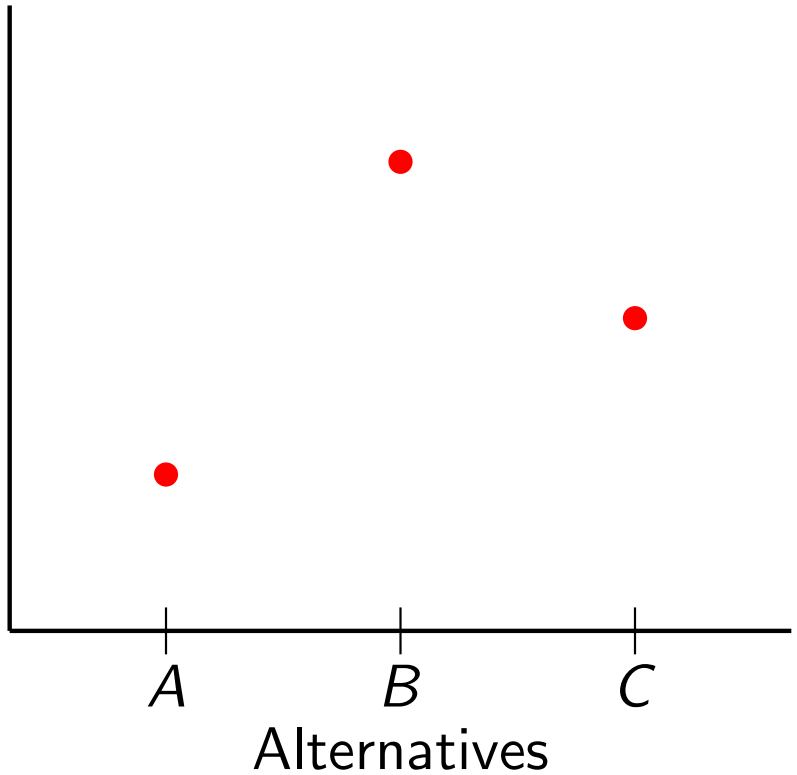

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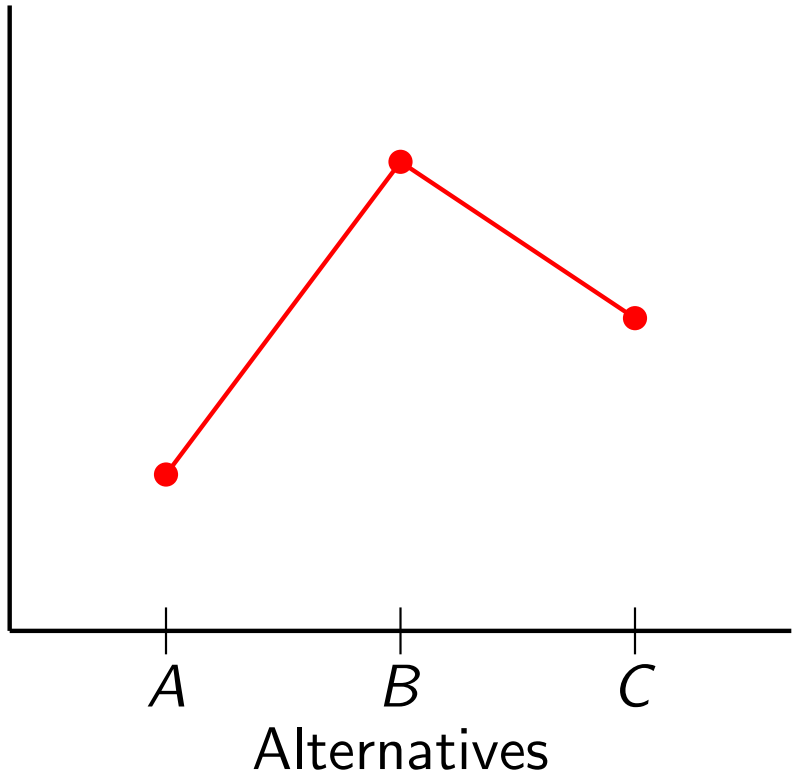

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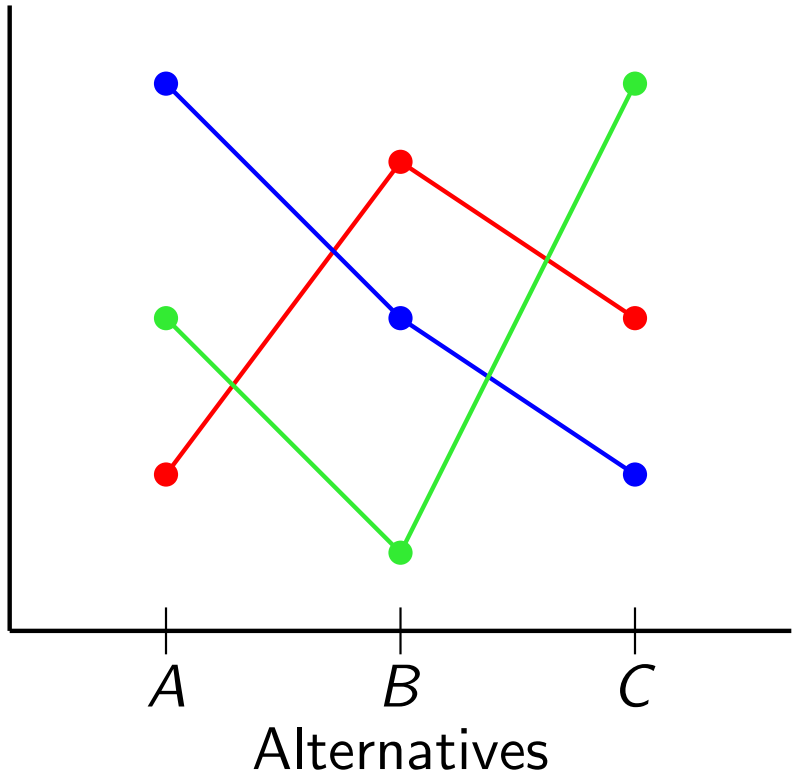
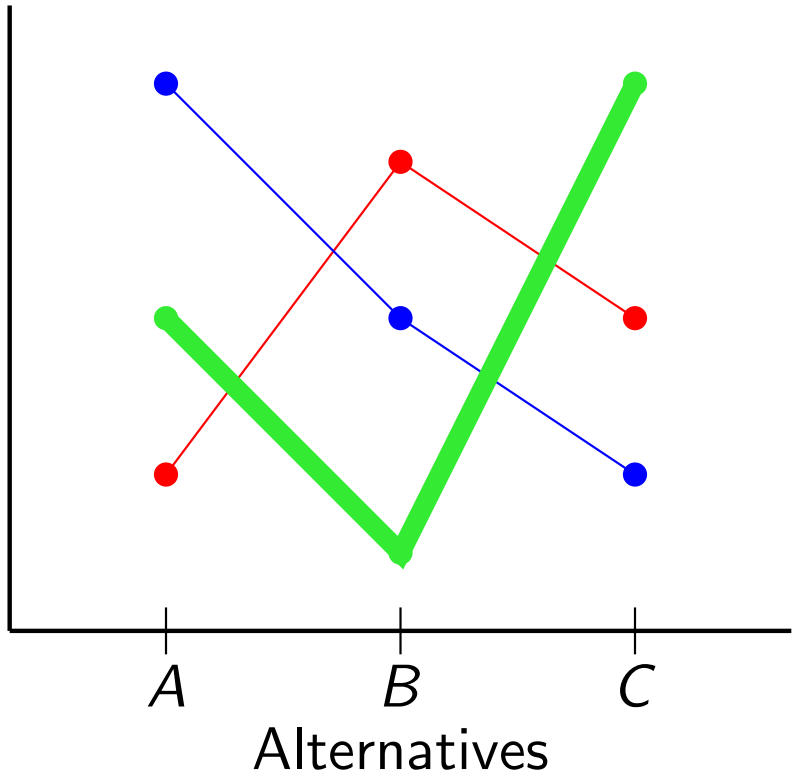


Plot A P_1 B P_1 C

Plot B P_2 C P_2 A

Plot C P_3 A P_3 B



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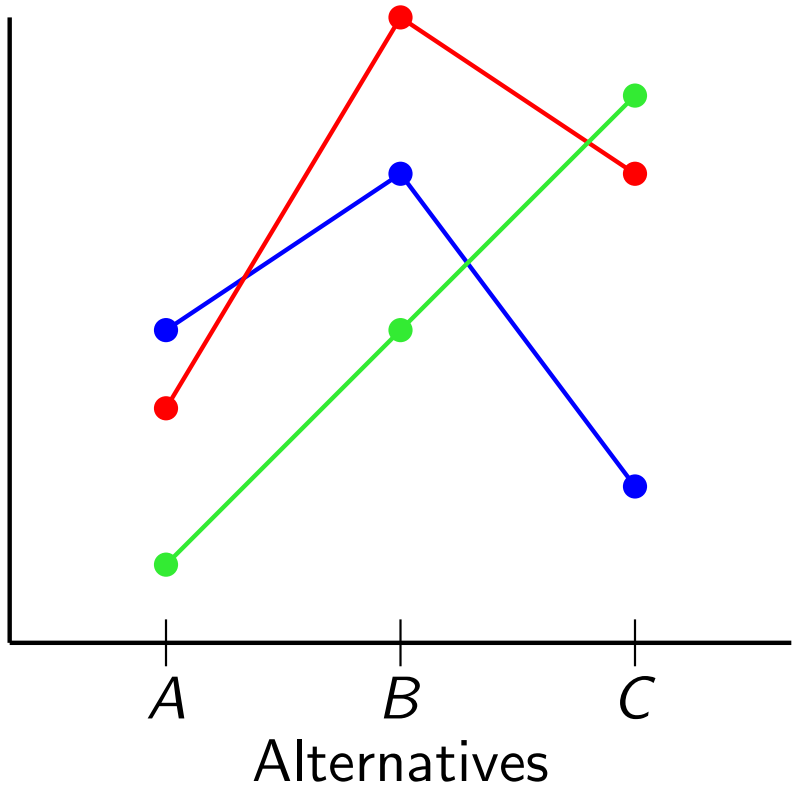

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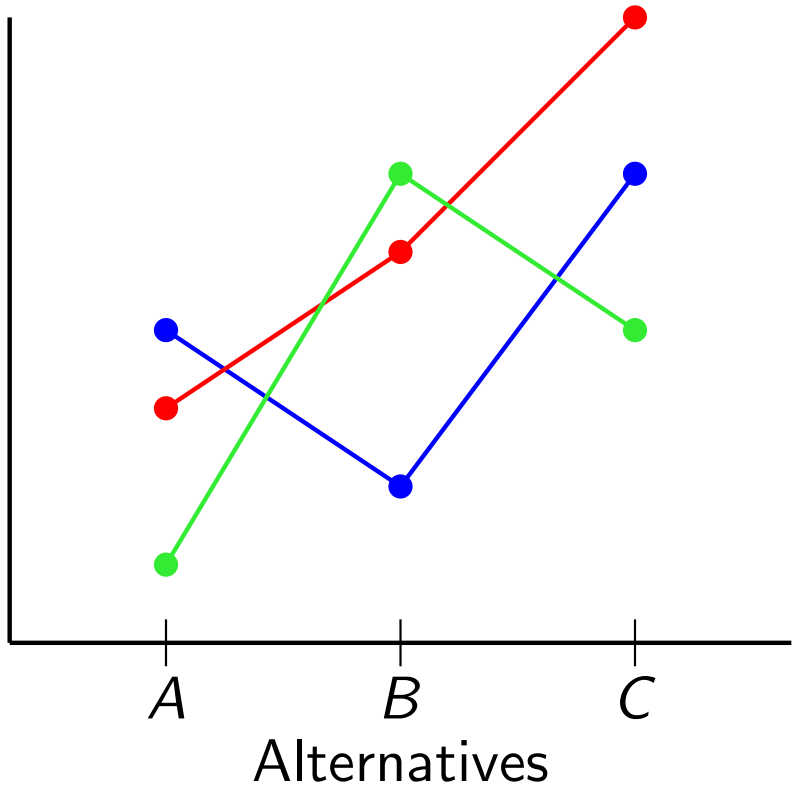

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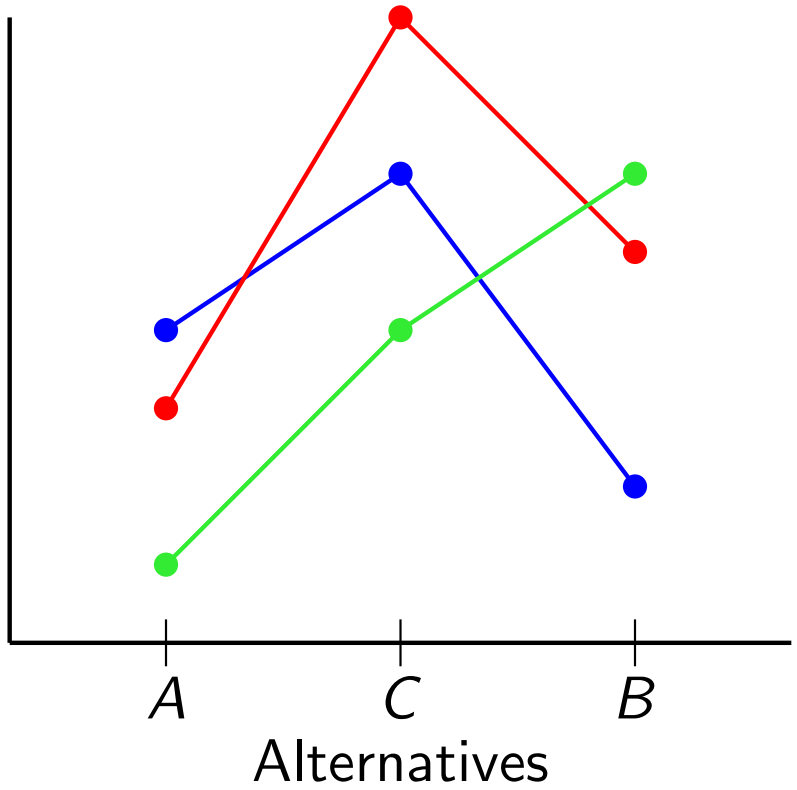

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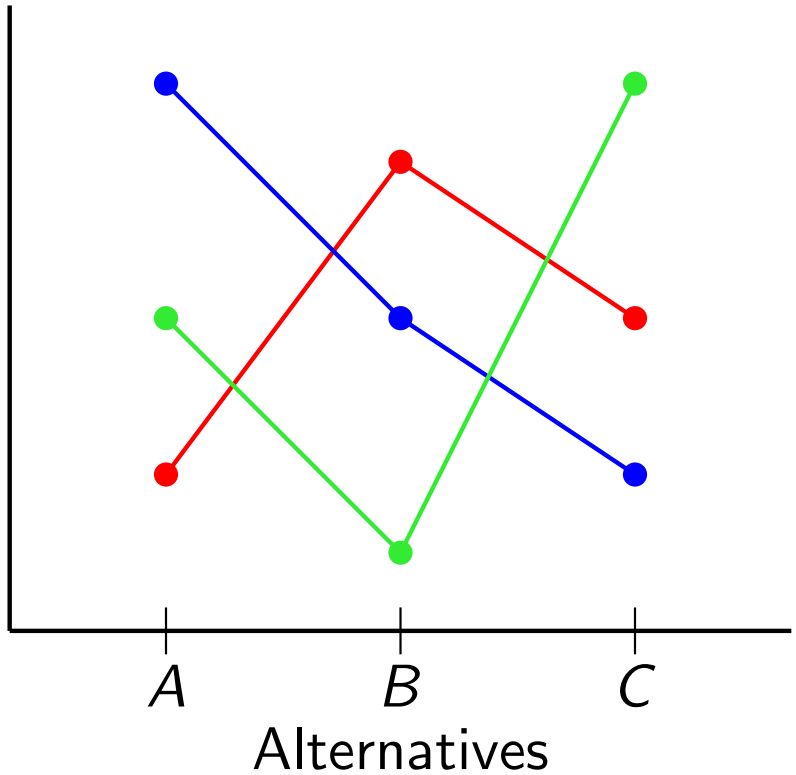

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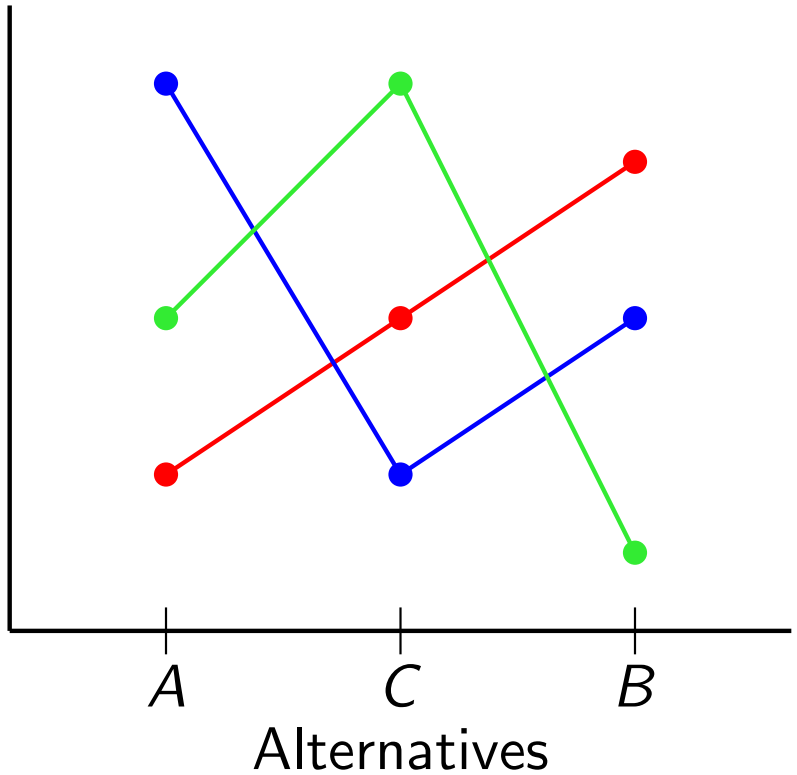

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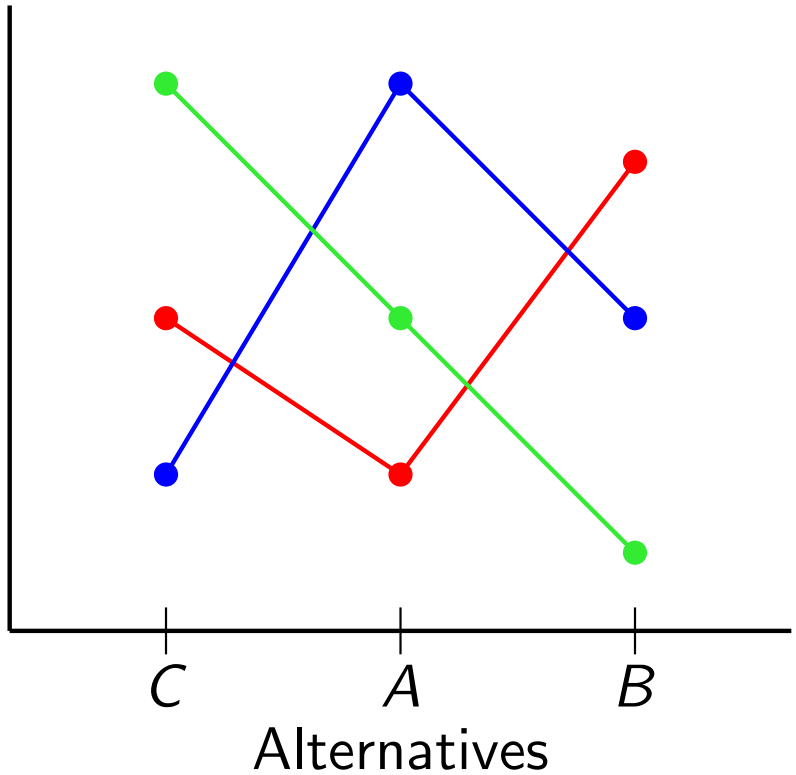

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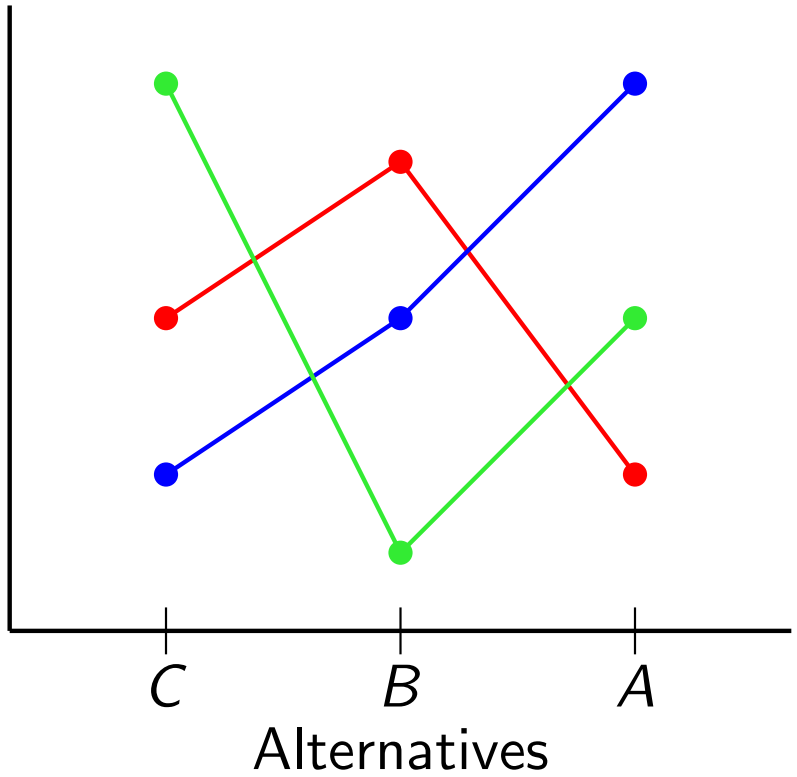

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D. Black. *On the rationale of group decision-making*. Journal of Political Economy, 56:1, pgs. 23 - 34, 1948.

Single-Peakedness: the preferences of group members are said to be single-peaked if the alternatives under consideration can be represented as points on a line and each of the utility functions representing preferences over these alternatives has a maximum at some point on the line and slopes away from this maximum on either side.

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Theorem. If there is an odd number of voters that display single-peaked preferences, then a Condorcet winner exists.