PHIL 308S: Voting Theory and Fair Division

Lecture 2

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The "rational choice" model

Chapter 2 of [Shepsle]

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Methodological individualism: The *individual* is our basic explanatory building block

- Individuals are *self-interested* (*not selfish*)
- Individuals are *instrumentally rational*: act in accord both with the decision maker's *preferences* and *beliefs*

Instrumental Rationality

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R. Nozick. *The Nature of Rationality*. Princeton University Press, 1993.

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We need to take the agent's beliefs into account

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What constraints should be placed on reasonable beliefs that underlie a rational choice?

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Deliberative Preferences: A person deliberates and (ideally) ranks all the possible "outcomes"

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3. xRy and yRx: The agent is *indifferent* between x and y
4. It is not the case that xRy and it is not the case that yRx: The agent *cannot compare* x and y

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xP_iy means "agent i strictly prefers x over y"

xI_iy means "agent i is indifferent between x and y

xR_iy iff xP_iy or xI_iy.

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Why satisfy these assumptions?

Why should we accept these axioms?

"Rather than trying to provide instrumental or pragmatic justifications for the axioms of ordinal utility, it is better...to see them as constitutive of our conception of a fully rational agent....those disposed to blatantly ignore transitivity are unintelligible to use: we can't understand their pattern of actions as sensible" (pg. 39)

Gerald Gaus. *On Philosophy, Politics and Economics*. Wadsworth Philosophical Topics, 2008.

Rationality is associated with both the capacity to order outcomes *and* to choose from the *top* of the order.

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"The formulation of maximizing behavior in economics has often parallels the modeling of maximization in physics an related disciplines. But maximizing *behavior* differs from nonvolitional maximization because of the fundamental relevance of the choice act, which has to be placed in a central position in analyzing maximizing behavior. A person's preferences over *comprehensive* outcomes (including the choice process) have to be distinguished form the conditional preferences over *culmination* outcomes *given* the act of choice." (pg. 745)

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To take another example, you may prefer mangoes to apples, but refuse to pick the last mango from a fruit basket, and yet be very pleased if someone else were to "force" that last mango on you. " (Sen, pg. 747)

Environmental Uncertainty and Beliefs

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Choice under

- *certainty*: highly confident about the relationship between actions and outcomes
- *risk*: clear sense of possibilities and their likelihoods
- *uncertainty*: the relationship between actions and outcomes is so imprecise that it is not possible to assign likelihoods

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What properties does this preference ordering have?

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4. Weak preference (and hence strict and indifference) is transitive

Utilities I

Fact. Suppose that X is finite and $\succeq$ is a complete and transitive ordering over X, then there is a utility function $u : X \to \mathfrak{R}$ that represents $\succeq$ ($x \succeq y$ iff $u(x) \geq u(y)$)

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Utility is *defined* in terms of preference (so it is an error to say that the agent prefers x to y *because* she assigns a higher utility to x than to y).

Utilities II

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Preference	u_1	u_2	u_3
x	3	10	1000
y	2	5	99
z	1	0	1

$x \succ y \succ z$ is represented by both $(3, 2, 1)$ and $(1000, 999, 1)$, so cannot say y is "closer" to x than to z.

The EU-Thesis

Expected Money/Value/Utility: Given an agent's beliefs and desires, the **expected utility** of an **action** leading to a set of outcomes *Out* is:

$$\sum_{o \in Out} [\text{how likely the act will lead to } o] \times [\text{how much the agent desires } o]$$

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1. principle of maximizing expected monetary value
2. principle of maximizing expected utility

Comments on Expected Utility

Options	$1/2$	$1/2$
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What numbers should we use in place of monetary value? (moral) value? personal utility?

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"Aristotle, Hobbes, Rousseau and other great political thinkers and philosophers have suggested that there is something special about politics —-that the collective choices for a nation, for example, are altogether different from choosing pizza toppings. Perhaps. Indeed, certainly this must bet the case. But the process of choosing *rationally* bears characterizing markings in all these contexts and so may be analyzed with the same intellectual framework."