PHIL 308S: Voting Theory and Fair Division

Lecture 17

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M. Balinski and R. Laraki. *Majority Judgement: Measuring, Ranking and Electing*. The MIT Press, 2010.

Voting Mechanism

- the specific form of the voters' and judges' *inputs*, the *messages* used to exert their wills, and
- the procedure by which the inputs or messages are amalgamated or transformed into a final decision, social choice, or *output*.

Grading

In many situations, people use measures or grades that are well defined **absolute common languages of evaluation** to define decision mechanisms:

- in figure skating (new system), diving and gymnastics competitions;
- in piano, ute and orchestra competitions;
- in classifying wines at wine competitions;
- in ranking university students;
- in classifying hotels and restaurants, e.g., the Michelin *

Grading

"Judges and voters have complex aims, ends, purposes, and wishes: their *preferences* or *utilities*. A judge's or a voter's preferences may depend on many factors, including his beliefs about what is right and wrong, about the common language, about the method that transforms input messages into decisions, about the other judges' or voters' acts and behaviors, in addition to his evaluations of the competitors or candidates. But the judges' or voters' input messages—the grades they give—are assuredly not their preferences: a judge may dislike a wine, a dive, or a part of a skater's performance yet give it a high grade because of its merits; or a judge may like it yet give it a low grade because of its demerits." (pg. 3)

Majority Grade

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$\{7, 7, 8, \mathbf{8}, 11, 11, 11, 13\}$: The majority interval is $\{8, 9, 10, 11\}$, so the majority grade is 8.

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Suppose A's grades are $\{7, 7, 8, 8, 11, 11, 11, 13\}$
Suppose B's grades are $\{9, 9, 9, 9, 9, 10, 10, 10\}$
B should be ranked higher than A

Dealing with ties

When two competitors have different majority-grades, the competitor with the higher grade is naturally ranked higher.

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B's grades: $\{8, 9, \mathbf{9}, 10, 11\}$

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When two competitors have different majority-grades, the competitor with the higher grade is naturally ranked higher.

A's grades: $\{7, 9, \mathbf{9}, 11, 11\}$
B's grades: $\{8, 9, \mathbf{9}, 10, 11\}$

The second majority grade is found:
A's grades: $\{7, \mathbf{9}, 9, 11, 11\}$
B's grades: $\{8, \mathbf{9}, 9, 10, 11\}$

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The third majority grade is found:
A's grades: $\{7, 9, 9, \mathbf{11}, 11\}$
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The third majority grade is found:
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So, A is ranked above B.

The Model

A common language Λ: a set of strictly ordered grades: $\alpha, \beta, \gamma, \ldots$;

A finite set of m competitors (alternatives, candidates, performances, competing goods) $C = \{A, B, ..., I, ..., Z\}$; and

A finite set of n judges $J = \{1, ..., j, ..., n\}$.

A problem is specified by a profile $\Phi = \Phi(C, J)$: $m \times n$ matrix of grades assigned to the competitors (rows) by the judges (columns).

Example

Suppose that there are five voters, $1, \ldots, 5$ and three candidates I,II, and III. The grades that the voters may assign are A, B, C, D, or F (from best to worst).

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I	A	A	C	D	D
II	B	B	F	B	F
III	D	C	B	A	D

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II	B	B	F	**B**	F
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Candidates I, II, and III receive *majority grades* (median grades) C, B and C, respectively. Thus Candidate II would be declared the winner.

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II	B	B	F	**B**	F
III	D	**C**	B	A	D

Candidate II is ranked above candidate I: Is this reasonable? *If asked their preference, then candidate I is preferred to II by 4 judges.*

The Thesis

1. The traditional model is a bad model, in theory and in practice
2. The majority judgement is a better alternative to all other known methods, in theory and in practice

Electing vs. Ranking: Condorcet's Ranking

Condorcet-ranking (also known as Kemeny's rule) associates a score to each possible rank ordering:

- A voter contributes k Condorcet-points to a rank-ordering if his input agrees in k pair-by-pair comparisons.
- The Condorcet-score of a rank-ordering is the sum of its Condorcet-points over all voters.
- The Condorcet-ranking is the ranking that maximizes the Condorcet-score.

Electing vs. Ranking

Is Bordas method good for designating a winner, or a ranking, or both?

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- Given a method of ranking, the first-placed candidate is the winner.
- Given a method of designating a winner (or loser), he is the first-ranked (or last-ranked); the second-ranked is the winner among the remaining candidates, and so on.

Are ranking and designating winners two sides of one coin?

333	333	333	1
A	B	C	A
B	C	A	C
C	A	B	B

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- No reasonable **ranking function** must choose $A \succ C \succ B$
- Any reasonable **choice function** must choose $A \succ C \succ B$

There is a fundamental incompatibility between electing and ranking.

Conclusion: The traditional model's inputs are inadequate messages and must be reformulated.

D. Felsenthal and M. Machover. *The Majority Judgement voting procedure: a critical evaluation*. Homo oeconomicus, 25, pp. 319 - 334.

S. Brams. *Grading Candidates*. Book Review, American Scientist, 2012.

M. Balinski. *A letter regarding Steven J. Bramss review of Majority Judgment* . American Scientist.

Grades: Excellent, Very Good, Good, Acceptable, Poor and Reject

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Situation I

- Median: A's grade is **reject**, B's grade is **poor**
- Mean: A's grade is $\frac{6+1+1+1+1}{5} = 2$, B's grade is $\frac{1+2+2+2+2}{5} = 1.8$

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Situation II

- Median: A's grade is **acceptable**, B's grade is **good**
- Mean: A's grade is $\frac{6+6+3+2+2}{5} = 3.8$, B's grade is $\frac{5+5+4+1+1}{5} = 20.2$

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5	poor	reject

Situation I

- Approval Winner: B

Grades: Approve, Not Approve

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Situation I

- Approval Winner: *B*

Situation II

- Approval Winner: *A*

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If the examiner's grade turns out to be lower than the median, there is no incentive to change: lowering the grade would not change the result, raising the grade may result in a *higher* median grade.

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If the examiner's grade turns out to be lower than the median, there is no incentive to change: lowering the grade would not change the result, raising the grade may result in a *higher* median grade.

Similarly, if the examiner's grade turns out to be higher than the median grade.

	1	2	3
x	A	C	C
y	D	A	D
z	B	D	A

	1	2	3
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z	B	D	A

- If the judges act sincerely, then z is the winner

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- If the judges act sincerely, then z is the winner
- Both 1 and 2 would prefer that x *wins*

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- 1 can down grade z to D, then x will win

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- 3 cannot do anything about this.

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- If the judges act sincerely, then z is the winner
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Region I

x	21 A	31 B	48 C	1 D	**Winner**: y
y	40 A	11 B	48 C	2 D	

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x	21 A	31 B	48 C	1 D	**Winner**: y
y	40 A	11 B	48 C	2 D	

Region II

x	1 A	46 B	14 C	40 D	**Winner**: y
y	1 A	45 B	33 C	22 D	

Region I

x	21 A	31 B	48 C	1 D	
y	40 A	11 B	48 C	2 D	**Winner**: y

Region II

x	1 A	46 B	14 C	40 D	
y	1 A	45 B	33 C	22 D	**Winner**: y

Region III

x	40 B	20 C	41 D	
y	48 B	3 C	50 D	**Winner**: y

Region I

x	21 A	31 B	48 C	1 D	**Winner**: y
y	40 A	11 B	48 C	2 D	

Region II

x	1 A	46 B	14 C	40 D	**Winner**: y
y	1 A	45 B	33 C	22 D	

Region III

x	40 B	20 C	41 D	**Winner**: y
y	48 B	3 C	50 D	

Merged

x	22 A	117 B	82 C	82 D	**Winner**: x
y	41 A	104 B	84 C	74 D	

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x	3	4	4
y	2	5	5

Winner: y

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x	3	4	4
y	2	5	5

Winner: y

x	1	1	3	4	4
y	1	1	2	5	5

Winner: x

No-Show Paradox, continued

Grades: Excellent, Very Good, Good, Acceptable, Poor and Reject

	A	B
1	excellent	very good
2	good	very good
3	good	acceptable
4	acceptable	poor
5	poor	reject

A is the winner under Majority Judgement (good), Highest Mean (3.8), Approval Voting

No-Show Paradox, continued

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5	poor	reject
6	excellent	very good
7	excellent	very good

A is the winner under Majority Judgement (good) and Approval Voting

B is the winner under Majority Judgement (very good), but A is the winner under Approval Voting

Balinski's Response

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- voters care about all the results of an election (the distributions of the votes, who is in second, third, down to last place, the spreads between candidates, and so on) and not merely who is the winner, and
- voters are dissatisfied with election results that do not reflect their true opinions.

No-Show, revisited

	A	B
1	excellent	very good
2	good	very good
3	good	acceptable
4	acceptable	poor
5	poor	reject
6	excellent	very good
7	excellent	very good

- Only the winner matters: Without that assumption, there cannot be a no-show paradox (the two new voters help to increase Bs Majority Grade from Acceptable to Very Good, exactly what they believe it should be).

No-Show, revisited

	A	B
1	excellent	very good
2	good	very good
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- If "Approve" means a grade of very good or above, then a thousand voters who judge A to be Excellent and B to be Very Good could be adjoined to the five, yet B would remain the AV-winner with a score of 1002 to As 1001. *With MJ a higher grade necessarily always helps*.

No-Show, revisited

	A	B
1	excellent	very good
2	good	very good
3	good	acceptable
4	acceptable	poor
5	poor	reject
6	excellent	very good
7	excellent	very good

- The no-show phenomenon cannot occur with AV or MJ when the voter sees a real difference between the candidates (for example, when the two give Very Good to A and Poor to B); it can occur only when the voter sees little difference between the candidates (Excellent and Very Good in the example).

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