PHIL 308S: Voting Theory and Fair Division

Lecture 15

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Approval Voting

S. Brams and P. Fishburn. *Approval Voting*. Birkhauser, 1983.

J.-F. Laslier and M. R. Sanver (eds). *Handbook of Approval Voting*. Studies in Social Choice and Welfare, 2010.

Approval Voting: Each voter selects a subset of candidates. The candidate with the most "approvals" wins the election.

Why Approval Voting?

Because the voter is given the opportunity to provide more information about her opinion than with a single-name ballot, adoption of Approval Voting might increase voter turnout in general elections. Given the generally accepted view that the quality of a democracy is linked to the number of voters participating and their level of satisfaction with the electoral process, this suggests that Approval Voting can contribute to strengthening democracy.

Why Approval Voting?

By eliminating the wasted-vote effect, Approval Voting might broaden the span of candidates running for office, thereby contributing to the richness of the political debate. This point is related to the standard observation that the one-round Plurality system makes third parties nonviable, a critical point in U.S. politics.

Why Approval Voting?

By eliminating the squeezing effect, Approval Voting would encourage the election of consensual candidates. The squeezing effect is typically observed in multiparty elections with a runoff. The runoff tends to prevent extremist candidates from winning, but a centrist candidate who would win any pairwise runoff (the Condorcet winner) is also often squeezed between the left-wing and the right-wing candidates and so eliminated in the first round. This point is critical in countries using two-round Plurality.

Why Approval Voting?

AV will reduce negative campaigning. AV induces candidates to try to mirror the views of a majority of voters, not just cater to minorities whose votes could give them a slight edge in a crowded plurality contest. AV is therefore likely to cut down on negative campaigning, because candidates will have an incentive to broaden their appeals by reaching out for approval to voters who might have a different first choice. Lambasting such a choice, rather than being more expansive, risks alienating this candidates supporters, thereby losing their approval.

Why Approval Voting?

AV is eminently practicable. Unlike more complicated ranking systems, which suffer from a variety of theoretical as well as practical defects, AV is simple for voters to understand and use. Although more votes must be tallied under AV than under PV, AV can readily be implemented on existing voting machines. Because AV does not violate any state constitutions in the United States (or, for that matter, the constitutions of most countries in the world), it requires only an ordinary statute to enact "approving of X" vs. "voting for X under approval voting"

An AV strategy S of a focal voter is *admissible* if it not dominated in a game-theoretic sense, i.e., there is no other strategy that gives outcomes at least as good as, and sometime better than, S for all strategy profiles of voters other than the focal voter.

Admissible strategies always involve voting for a most-preferred candidate

An AV strategy is *sincere* if, given the lowest-ranked candidate that a voter approves of, he or she also approves of all candidates ranked higher.

Exclude “vote for everybody”

3	2	2
a	b	d
b	c	b
c	a	c
d	d	a

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Voters of type 1 have 3 sincere strategies: $\{a\}, \{a, b\}, \{a, b, c\}$

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A sincere strategy profile is $(a, a, a, bc, bc, dbc, dbc)$. AV selects $\{b, c\}$.

AV is more flexible

Fact There is no fixed rule that always elects a unique Condorcet winner.

# voters	2	2	1
	a	b	c
	d	d	a
	b	a	b
	c	c	d

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The unique Condorcet winner is *a*.

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# voters	2	2	1
	a	b	c
	d	d	a
	b	a	b
	c	c	d

Vote-for-1 elects $\{a, b\}$, vote-for-2 elects $\{d\}$, vote-for-3 elects $\{a, b\}$.

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# voters	2	2	1
	a	b	c
	d	d	a
	b	a	b
	c	c	d

$(\{a\}, \{b\}, \{c, a\})$ elects a under AV.

AV is more flexible

Fact Condorcet winners are always AV outcomes, but a Condorcet looser may or may not be an AV outcome.

3	2	2
a	b	c
b	c	b
c	a	a

Fact There is a profile where a candidate is not chosen by any scoring rule, but may be an AV outcome.

3	2	1	1
a	b	b	c
b	c	a	a
c	a	c	b

a is a Condorcet winner, but *b* will be elected by any scoring rule.

E. Aragones, I. Gilboa and A. Weiss. *Making Statements and Approval Voting*. Theory and Decision, Volume 71, Number 4 (2011), pp. 461 – 472.

E. Aragones, I. Gilboa and A. Weiss. *Making Statements and Approval Voting*. Theory and Decision, Volume 71, Number 4 (2011), pp. 461 – 472.

Abstract. We assume that people have a need to make statements, and construct a model in which this need is the sole determinant of voting behavior. In this model, an individual selects a ballot that makes as close a statement as possible to her ideal point, where abstaining from voting is a possible (null) statement. We show that in such a model, a political system that adopts approval voting may be expected to enjoy a significantly higher rate of participation in elections than a comparable system with plurality rule.

A set of parties $T = \{1, \ldots, m\}$, each party $j \in T$ is characterized by its positions on the various issues of concern $I = \{1, \ldots, n\}$.

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The voter considers every possible ballot she may cast as a vectot $x = (x_1, \ldots, x_m) \in \mathbb{R}_+^m$, where x_j is the degree to which the ballot supports party j. The voting system determines a subset $F \subset \mathbb{R}_+^m$ of feasible values for x.

1. Abstention corresponds to the null statement $0 \in \mathbb{R}^n$;
2. A voter for a single party corresponds to the position $v^j \in \mathbb{R}^n$ of that party; and
3. A voter approving of a non-empty set of parties corresponds ot the arithmetic average of their positions.

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A ballot $x \in \mathbb{R}^m_+$ makes the statement

$$Vx = \sum_{j \in T} x_j v^j \in \mathbb{R}^n$$

Plurality rule, in which a voter selects a single party. In this case the degree of support is $x^j = 1$ for the selected party and $x^j = 0$ for the others. $F^M = \{0\} \cup \{e^j\}_{j \leq m}$ where e^j is the vector with 1 in the jth position.

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Approval voting, in which a voter may choose any subset of parties as her vote. We model an individual who selects a non-empty subset $S \subset T$ as choosing the vector

$$x = \frac{1}{|S|} \sum_{j \in S} e^j$$

that is, as supporting each party to degree $\frac{1}{|S|}$.

This reflects the fact that the strength of the statement made in favor of a party by endorsing it depends on the other parties one endorses.

A voter with an ideal point $w \in \mathbb{R}^n$ may be modeled as solving the following problem:

$$\min_{x \in F} || \sum_{j \in A} x^j v^j - w ||$$

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If the solution to this problem is $x = 0 \in \mathbb{R}^m$, the voter will abstain.

Abstention does not result from the cost of voting, but rather from the fact that the voter feels dissatisfied with any statement that the political systems allows her to make.