PHIL 308S: Voting Theory and Fair Division

Lecture 12

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Manipulation

- Agenda manipulation
- Misrepresenting preferences
- Sophisticated voting
- What is *wrong* with manipulation?

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If $V(\vec{P})$ and $V(\vec{P}')$ are singletons, then "i **prefers** $V(\vec{P}')$ **to** $V(\vec{P})$" means $V(\vec{P}')P_iV(\vec{P})$

What if $V(\vec{P})$ and $V(\vec{P}')$ are not singletons?

Preference Lifting, I

Given a preference ordering $\preceq$ over a set of objects X, we want to **lift** this to an ordering $\hat{\preceq}$ over $\wp(X)$.

Given $\preceq$, what reasonable properties can we infer about $\hat{\preceq}$?

S. Barberá, W. Bossert, and P.K. Pattanaik. *Ranking sets of objects*. In Handbook of Utility Theory, volume 2. Kluwer Academic Publishers, 2004.

Preference Lifting, II

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  Can you infer that $\{x, y\} \hat{\prec} \{z\}$?

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Preference Lifting, III

There are different interpretations of $X \mathrel{\hat{\preceq}} Y$:

- You will get one of the elements, but cannot control which.
- You can choose one of the elements.
- You will get the full set.

Preference Lifting, IV

Kelly Principle

(EXT) $\{x\} \hat{\preceq} \{y\}$ provided $x \prec y$

(MAX) $A \hat{\preceq} Max(A)$

(MIN) $Min(A) \hat{\preceq} A$

J.S. Kelly. *Strategy-Proofness and Social Choice Functions without Single-Valuedness*. Econometrica, 45(2), pp. 439 - 446, 1977.

Preference Lifting, IV

Gärdenfors Principle

(G1) $A \mathrel{\hat{\preceq}} A \cup \{x\}$ if $a \prec x$ for all $a \in A$

(G2) $A \cup \{x\} \mathrel{\hat{\preceq}} A$ if $x \prec a$ for all $a \in A$

P. Gärdenfors. *Manipulation of Social Choice Functions*. Journal of Economic Theory. 13:2, 217 - 228, 1976.

Preference Lifting, IV

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Independence

(IND) $A \cup \{x\} \mathrel{\hat{\preceq}} B \cup \{x\}$ if $A \mathrel{\hat{\prec}} B$ and $x \notin A \cup B$

Preference Lifting, V

Theorem (Kannai and Peleg). If $|X| \geq 6$, then no weak order satisfies both the Gärdenfors principle and independence.

Y. Kannai and B. Peleg. *A Note on the Extension of an Order on a Set to the Power Set*. Journal of Economic Theory, 32(1), pp. 172 - 175, 1984.

Suppose that $V(\vec{P})$ and $V(\vec{P}')$ are not singletons

- X is **weakly dominates** Y for i provided

$$\forall x \in X \forall y \in Y \quad xR_i y \quad \text{and} \quad \exists x \in X \exists y \in Y \quad xP_i y$$

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- X is preferred by a *pessimist* to Y: $\min_i(X, P) P_i \min_i(Y, P)$
- X has higher "expected utility": There exists a utility function representing P_i such that, if $p(x) = \frac{1}{|X|}$ and $p(y) = \frac{1}{|Y|}$, then

$$\sum_{x \in X} p(x) \cdot u(x) > \sum_{y \in Y} p(y) \cdot u(y)$$

The Gibbard-Satterthwaite Theorem

A social choice function is **strategy-proof** if for no individual i there exists a profile $\vec{R}$ and a linear order R_i' such that $V(\vec{R}_{-i}, R_i')$ is ranked above $V(\vec{R})$ according to i.

Theorem. Any social choice function for three or more alternatives that is Pareto and strategy-proof must be a dictatorship.

M. A. Satterthwaite. *Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions*. Journal of Economic Theory, 10(2):187-217, 1975.

A. Gibbard. *Manipulation of voting schemes: A general result*. Econometrica, 41(4):587-601, 1973.

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Example I

The following example is due to [Brams & Fishburn]

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\begin{aligned}
P_A^* &= o_1 > o_3 > o_2 \\
P_B^* &= o_2 > o_3 > o_1 \\
P_C^* &= o_3 > o_1 > o_2
\end{aligned}
$$

Size	Group	I	II
4	A	$\mathbf{o_1}$	$\mathbf{o_1}$
3	B	o_2	o_2
2	C	o_3	$\mathbf{o_1}$

If the current winner is o, then agent i will switch its vote to some candidate o' provided

1. o' is one of the top two candidates as indicated by a poll
2. o' is preferred to the other top candidate

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P_A^* &= (o_1, o_4, o_2, o_3) \\
P_B^* &= (o_2, o_1, o_3, o_4) \\
P_C^* &= (o_3, o_2, o_4, o_1) \\
P_D^* &= (o_4, o_1, o_2, o_3) \\
P_E^* &= (o_3, o_1, o_2, o_4)
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Size	Group	I	II	III	IV
40	A	$\mathbf{o_1}$	o_1	o_4	$\mathbf{o_1}$
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15	C	o_3	$\mathbf{o_2}$	$\mathbf{o_2}$	o_2
8	D	o_4	o_4	o_1	o_4
7	E	o_3	o_3	o_1	$\mathbf{o_1}$

If the current winner is o, then agent i will switch its vote to some candidate o' provided

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Sophisticated Voting

Consider a legislator voting on a pay raise.

(pass and vote nay) P_i (pass and vote yea) P_i (fail and vote nay) P_i (fail and vote yea)

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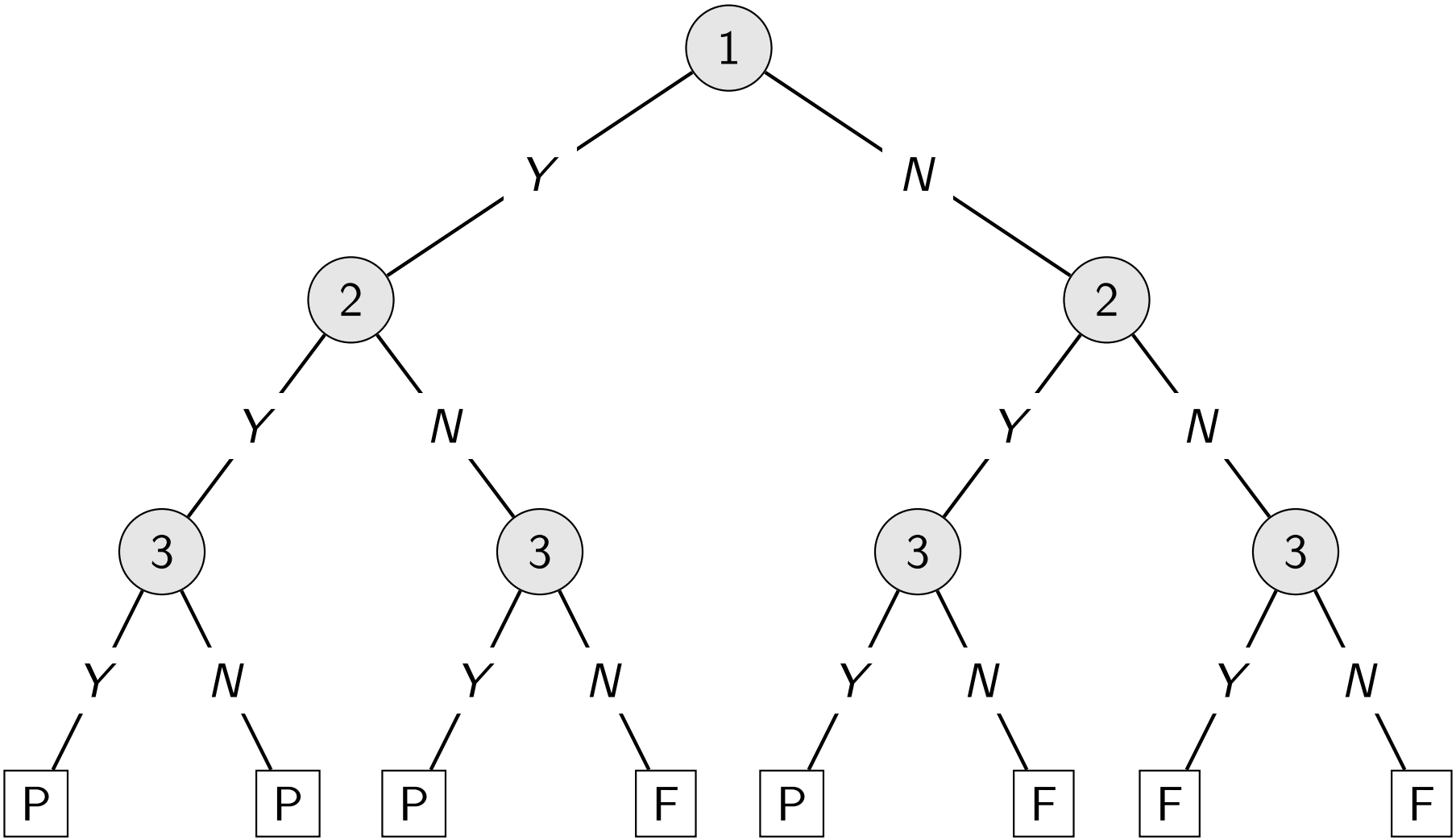
If there are three voters who voter in turn, what will the first legislator choose?

$$(P \ \& \ N) \ P_i \ (P \ \& \ Y) \ P_i \ (F \ \& \ N) \ P_i \ (F \ \& \ Y)$$



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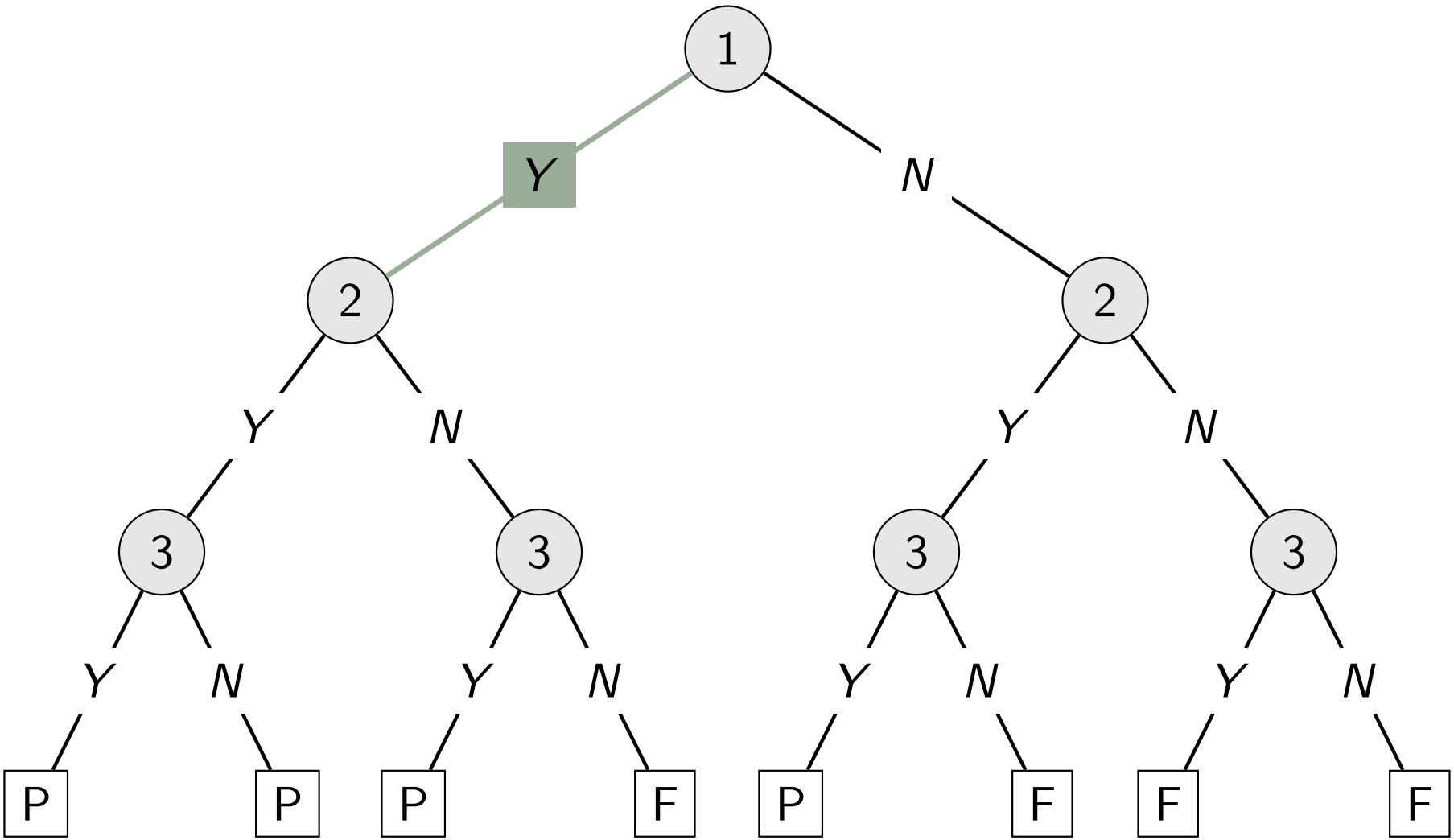

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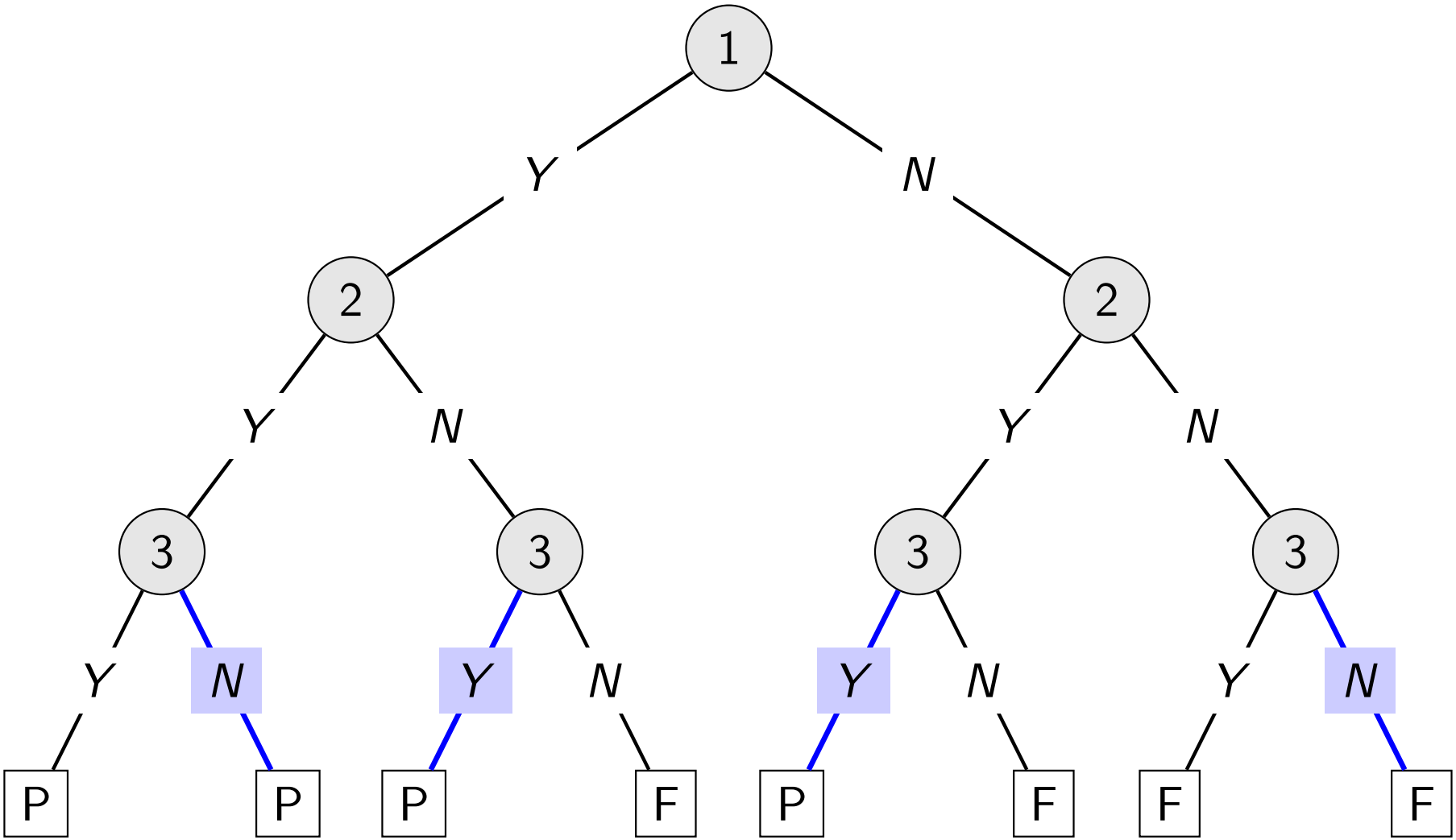

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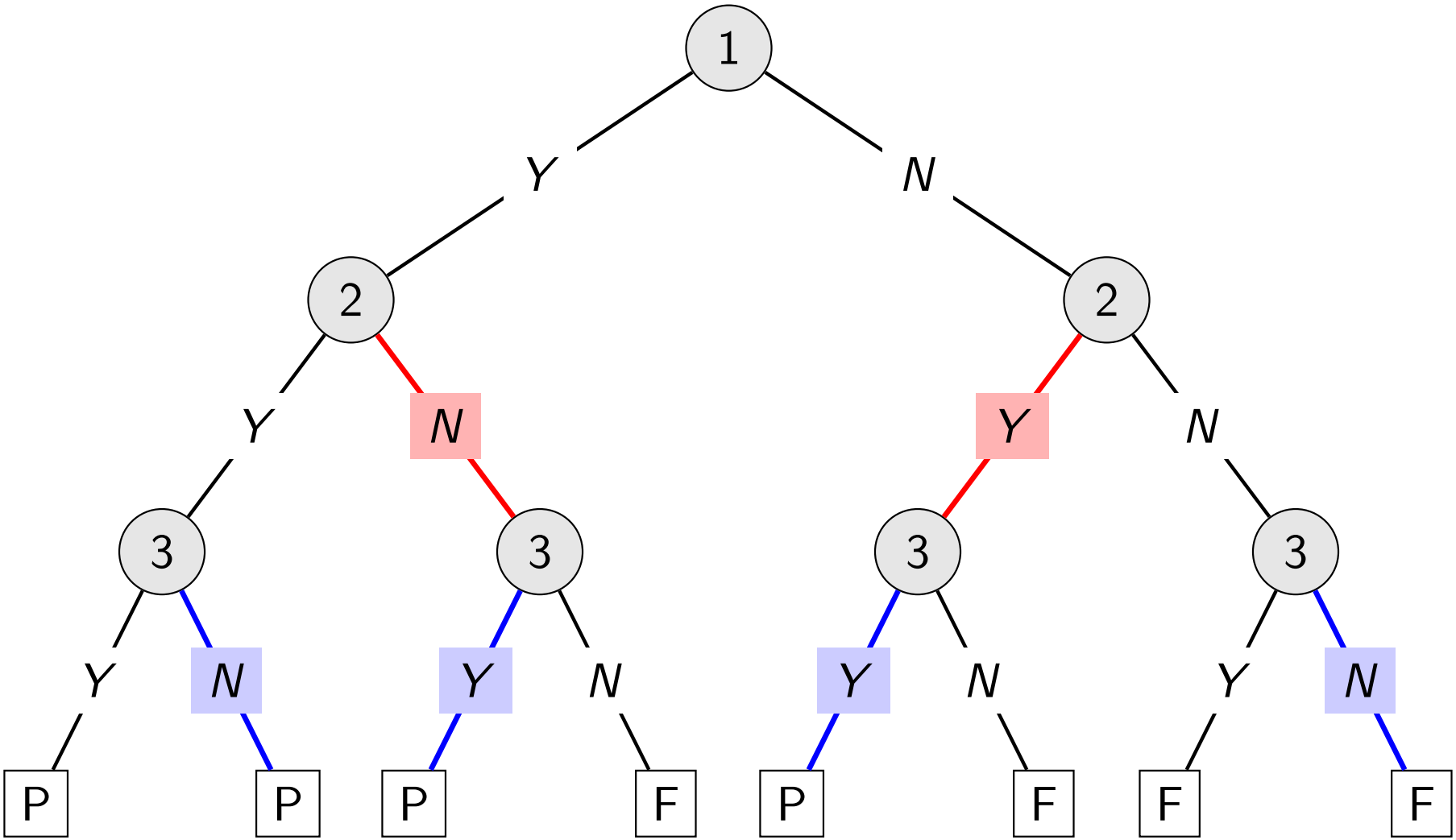

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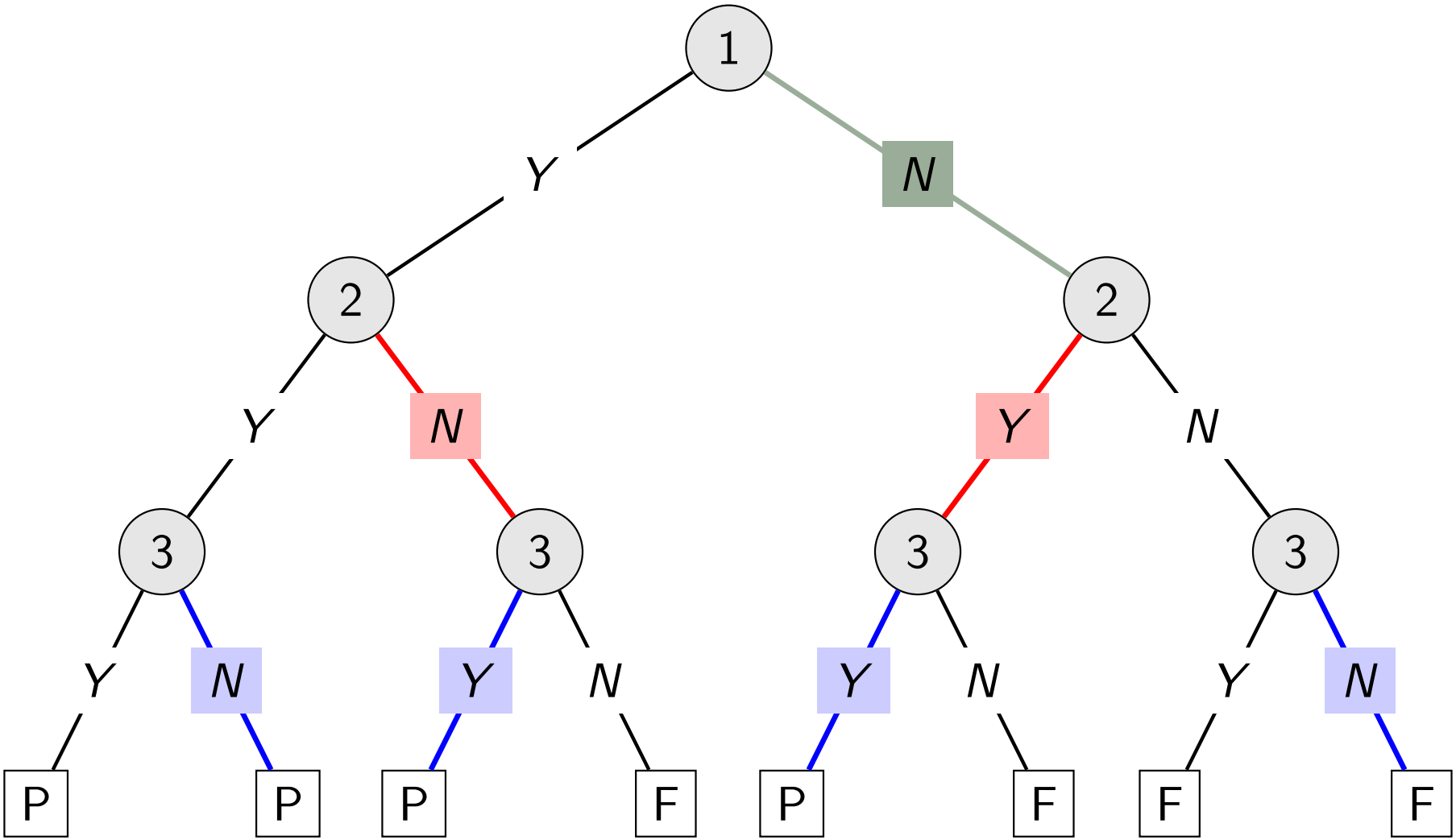

What does it *mean* to vote strategically?

- Voting as a game vs. voting as an act of communication

K. Dowding and M. van Hees. *In Praise of Manipulation*. British Journal of Political Science, 38 : pp 1-15, 2008.