PHIL 308S: Voting Theory and Fair Division

Lecture 11

Eric Pacuit

Department of Philosophy
University of Maryland, College Park
ai.stanford.edu/∼epacuit
epacuit@umd.edu

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Manipulation

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(Taken from A. Taylor *Social Choice and the Mathematics of Manipulation* who took it from D. Black *A Theory of Committees and Elections* who took it from Dodgson.)

"If we assume society discourages the concentration of power, then at least two methods of manipulation are always available, no matter what method of voting is used: First, those in control of procedures can manipulate the agenda (by, for example, restricting alternatives or by arranging the order in which they are brought up). Second, those not in control can still manipulate by false revelation of values." (Riker, p. 137)

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W. Poundstone. *Gaming the Vote.* Hill and Wang Publishers, 2008.

- Agenda manipulation
- Misrepresenting preferences
- Sophisticated voting
- What is *wrong* with manipulation?

Manipulation: setting the agenda

# voters	1	1	1
	B	A	C
	D	B	A
	C	D	B
	A	C	D

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	B	A	C
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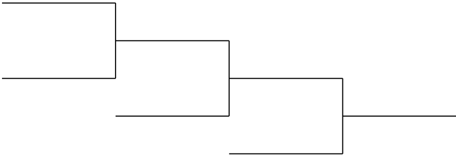

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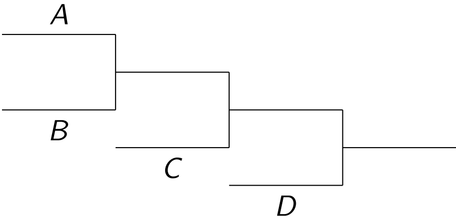

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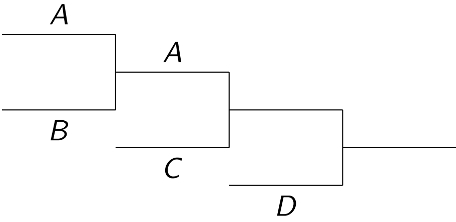

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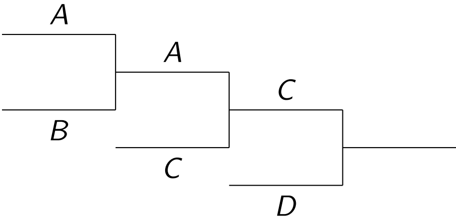

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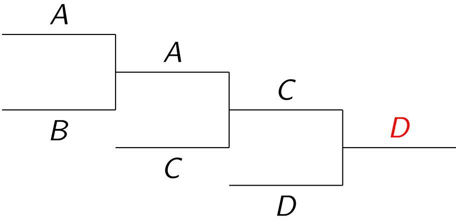

Manipulation: misrepresenting preferences

# voters	3	3	1
	A	B	C
	B	A	A
	C	C	B

Borda Winner: A

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Borda Winner: B

Manipulation: misrepresenting preferences

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	C	C	B

Borda Winner: *A*

# voters	3	3	1
	A	B	C
	B	C	A
	C	A	B

Borda Winner: *B*

Borda: "My procedure is only meant for honest men!"

Manipulation by a *group*.

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Intuition: P_i is voter i's "true preference".

If $V(\vec{P})$ and $V(\vec{P'})$ are singletons, then "i **prefers** $V(\vec{P'})$ **to** $V(\vec{P})$" means $V(\vec{P'}) P_i V(\vec{P})$

What if $V(\vec{P})$ and $V(\vec{P'})$ are not singletons?

Preference Lifting, I

Given a preference ordering $\preceq$ over a set of objects X, we want to **lift** this to an ordering $\hat{\preceq}$ over $\wp(X)$.

Given $\preceq$, what reasonable properties can we infer about $\hat{\preceq}$?

S. Barberá, W. Bossert, and P.K. Pattanaik. *Ranking sets of objects*. In Handbook of Utility Theory, volume 2. Kluwer Academic Publishers, 2004.

Preference Lifting, II

- You know that $x \prec y \prec z$
  Can you infer that $\{x, y\} \mathrel{\hat{\prec}} \{z\}$?

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Preference Lifting, III

There are different interpretations of $X \hat{\precsim} Y$:

- You will get one of the elements, but cannot control which.
- You can choose one of the elements.
- You will get the full set.

Preference Lifting, IV

Kelly Principle

(EXT) $\{x\} \hat{\precsim} \{y\}$ provided $x \prec y$

(MAX) $A \hat{\precsim} Max(A)$

(MIN) $Min(A) \hat{\precsim} A$

J.S. Kelly. *Strategy-Proofness and Social Choice Functions without Single-Valuedness*. Econometrica, 45(2), pp. 439 - 446, 1977.

Preference Lifting, IV

Gärdenfors Principle

(G1) $A \hat{\precsim} A \cup \{x\}$ if $a \prec x$ for all $a \in A$

(G2) $A \cup \{x\} \hat{\precsim} A$ if $x \prec a$ for all $a \in A$

P. Gärdenfors. *Manipulation of Social Choice Functions*. Journal of Economic Theory. 13:2, 217 - 228, 1976.

Preference Lifting, IV

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Independence

(IND) $A \cup \{x\} \hat{\preceq} B \cup \{x\}$ if $A \hat{\precsim} B$ and $x \notin A \cup B$

Preference Lifting, V

Theorem (Kannai and Peleg). If $|X| \geq 6$, then no weak order satisfies both the Gärdenfors principle and independence.

Y. Kannai and B. Peleg. *A Note on the Extension of an Order on a Set to the Power Set*. Journal of Economic Theory, 32(1), pp. 172 - 175, 1984.

Suppose that $V(\vec{P})$ and $V(\vec{P'})$ are not singletons

- X is **weakly dominates** Y for i provided

$$\forall x \in X \forall y \in Y \quad xR_iy \quad \text{and} \quad \exists x \in X \exists y \in Y \quad xP_iy$$

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- X is preferred by an *optimist* to Y: $\max_i(X, P) P_i \max_i(Y, P)$

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- X is preferred by a *pessimist* to Y: $\min_i(X, P) P_i \min_i(Y, P)$
- X has higher "expected utility": There exists a utility function representing P_i such that, if $p(x) = \frac{1}{|X|}$ and $p(y) = \frac{1}{|Y|}$, then

$$\sum_{x \in X} p(x) \cdot u(x) > \sum_{y \in Y} p(y) \cdot u(y)$$

Fact. Borda count is single-winner manipulable.

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3	3	1
A	B	C
B	A	A
C	C	B

Borda Winner: A

3	3	1
A	B	C
B	C	A
C	A	B

Borda Winner: B

Fact. Plurality rules is weak dominance manipulable, but is never single-winner manipulable.

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1	2	1
A	C	B
B	A	A
C	B	C

Plurality Winner: C

1	2	1
B	C	B
A	A	A
C	B	C

Plurality Winners: $\{B, C\}$

Fact. Condorcet rule is manipulable by both optimists and pessimists, but is never weak dominance manipulable.

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1	1	1
A	B	C
C	C	A
B	A	B

Condorcet Winner: C

1	1	1
A	B	C
B	C	A
C	A	B

Condorcet Winners: $\{A, B, C\}$

Fact. The "near-unanimity rule" is manipulable by pessimists, but is never by optimists.

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1	1	1
A	B	C
B	A	A
C	C	B

Near-Unanimity Winner: $\{A, B, C\}$

1	1	1
B	B	C
A	A	A
C	C	B

Near-Unanimity Winner: $\{B\}$

Fact. The "Pareto rule" is expected-utility manipulable, but never manipulable by optimists or pessimists.

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1	1	1
A	A	C
B	C	B
C	B	A

Pareto Winner: $\{A, B, C\}$

1	1	1
A	A	C
C	C	B
B	B	A

Pareto Winner: $\{A, C\}$

Fact. The "Pareto rule" is expected-utility manipulable, but never manipulable by optimists or pessimists.

1	1	1
A	A	C
B	C	B
C	B	A

Pareto Winner: $\{A, B, C\}$

1	1	1
A	A	C
C	C	B
B	B	A

Pareto Winner: $\{A, C\}$

Let $u_1(A) = 18, u_1(B) = 9$, and $u_1(C) = 6$

$$18 \cdot \frac{1}{3} + 9 \cdot \frac{1}{3} + 6 \cdot \frac{1}{3} = 11 \qquad 18 \cdot \frac{1}{2} + 6 \cdot \frac{1}{2} = 12$$

The Gibbard-Satterthwaite Theorem

A social choice function is **strategy-proof** if for no individual i there exists a profile $\vec{R}$ and a linear order R_i' such that $V(\vec{R}_{-i}, R_i')$ is ranked above $V(\vec{R})$ according to i.

Theorem. Any social choice function for three or more alternatives that is Pareto and strategy-proof must be a dictatorship.

M. A. Satterthwaite. *Strategy-proofness and Arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions*. Journal of Economic Theory, 10(2):187-217, 1975.

A. Gibbard. *Manipulation of voting schemes: A general result*. Econometrica, 41(4):587-601, 1973.

- ✓ Agenda manipulation
- ✓ Misrepresenting preferences
- Sophisticated voting
- What is *wrong* with manipulation?

Voting Strategically

Consider a legislator voting on a pay raise.

(pass and vote nay) P_i (pass and vote yea) P_i (fail and vote nay) P_i (fail and vote yea)

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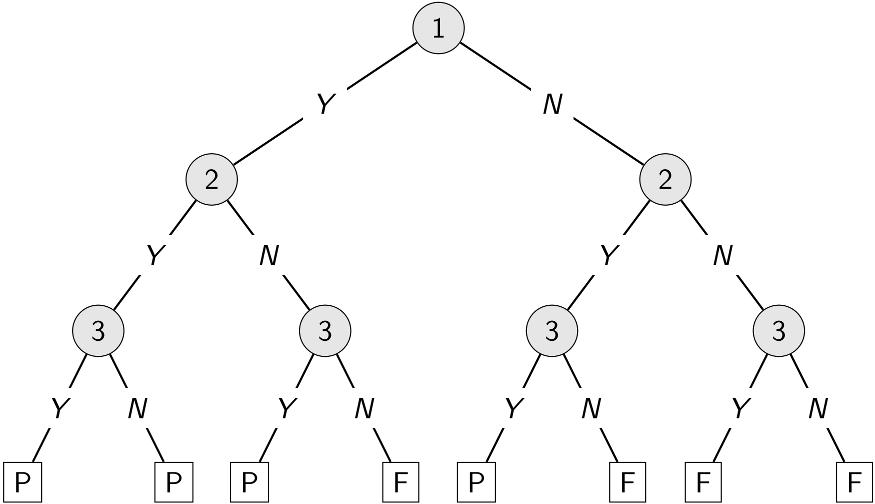
If there are three voters who voter in turn, what will the first legislator choose?

$(P \,\&\, N)\ P_i\ (P \,\&\, Y)\ P_i\ (F \,\&\, N)\ P_i\ (F \,\&\, Y)$



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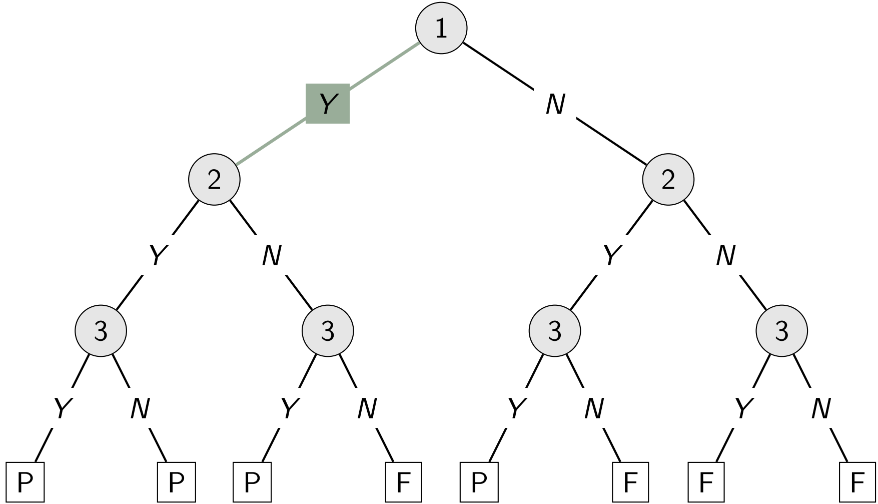
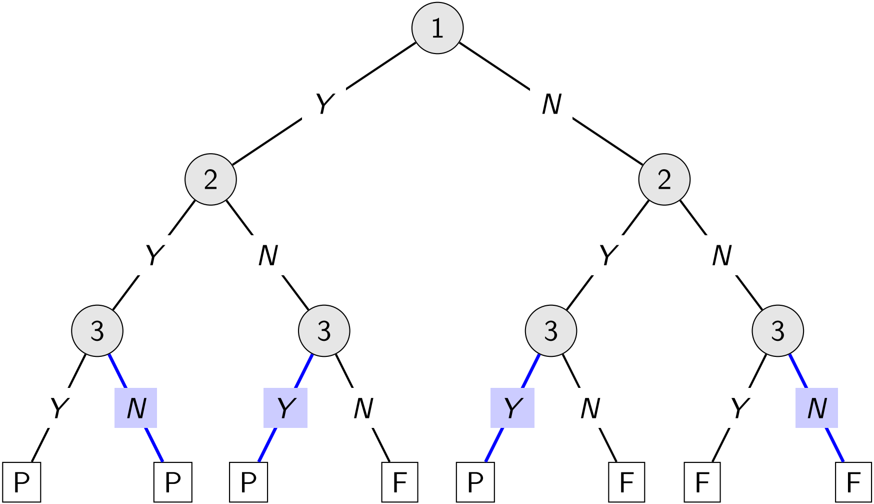
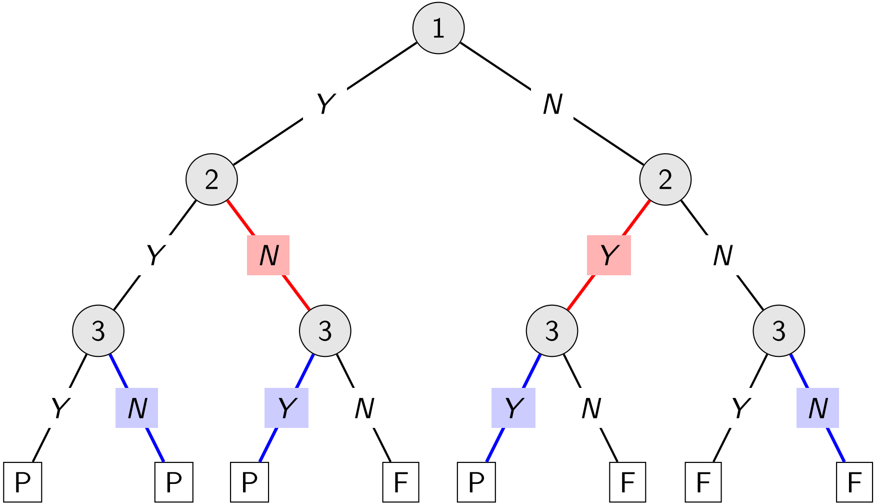


$(P \ \& \ N) \ P_i \ (P \ \& \ Y) \ P_i \ (F \ \& \ N) \ P_i \ (F \ \& \ Y)$



$$(P \,\&\, N)\ P_i\ (P \,\&\, Y)\ P_i\ (F \,\&\, N)\ P_i\ (F \,\&\, Y)$$



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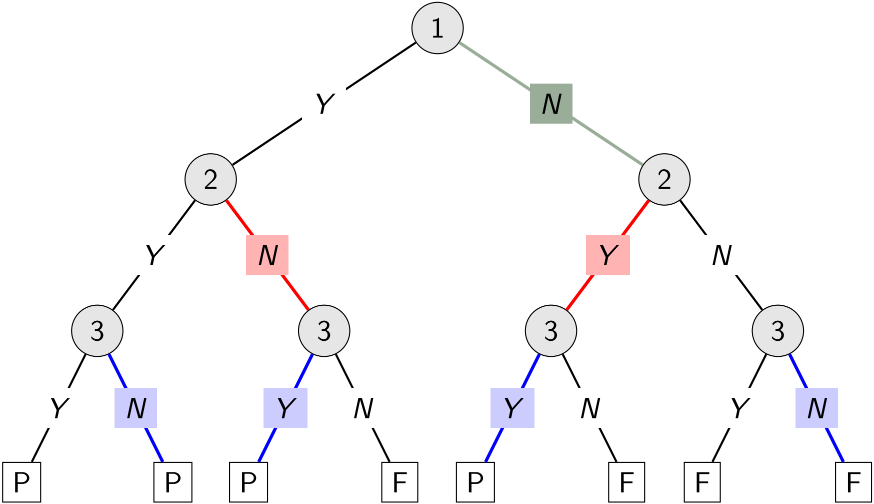

What does it *mean* to vote strategically?

- Voting as a game vs. voting as an act of communication

K. Dowding and M. van Hees. *In Praise of Manipulation*. British Journal of Political Science, 38 : pp 1-15, 2008.