PHIL 308S: Voting Theory and Fair Division

Lecture 10

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Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

YYY	YYN	YNY	YNN	NYY	NYN	NNY	NNN
1	1	1	3	1	3	3	0

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Proposition 1: N (7 - 6)

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Proposition 3: N (7 - 6)

But there is no support for NNN!

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Anscombe's Paradox

	Issue 1	Issue 2	Issue 3
Voter 1	Yes	Yes	No
Voter 2	No	No	No
Voter 3	No	Yes	Yes
Voter 4	Yes	No	Yes
Voter 5	Yes	No	Yes
Majority	Yes	No	Yes

G. E. M. Anscombe. *On Frustration of the Majority by Fulfillment of the Majority's Will*. Analysis, 36(4): 161-168, 1976.

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Voters 1,2 & 3 do not support the outcome on a majority of issues

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Voter 5	Yes	No	Yes
Majority	Yes	No	Yes

Voters 4 & 5 support the outcome on a majority of issues
Voters 1,2 & 3 do not support the outcome on a majority of issues

A majority of voters do not support the majority outcome on a majority of issues.

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Judgement Aggregation

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Preference aggregations vs. *judgement aggregation*

- Judgements of preference, value judgements, beliefs

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- What should be done? What is the best alternative?

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Preference aggregations vs. *judgement aggregation*

- Judgements of preference, value judgements, beliefs
- What should be done? What is the best alternative?
- The Pareto conditions (see forthcoming work by W. Rabinowicz, S. Hartmann and S. Rafiee Rad)

Doctrinal Paradox

Suppose that three experts *independently* formed opinions about three propositions. For example,

1. p: "Carbon dioxide emissions are above the threshold x"
2. $p \rightarrow q$: "If carbon dioxide emissions are above the threshold x, then there will be global warming"
3. q: "There will be global warming"

Doctrinal Paradox

	p	$p \rightarrow q$	q
Expert 1			
Expert 2			
Expert 3			

Doctrinal Paradox

	p	$p \rightarrow q$	q
Expert 1	True	True	
Expert 2			
Expert 3			

Doctrinal Paradox

	p	$p \to q$	q
Expert 1	True	True	True
Expert 2			
Expert 3			

Doctrinal Paradox

	p	$p \to q$	q
Expert 1	True	True	True
Expert 2	True		False
Expert 3			

Doctrinal Paradox

	p	$p \rightarrow q$	q
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3			

Doctrinal Paradox

	p	$p \rightarrow q$	q
Expert 1	True	True	True
Expert 2	True	False	False
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Doctrinal Paradox

	p	$p \to q$	q
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group			

Doctrinal Paradox

	p	$p \to q$	q
Expert 1	True	True	True
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Group	True	True	False

The Logic of Group Decisions, II

(Kornhauser and Sager 1993)

p: a valid contract was in place

q: there was a breach of contract

r: the court is required to find the defendant liable.

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

The Logic of Group Decisions, II

(Kornhauser and Sager 1993)

Should we accept r?

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

The Logic of Group Decisions, II

(Kornhauser and Sager 1993)

Should we accept r? No, a simple majority votes no.

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

The Logic of Group Decisions, II

(Kornhauser and Sager 1993)

Should we accept r? Yes, a majority votes yes for p and q and $(p \wedge q) \leftrightarrow r$ is a legal doctrine.

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

Many Variants!

See
`http://personal.lse.ac.uk/LIST/doctrinalparadox.htm`
for many generalizations!

Kornhauser and Sager. *Unpacking the court*. Yale Law Journal, 1986.

C. List and P. Pettit. *Aggregating Sets of Judgments: An Impossibility Result*. Economics and Philosophy 18: 89-110, 2002.

The Judgement Aggregation Model: The Propositions

Propositions: Let $\mathcal{L}$ be a logical language (called **propositions** in the literature) with the usual boolean connectives.

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Consistency: The standard notion of logical consistency.

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Consistency: The standard notion of logical consistency.

Aside: We actually need

1. $\{p, \neg p\}$ are inconsistent
2. all subsets of a consistent set are consistent
3. $\emptyset$ is consistent and each $S \subseteq \mathcal{L}$ has a consistent maximal extension (not needed in all cases)

The Judgement Aggregation Model: The Agenda

Definition The **agenda** is a non-empty set $X \subseteq \mathcal{L}$, interpreted as the set of propositions on which judgments are made (note: X is a union of proposition-negation pairs $\{p, \neg p\}$).

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Example: In the discursive dilemma:
$X = \{a, \neg a, b, \neg b, a \to b, \neg(a \to b)\}$.

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Definition: Given an agenda X, each individual i's judgement set is a subset $A_i \subseteq X$.

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Rationality Assumptions:

1. A_i is **consistent**
2. A_i is **complete**, if for each $p \in X$, either $p \in A_i$ or $\neg p \in A_i$

The Judgement Aggregation Model: Aggregation Rules

Let X be an agenda, $N = \{1, \ldots, n\}$ a set of voters, a **profile** is a tuple $(A_i, \ldots, A_n)$ where each A_i is a judgement set. An **aggregation function** is a map from profiles to judgment sets. I.e., $F(A_1, \ldots, A_n)$ is a judgement set.

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Examples:

- **Propositionwise majority voting**: for each $(A_1, \ldots, A_n)$,

$$F(A_1, \ldots, A_n) = \{p \in X \mid |\{i \mid p \in A_i\}| \geq |\{i \mid p \notin A_i\}|\}$$

- **Dictator of** i: $F(A_1, \ldots, A_n) = A_i$
- **Reverse Dictator of** i: $F(A_1, \ldots, A_n) = \{\neg p \mid p \in A_i\}$

The Judgement Aggregation Model: Input Condition

Universal Domain: The domain of F is the set of all possible profiles of consistent and complete judgement sets.

The Judgement Aggregation Model: Output Condition

Collective Rationality: F generates consistent and complete collective judgment sets.

The Judgement Aggregation Model: Responsiveness Conditions

Systematicity: For any $p, q \in X$ and all $(A_1, \ldots, A_n)$ and $(A_1^*, \ldots, A_n^*)$ in the domain of F,

$$\text{if [for all } i \in N,\ p \in A_i \text{ iff } q \in A_i^*]$$
$$\text{then } [p \in F(A_1, \ldots, A_n) \text{ iff } q \in F(A_1^*, \ldots A_n^*)\ ].$$

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- independence
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Independence: For any $p \in X$ and all $(A_1, \ldots, A_n)$ and $(A_1^*, \ldots, A_n^*)$ in the domain of F,

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$$\text{then } [p \in F(A_1, \ldots, A_n) \text{ iff } p \in F(A_1^*, \ldots A_n^*)\ ].$$

The Judgement Aggregation Model: Responsiveness Conditions

Anonymity: For all profiles $(A_1, \ldots, A_n)$, $F(A_1, \ldots, A_n) = F(A_{\pi(1)}, \ldots, A_{\pi(n)}$ where π is a permutation of the voters.

Unanimity: For all profiles $(A_1, \ldots, A_n)$ if $p \in A_i$ for each i then $p \in F(A_1, \ldots, A_n)$

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Monotonicity: For any $p \in X$ and all $(A_1, \ldots A_i, \ldots, A_n)$ and $(A_1, \ldots, A_i^*, \ldots, A_n)$ in the domain of F,

$$\text{if } [p \notin A_i,\ p \in A_i^* \text{ and } p \in F(A_1, \ldots, A_i, \ldots A_n)]$$
$$\text{then } [p \in F(A_1, \ldots, A_i^*, \ldots A_n)].$$

The Judgement Aggregation Model: Responsiveness Conditions

Non-dictatorship: There exists no $i \in N$ such that, for any profile $(A_1, \ldots, A_n)$, $F(A_1, \ldots, A_n) = A_i$

Baseline Result

Theorem (List and Pettit, 2001) If $X \subseteq \{a, b, a \wedge b\}$, there exists no aggregation rule satisfying universal domain, collective rationality, systematicity and anonymity.

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See `personal.lse.ac.uk/LIST/doctrinalparadox.htm` for many generalizations!

Sen's Liberal Paradox

Two members of a small society Lewd and Prude each have a personal copy of *Lady Chatterley's Lover*, consider

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l: Lewd reads the book;
p: Prude reads the book;
$l \to p$: If Lewd reads the book, then so does Prude.

A. Sen. *The Impossibility of a Paretian Liberal*. Journal of Political Economy, 78:1, pp. 152 - 157, 1970.

Sen's Liberal Paradox

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Lewd desires to read the book, and if he reads it, then so does Prude (Lewd enjoys the thought of Prude's moral outlook being corrupted)

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Lewd desires to read the book, and if he reads it, then so does Prude (Lewd enjoys the thought of Prude's moral outlook being corrupted)

Prude desires to not read the book, and that Lewd not read it either, but in case Lewd does read the book, Prude wants to read the book to be informed about the dangerous material Lewd has read.

Sen's Liberal Paradox

	l	p	$l \rightarrow p$

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	l	p	$l \rightarrow p$
Lewd	True	True	True

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	l	p	$l \rightarrow p$
Lewd	True	True	True
Prude	False	False	True

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1. Society assigns to each individual the liberal right to determine the collective desire on those propositions that concern only the individual's private sphere
   l is Lewd's case, p is Prude's case

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2. Unanimous desires of all individuals must be respected.

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So, society must be inconsistent!

"What is the moral?

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"What is the moral? It is that in a very basic sense liberal values conflict with the Pareto principle. If someone takes the Pareto principle seriously, as economists seem to do, then he has to face problems of consistency in cherishing liberal values, even very mild ones.... While the Pareto criterion has been thought to be an expression of individual liberty, it appears that in choices involving more than two alternatives it can have consequences that are, in fact, deeply illiberal." (pg. 157)