

Problem Set 4

1. Ann, Bob, Charles and David are sharing a cake consisting of four homogenous components: chocolate, vanilla, strawberry and walnut. Ann likes only the vanilla piece. Bob likes the chocolate and strawberry components equally, but does not like vanilla or walnuts at all. Charles likes nothing other than the chocolate piece. David likes strawberries and walnuts equally, but does not like chocolate or vanilla at all. We give to Ann the entire vanilla component, to Bob two-thirds of the chocolate and the entire strawberry component, to Charles one-third of the chocolate component and to David the entire walnut component.
 - (a) Is this *fair* (i.e., is the division proportional)?
 - (b) Is it envy-free? If not, list all incidents of envy.
 - (c) Is it equitable?
2. Explain why for any number N of people sharing a cake, any envy-free division is fair.
3. Give an example of an equitable division that is not fair (i.e., is not proportional).
4. Who among Phil, Jill and Will is guaranteed $1/3$ by each of the following algorithms (explain your answer):
 - (a) Phil cuts halves and Jill chooses. Then each cuts the other's piece into what the cutter considers equal thirds. Finally Will takes a piece from each player.
 - (b) Jill cuts equal thirds, and then Will cuts each of those pieces in halves. The six pieces are chosen in the order Phil, Will, Jill, Jill, Will, Phil.
 - (c) Cut as in (b) but choose in the order Phil, Jill, Will, Will, Jill, Phil.