Notes on Natural Logic

Notes for PHIL370

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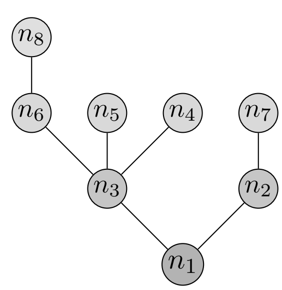
1 Preliminaries: Trees

A **tree** is a structure $\mathcal{T} = (T, E)$, where T is a nonempty set whose elements are called *nodes* and E is a relation on T, $E \subseteq T \times T$, called the **immediate edge relation**, satisfying the following conditions: for all nodes, $n, n', m \in T$,

- Every node has a unique predecessor: If nEm and $n'Em$, then $n = n'$,
- There are no cycles: If $(n_1, \ldots, n_k)$ is a sequence of nodes where for each $i = 1, \ldots, k-1$, $n_i E n_{i+1}$, then $n_1 \neq n_k$, and
- Between any two nodes there is a unique path: For each $n, n' \in T$ there is a unique sequence $n_1, n_2, \ldots, n_m$ such that $n = n_1 E n_2 \cdots E n_m = n'$.

Let n be a node, then $\mathbf{succ}(n) = \{n' \mid nEn'\}$ are the successors of n and $\mathbf{pred}(n) = \{n' \mid n'En\}$ are the predecessors of n. A node r is called the **root** of the tree provided $\mathbf{pred}(r) = \emptyset$. A node n is called a **leaf** provided $\mathbf{succ}(n) = \emptyset$. A **path** in a tree is a sequence of nodes connected by an edge relation: a path is a sequence $(n_1, n_2, \ldots, n_k)$ such that for each $i = 1, \ldots, k-1$, $n_i E n_{i+1}$. The **length** of a path is equal to the number of edges along that path (equivalently, one minus the number of nodes). The **height** of a tree is the length of the longest path. Here is an example:



The root of this tree is n_1 and the leaves are n_8, n_5, n_4, and n_7. We have $\mathbf{succ}(n_3) = \{n_6, n_5, n_4\}$ and $\mathbf{pred}(n_3) = \{n_1\}$. Two paths in the tree are (n_3, n_5) and (n_1, n_3, n_6). The height of the tree is 3 (the longest path is (n_1, n_3, n_6, n_8) which has length 3).

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2 The logic of "All" and "Some"

Definition 2.1 (Sentences) Let $\mathcal{V} = \{X, Y, Z, \ldots\}$ be a set of *variables*. The *All* and *Some* language $\mathcal{L}_{AS}$ are all the sentences of the form:

- $\mathsf{All}\ X\ \mathsf{are}\ Y$
- $\mathsf{Some}\ X\ \mathsf{are}\ Y$ ◁

Definition 2.2 (Models) A model for $\mathcal{L}_{AS}$ is a pair $\mathcal{M} = \langle W, [\![\cdot]\!]\rangle$, where W is a non-empty set and $[\![\cdot]\!] : \mathcal{V} \to \wp(W)$ is a function assigning to each variable a subset of W. ◁

Truth of sentences is defined as follows:

1. $\mathcal{M} \models \mathsf{All}\ X\ \mathsf{are}\ Y$ iff $[\![X]\!] \subseteq [\![Y]\!]$
2. $\mathcal{M} \models \mathsf{Some}\ X\ \mathsf{are}\ Y$ iff $[\![X]\!] \cap [\![Y]\!] \neq \emptyset$

Example 2.3 Suppose $\mathcal{M} = \langle W, [\![\cdot]\!]\rangle$, where $W = \{a, b, c\}$, $[\![X]\!] = \{a\}$, $[\![Y]\!] = \{a, c\}$, $[\![U]\!] = \{c, b\}$ and $[\![Z]\!] = \emptyset$. Then, it is easy to verify that:

1. $\mathcal{M} \models \mathsf{All}\ X\ \mathsf{are}\ Y$ and $\mathcal{M} \not\models \mathsf{All}\ Y\ \mathsf{are}\ X$
2. $\mathcal{M} \models \mathsf{Some}\ X\ \mathsf{are}\ Y$, $\mathcal{M} \models \mathsf{Some}\ U\ \mathsf{are}\ Y$, $\mathcal{M} \models \mathsf{Some}\ Y\ \mathsf{are}\ U$, and $\mathcal{M} \not\models \mathsf{Some}\ U\ \mathsf{are}\ X$
3. $\mathcal{M} \models \mathsf{All}\ Z\ \mathsf{are}\ Y$, $\mathcal{M} \models \mathsf{All}\ Z\ \mathsf{are}\ X$, and $\mathcal{M} \models \mathsf{All}\ Z\ \mathsf{are}\ Z$
4. $\mathcal{M} \not\models \mathsf{Some}\ Z\ \mathsf{are}\ Y$, $\mathcal{M} \not\models \mathsf{Some}\ Z\ \mathsf{are}\ X$, and $\mathcal{M} \not\models \mathsf{Some}\ Z\ \mathsf{are}\ Z$

◁

Let $\mathcal{M}$ be a model and Γ a set of sentences. We write $\mathcal{M} \models \Gamma$ provided $\mathcal{M} \models S$ for each $S \in \Gamma$.

Definition 2.4 (Semantic Consequence) Let Γ be a set of sentences and S a sentence. We write $\Gamma \models S$ (read "S semantically follows from Γ") provided for all models $\mathcal{M}$ if $\mathcal{M} \models \Gamma$ then $\mathcal{M} \models S$. ◁

A **proof rule** is a relation between a (possibly empty) set of sentences and a sentence. We denote proof rules as follows:

$$\frac{S_1 \quad S_2 \quad \cdots \quad S_n}{S}$$

Alternatively, we may write $R(\{S_1, \ldots, S_n\}, S)$ (where R is the name of the rule). The intended interpretation is that from S_1, S_2, $\ldots$, and S_n, one can infer S. An axiom system is a set of proof rules.

Definition 2.5 (Proof Tree) Let $\mathbf{A}$ be an axiom system and Γ a set of sentences. We write $\Gamma \vdash_{\mathbf{A}} S$ provided there exists a tree $\mathcal{T} = (T, E)$ where the set of nodes T are sentences in $\mathcal{L}_{AS}$, the root of T is S, and and for each $S' \in T$:

1. If S' is a leaf node, then $S' \in \Gamma$, or

2. there is a rule R in $\mathbf{A}$ such that $R(\mathbf{succ}(S'), S')$.

The tree $\mathcal{T}$ is called a proof tree. We say $\mathcal{T}$ is a proof tree for $(\Gamma, \mathbf{A})$ provided that it satisfies the above to properties for Γ and $\mathbf{A}$. ◁

The axiom system for the logic of "All" and "Some", denoted $\mathbf{A}_{AS}$ contains the following rules:

$$\frac{}{\textsf{All } X \textsf{ are } X} \qquad \frac{\textsf{All } X \textsf{ are } Y \quad \textsf{All } Y \textsf{ are } Z}{\textsf{All } X \textsf{ are } Z} \qquad \frac{\textsf{All } X \textsf{ are } Z \quad \textsf{Some } X \textsf{ are } Y}{\textsf{Some } Y \textsf{ are } Z}$$

$$\frac{\textsf{Some } X \textsf{ are } Y}{\textsf{Some } Y \textsf{ are } X} \qquad \frac{\textsf{Some } X \textsf{ are } Y}{\textsf{Some } X \textsf{ are } X}$$

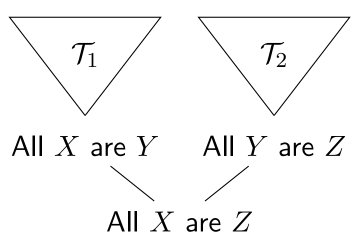
Theorem 2.6 (Soundness of $\mathbf{A}_{AS}$) *Let Γ be a set of sentences and S a setence. Then, $\Gamma \vdash_{\mathbf{A}_{AS}} S$ implies $\Gamma \models S$.*

Proof. We say that $\mathcal{T}$ is a proof tree for $(\Gamma, \mathbf{A}_{AS})$ if $\mathcal{T}$ satisfies the two conditions of Definition 2.5 for Γ and $\mathbf{A}_{AS}$. Given this notion, we want to prove a fact about all proof trees: For all proof tree $\mathcal{T}$, for all sentences S if $\mathcal{T}$ is a proof tree for $(\Gamma, \mathbf{A}_{AS})$ with root S, then $\Gamma \models S$. The proof is by induction on the height of trees.

Base Case: Suppose that $\mathcal{T}$ is a proof tree of height 0. Then $\mathcal{T}$ consists of exactly one node. Since $\mathcal{T}$ is a proof tree for $(\Gamma, \mathbf{A}_{AS})$, either $S \in \Gamma$ or S is of the form $\textsf{All } X \textsf{ are } X$. If $S \in \Gamma$ and $\mathcal{M} \models \Gamma$, then it is obvious that $\mathcal{M} \models S$. If S is of the form $\textsf{All } X \textsf{ are } X$, then any $\mathcal{M}$ makes S true (this follows from the fact that $[\![X]\!] \subseteq [\![X]\!]$ for any variable X). Hence, $\Gamma \models S$.

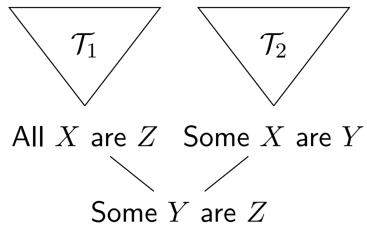
Induction Step: The Induction hypothesis is: for any proof tree $\mathcal{T}$ for $(\Gamma, \mathbf{A}_{AS})$ of height less than k with the sentence S as the root, $\Gamma \models S$. Suppose that $\mathcal{T}$ is a proof tree of height k with root S. There are four cases.

Case 1 The proof tree $\mathcal{T}$ is of the form:



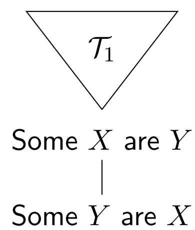
where $\mathcal{T}_1$ and $\mathcal{T}_2$ are proof trees of height less than k. By the induction hypothesis, $\Gamma \models \textsf{All } X \textsf{ are } Y$ and $\Gamma \models \textsf{All } Y \textsf{ are } Z$. Suppose that $\mathcal{M} \models \Gamma$. Then $\mathcal{M} \models \textsf{All } X \textsf{ are } Y$ and $\mathcal{M} \models \textsf{All } Y \textsf{ are } Z$. We must show $\mathcal{M} \models \textsf{All } X \textsf{ are } Z$. Suppose that $a \in [\![X]\!]$. Then, since $[\![X]\!] \subseteq [\![Y]\!]$, $a \in [\![Y]\!]$ and since $[\![Y]\!] \subseteq [\![Z]\!]$, we have $a \in [\![Z]\!]$. Hence, $\mathcal{M} \models \textsf{All } X \textsf{ are } Z$.

Case 2 The proof tree $\mathcal{T}$ is of the form:



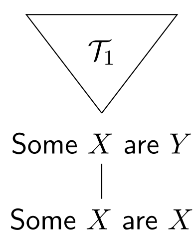
where $\mathcal{T}_1$ and $\mathcal{T}_2$ are proof trees of height less than k. By the induction hypothesis, $\Gamma \models$ All X are Z and $\Gamma \models$ Some X are Y. Suppose that $\mathcal{M} \models \Gamma$. Then $\mathcal{M} \models$ All X are Z and $\mathcal{M} \models$ Some X are Y. We must show $\mathcal{M} \models$ Some Y are Z. Since $[\![X]\!] \cap [\![Y]\!] \neq \emptyset$, there is a $a \in [\![X]\!] \cap [\![Y]\!]$. Then, since $[\![X]\!] \cap [\![Y]\!] \subseteq [\![X]\!] \subseteq [\![Z]\!]$, $a \in [\![Z]\!]$. Since, we also have $[\![X]\!] \cap [\![Y]\!] \subseteq [\![Y]\!]$, we have $a \in [\![Y]\!] \cap [\![Z]\!]$. Hence, $\mathcal{M} \models$ Some X are Z.

<u>Case 3</u> The proof tree $\mathcal{T}$ is of the form:



where $\mathcal{T}_1$ is a proof tree of height less than k. By the induction hypothesis, $\Gamma \models$ Some X are Y. Suppose that $\mathcal{M} \models \Gamma$. Then $\mathcal{M} \models$ Some X are Y. Hence, $[\![Y]\!] \cap [\![X]\!] = [\![X]\!] \cap [\![Y]\!] \neq \emptyset$. Therefore, $\mathcal{M}$Some Y are X.

<u>Case 4</u> The proof tree $\mathcal{T}$ is of the form:

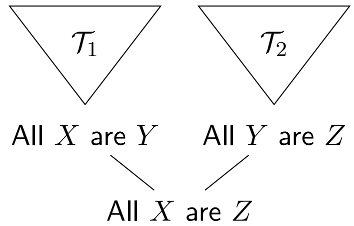


where $\mathcal{T}_1$ is a proof tree of height less than k. By the induction hypothesis, $\Gamma \models$ Some X are Y. Suppose that $\mathcal{M} \models \Gamma$. Then $\mathcal{M} \models$ Some X are Y. Since $[\![X]\!] \cap [\![Y]\!] \neq \emptyset$ there is an $a \in [\![X]\!] \cap [\![Y]\!] \subseteq [\![X]\!]$. So, $a \in [\![X]\!] = [\![X]\!] \cap [\![X]\!]$. Therefore, $\mathcal{M} \models$ Some X are X. QED

Definition 2.7 Suppose that Γ is a set of sentences. For $U, V \in \mathcal{V}$, define $U \leq_\Gamma V$ iff $\Gamma \vdash$ All U are V. When Γ is clear from the context, I write $U \leq V$.

Lemma 2.8 *For each Γ, the relation $\leq_\Gamma$ is reflexive and transitive.*

Proof. Since All X are X is a proof tree, we have $\Gamma \vdash$ All X are X for each variable X. Hence, for all $X \in \mathcal{V}$, $X \leq X$. For transitivity, suppose that $\Gamma \vdash$ All X are Y and $\Gamma \vdash$ All Y are Z. Then, the following is a proof tree:



Therefore, $\Gamma \vdash$ All X are Z. QED

If X is a variable, let $[X]{\downarrow} = \{Y \mid Y \leq X\}$ be the downset of X.

Theorem 2.9 $\Gamma \models S$ *implies* $\Gamma \vdash S$.

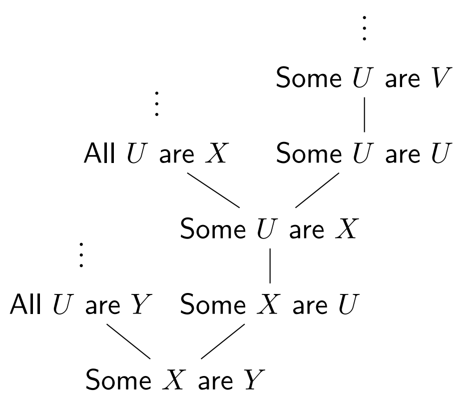
Proof. Let Γ be a set of sentences and suppose that $\Gamma \models S$. The proof splits into two cases depending on the form of S. The first case is when S is of the form Some X are Y. Construct a model $\mathcal{M}^S = \langle W^S, [\![\cdot]\!] \rangle^S$ as follows:

- $W^S = \{$Some U are $V \mid$ Some U are $V \in \Gamma\}$
- $[\![X]\!]^S = \{$Some U are $V \mid U \leq X$ or $V \leq X\}$

We first show that $\mathcal{M}^S \models \Gamma$. Suppose that All X are $Y \in \Gamma$. If $[\![X]\!]^S = \emptyset$, then clearly $[\![X]\!]^S \subseteq [\![Y]\!]^S$. Suppose that Some U are $V \in [\![X]\!]^S$. Then either $U \leq X$ or $V \leq X$. Since $\Gamma \vdash$ All X are Y, we also have $X \leq Y$. Then, since $\leq$ is transitive, we have either $U \leq Y$ or $V \leq Y$. Hence, Some U are $V \in [\![Y]\!]^S$. Thus, $\mathcal{M}^S \models$ All X are Y. Suppose that Some X are $Y \in \Gamma$. Then, since $X \leq X$, we have Some X are $Y \in [\![X]\!]^S$ and since $Y \leq Y$, we have Some X are $Y \in [\![Y]\!]^S$. Hence, Some X are $Y \in [\![X]\!]^S \cap [\![Y]\!]^S \neq \emptyset$. Therefore, $\mathcal{M}^S \models$ Some X are Y.

Next, we show that if $\mathcal{M}^S \models$ Some X are Y, then $\Gamma \vdash$ Some X are Y. Suppose that $\mathcal{M}^S \models$ Some X are Y. Then, we have Some U are $V \in [\![X]\!] \cap [\![Y]\!]$ for some Some U are $V \in \Gamma$. Since Some U are $V \in \Gamma$, we have $\Gamma \vdash$ Some U are V. Since, Some U are $V \in [\![X]\!] \cap [\![Y]\!]$, we have $U \leq X$ or $V \leq X$ and $U \leq Y$ or $V \leq Y$. There are four cases.

Case 1 Suppose that $U \leq X$ and $U \leq Y$, the $\Gamma \vdash$ All U are X and $\Gamma \vdash$ All U are Y. Then, the following is a proof tree for Some X are Y:



<u>Case 2</u> Suppose that $U \leq X$ and $V \leq Y$. Then $\Gamma \vdash$ All U are X and $\Gamma \vdash$ All V are Y. Then, the following is a proof tree for Some X are Y:

$$\dfrac{\begin{array}{c}\vdots\\ \textsf{All } V \textsf{ are } Y\end{array} \qquad \dfrac{\dfrac{\begin{array}{c}\vdots\\ \textsf{All } U \textsf{ are } X\end{array} \qquad \dfrac{\begin{array}{c}\vdots\\ \textsf{Some } U \textsf{ are } V\end{array}}{\textsf{Some } V \textsf{ are } U}}{\textsf{Some } V \textsf{ are } X}}{\textsf{Some } X \textsf{ are } V}}{\textsf{Some } X \textsf{ are } Y}$$

<u>Case 3</u> Suppose that $V \leq X$ and $U \leq Y$. Then, $\Gamma \vdash$ All V are X and $\Gamma \vdash$ All U are Y. Then, the following is a proof tree for Some X are Y:

$$\dfrac{\begin{array}{c}\vdots\\ \textsf{All } U \textsf{ are } Y\end{array} \qquad \dfrac{\dfrac{\begin{array}{c}\vdots\\ \textsf{All } V \textsf{ are } X\end{array} \qquad \begin{array}{c}\vdots\\ \textsf{Some } U \textsf{ are } V\end{array}}{\textsf{Some } U \textsf{ are } X}}{\textsf{Some } X \textsf{ are } U}}{\textsf{Some } X \textsf{ are } Y}$$

<u>Case 4</u> Suppose that $V \leq X$ and $V \leq Y$. Then $\Gamma \vdash$ All U are X and $\Gamma \vdash$ All V are Y. Then, the following is a proof tree for Some X are Y:

$$\dfrac{\begin{array}{c}\vdots\\ \textsf{All } V \textsf{ are } Y\end{array} \qquad \dfrac{\dfrac{\begin{array}{c}\vdots\\ \textsf{All } V \textsf{ are } X\end{array} \qquad \dfrac{\dfrac{\begin{array}{c}\vdots\\ \textsf{Some } U \textsf{ are } V\end{array}}{\textsf{Some } V \textsf{ are } U}}{\textsf{Some } V \textsf{ are } V}}{\textsf{Some } V \textsf{ are } X}}{\textsf{Some } X \textsf{ are } V}}{\textsf{Some } X \textsf{ are } Y}$$

In all cases, we have $\Gamma \vdash \textsf{Some } X \textsf{ are } Y$, as desired.

The proof of the statement follows from these two observations: Suppose that $\Gamma \models \textsf{Some } X \textsf{ are } Y$. Then, since $\mathcal{M}^S \models \Gamma$, we must have $\mathcal{M}^S \models \textsf{Some } X \textsf{ are } Y$. Hence, $\Gamma \vdash \textsf{Some } X \textsf{ are } Y$.

The second case is when S is of the form $\textsf{All } X \textsf{ are } Y$. Construct a model $\mathcal{M}^A = \langle W^A, [\![\cdot]\!]^A \rangle$ as follows:

- $W^A = \mathcal{V} \cup \{*\}$ where $* \notin \mathcal{V}$.
- $[\![X]\!]^A = \begin{cases} [X]\!\downarrow \cup \{*\} & \text{if } \textsf{Some } X \textsf{ are } Y \in \Gamma \text{ or } \textsf{Some } Y \textsf{ are } X \in \Gamma \\ [X]\!\downarrow & \text{otherwise} \end{cases}$

We first show that $\mathcal{M}^A \models \Gamma$. Suppose that $\textsf{All } X \textsf{ are } Y \in \Gamma$. Then $X \leq Y$. Suppose that $V \in [\![X]\!]^A$. Then, either V is a variable such that $V \leq X$ or V is $*$. In the first case, we have $V \leq X$ and $X \leq Y$. Since, $\leq$ is transitive, we have $V \leq Y$, and so $V \in [\![Y]\!]^A$. In the second case, we have either $\textsf{Some } X \textsf{ are } Y \in \Gamma$ or $\textsf{Some } Y \textsf{ are } X \in \Gamma$. In either case, we also have $* \in [\![Y]\!]^A$. Hence, $[\![X]\!]^A \subseteq [\![Y]\!]^A$. Suppose that $\textsf{Some } X \textsf{ are } Y \in \Gamma$. Then, we have $* \in [\![X]\!]^A$ and $* \in [\![Y]\!]^A$. Therefore, $[\![X]\!]^A \cap [\![Y]\!]^A \neq \emptyset$, so $\mathcal{M}^A \models \textsf{Some } X \textsf{ are } Y$.

We now show that if $\mathcal{M}^A \models \textsf{All } X \textsf{ are } Y$, then $\Gamma \vdash \textsf{All } X \textsf{ are } Y$. Suppose that $\mathcal{M}^A \models \textsf{All } X \textsf{ are } Y$. Then, $[\![X]\!]^A \subseteq [\![Y]\!]^A$. Since $X \in [\![X]\!]^A$ (this follows from the fact that $X \leq X$ for all variables), we have $X \in [\![Y]\!]^A$. Hence, $X \leq Y$. But this means, $\Gamma \vdash \textsf{All } X \textsf{ are } Y$, as desired.

The proof of the statement follows from these two observations: Suppose that $\Gamma \models \textsf{All } X \textsf{ are } Y$. Then, since $\mathcal{M}^A \models \Gamma$, we must have $\mathcal{M}^A \models \textsf{All } X \textsf{ are } Y$. Hence, $\Gamma \vdash \textsf{All } X \textsf{ are } Y$. QED

2.1 Completeness for a subclass of models

In this section, we show how to use the above completeness theorem to prove completeness for a subclass of models. Let $\mathbb{M} = \{\mathcal{M} \mid \text{ for all } X \in \mathcal{V}, [\![X]\!] \neq \emptyset\}$ be the class of models that assign *nonempty* subsets to variables. We write $\Gamma \models_{\mathbb{M}} S$ provided for each $\mathcal{M} \in \mathbb{M}$, if $\mathcal{M} \models \Gamma$ then $\mathcal{M} \models S$. Obviously, the above set of rules $\mathbf{A}_{AS}$ are sound for this class of models. The question is: Are they complete? It is not hard to see that the answer is "no". If we restrict to the models in $\mathbb{M}$, then the following rule becomes valid:

$$\frac{\textsf{All } X \textsf{ are } Y}{\textsf{Some } X \textsf{ are } Y}$$

Let $\mathbf{A}'$ be the set of rules in the axiom system $\mathbf{A}_{AS}$ together with the above rule. We now show that this axiom system is complete with respect to $\mathbb{M}$.

Theorem 2.10 $\Gamma \models_{\mathbb{M}} S$ *implies* $\Gamma \vdash_{\mathbf{A}'} S$.

Proof. Let $\overline{\Gamma} = \Gamma \cup \{\textsf{Some } X \textsf{ are } Y \mid \Gamma \vdash_{\mathbf{A}_{AS}} \textsf{All } X \textsf{ are } Y\}$. We first prove two claims.

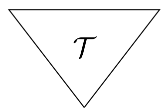
Claim 2.11 *If* $\Gamma \models_{\mathbb{M}} S$, *then* $\overline{\Gamma} \models S$.

Suppose that $\Gamma \models_{\mathbb{M}} S$ and $\mathcal{M} \models \overline{\Gamma}$. First, note that for each each $X \in \mathcal{V}$, we have $\textsf{Some } X \textsf{ are } X \in \overline{\Gamma}$ (this follows from the fact that for each $X \in \mathcal{V}$, $\Gamma \vdash \textsf{All } X \textsf{ are } X$). Since $\mathcal{M} \models \overline{\Gamma}$, for each $X \in \mathcal{V}$, $\mathcal{M} \models \textsf{Some } X \textsf{ are } X$. Hence $\emptyset \neq [\![X]\!] \cap [\![X]\!] = [\![X]\!]$. Therefore, $\mathcal{M} \in \mathbb{M}$. Furthermore, since $\Gamma \subseteq \overline{\Gamma}$, we have $\mathcal{M} \models \Gamma$. Since $\Gamma \models_{\mathbb{M}} S$. This implies, $\mathcal{M} \models S$. Hence, $\overline{\Gamma} \models S$. This completes the proof of the first claim.

Claim 2.12 *If* $\overline{\Gamma} \vdash_{\mathbf{A}_{AS}} S$*, then* $\Gamma \vdash_{\mathbf{A}'} S$.

Suppose that $\overline{\Gamma} \vdash_{\mathbf{A}_{AS}} S$. The proof is by induction on the height of proof trees.

Base Case: Suppose that $\mathcal{T}$ is a proof tree of height 0. This means that either S is $\textsf{All } X \textsf{ are } X$ or $S \in \overline{\Gamma}$. Suppose that S is $\textsf{All } X \textsf{ are } X$. Since $\dfrac{}{\textsf{All } X \textsf{ are } X}$ is a proof rule in $\mathbf{A}'$, we have $\Gamma \vdash_{\mathbf{A}'} S$. If $S \in \Gamma$, then we are also done since this immediately implies $\Gamma \models S$. Suppose that S is of the from $\textsf{Some } X \textsf{ are } Y$ with $\Gamma \vdash_{\mathbf{A}_{AS}} S$. Since all the $\mathbf{A}'$ contains all the rules of $\mathbf{A}_{AS}$, the following is a proof tree in $\mathbf{A}'$:



$$\textsf{All } X \textsf{ are } Y$$
$$|$$
$$\textsf{Some } X \textsf{ are } Y$$

where $\mathcal{T}$ is the proof tree for $\textsf{All } X \textsf{ are } Y$ using the rules from $\mathbf{A}_{AS}$. Hence $\Gamma \vdash_{\mathbf{A}'} S$.

Induction Step: Since $\mathbf{A}'$ contains all the rules of $\mathbf{A}_{AS}$, all extensions of proof trees for the axiom system $\mathbf{A}'$ using rules from $\mathbf{A}_{AS}$ are still proof trees in the axiom system $\mathbf{A}'$.

This completes the proof of the claim.

Putting everything together: Suppose that $\Gamma \models_{\mathbb{M}} S$. Then, $\overline{\Gamma} \models S$ by Claim 2.11. By the completeness theorem for $\mathbf{A}_{AS}$, we have $\overline{\Gamma} \vdash_{\mathbf{A}_{AS}} S$. By Claim 2.12, $\Gamma \vdash_{\mathbf{A}'} S$, as desired. QED