Problem Set 3

1. Show by example that $\varphi[y/x][x/y]$ is not in general equal to φ.
2. Prove that $\vdash \forall x(\alpha \wedge \beta) \leftrightarrow \forall x\alpha \wedge \forall x\beta$
3. For each of the following sentences, check whether they are valid or not. If a sentence is valid, show that it has a deduction (you can use the meta-theorems given in the notes) If a sentence is invalid, give a structure where that sentence is false.
   (a) $\exists x(\forall y Qy \rightarrow Qx)$
   (b) $(\exists x Px \rightarrow \forall y Qy) \rightarrow \forall z(Pz \rightarrow Qz)$
   (c) $\forall z(Pz \rightarrow Qz) \rightarrow (\exists x Px \rightarrow \forall y Qy)$
   (d) $\exists y \forall x(Pxy \leftrightarrow \neg Pxx)$

The midterm is DUE Tuesday, December 4 in class.