Rationality

Lecture 3

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February 7, 2011

What does it mean to be *rational* or *reasonable* as opposed to *irrational* or unreasonable?

Rationality designates a capacity or set of capacities: an agent is **rational** to the degree that he or she possesses and manifests the relevant range of capacities.

- the capacity to recognize or make correct judgements about reasons and other normative facts or truths
- the capacity to *reason* well — to engage in "rational forms of reasoning", to have one's reflections and deliberations proceed in ways that satisfy various formal constraints.

Reasoning

We have already distinguished between **practical** and **theoretical** reasoning:

- Practical reasoning is reasoning directed toward action: figuring out what to *do*
- Theoretical reasoning is reasoning directed towards an *informational state*: figuring out how the facts stand.

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Rational Beliefs

Beliefs can represent the world more or less accurately....the more accurate the better.

But we can also judge some beliefs as being more *rational* than others.

Accuracy and rationality are linked, they are not the same: a fool may hold a belief irrationally — as a result of a lucky guess or wishful thinking — yet it might happen to be correct. Conversely, a detective might hold a belief on the basis of a careful and exhaustive examination of all the evidence and yet the evidence may be misleading, and the belief may turn out to be wrong.

Theoretical Reasoning

Rational beliefs are those that arise from **good thinking**, whether or not that thinking was successful in latching on to the truth.

But, what is **good thinking**?

- classical logic (modus ponens, modus tollens, etc.)
- non-monotonic/default logic
- closed-world reasoning
- induction (induction from examples)
- Abdunction (inference to the best explanation)
- Bayesian inference
- case-based reasoning/reasoning by analogy
- fast and frugal heuristics

Reasoning

Reasoning is a "transition in thought, where some beliefs (or thoughts) provide the ground or reasons for coming to another"

J. Adler. *Introduction: Philosophical Foundations*. in *Reasoning: Studies in Human Inference and its Foundations*, Cambridge University Press, 2008.

(1) Ann believes that Bill's final grade is either a 6 or a 9.

(2) Ann believes that Bill's final grade is not a 6.

So, (3) Ann believes that Bill's final grade is a 9.

(1') Bill's final grade is either a 6 or a 9.

(2') Bill's final grade is not a 6.

So, (3') Bill's final grade is a 9.

(1) Bill brought his backpack to class every day of the semester.

So, [probably] (2) Bill will bring it to the next class.

(1) I need to catch the train at 9.09

Oh, (2) I better put the slides on the website.

What are the rules or formal constraints that govern *rational* transitions in thought?

Classical Logic and Rational Beliefs

- Cognitive limitations
- Are logically omniscient agents rational?
- Deduction reasoning may lead to *revising*
- Foundational issues
- Ordinary language challenges
- Psychology of reasoning

Classical Logic: Inference Rules

Deductive cogency: a rational agent's beliefs are logically **consistent** and **closed under deduction**

Rules of inference:

- *Modus Ponens*: P, $P \rightarrow Q \vdash Q$
- *Modus Tollens*: $\neg Q$, $P \rightarrow Q \vdash \neg P$
- *Disjunctive Syllogism*: $P \vee Q$, $\neg P \vdash Q$
- *Adjunction*: P_1, P_2,... $P_n \vdash P_1 \wedge \cdots \wedge P_n$
- *Noncontradiction*: P, $\neg P \vdash Q$
- *Monotonicity*: $P \rightarrow Q \vdash (P \wedge R) \rightarrow Q$; $P \vdash Q$ implies $P, R \vdash Q$

Inference and Reasoning vs. Implication and Consistency

The relationship between logical implication and what is reasonable to believe is very complex!

1. Ann believes that P is true; Ann believes that $P \rightarrow Q$ is true; So, Ann (ought to, may, should, is rationally required to) believes that Q is true
2. P is true; $P \rightarrow Q$ is true; So, Q is true.

A set of formulas is **inconsistent** if there is no way of making all of the formulas true

1. Ann recognizes that $\{P, Q, R\}$ are inconsistent
2. $\{P, Q, R\}$ are inconsistent

Rationality versus genius

A, B, C imply D. Sam believes A, B and C. But some does nto realize that A, B, C imply D. In fact, it would take a genius to recognize that $A, B, C \vdash D$. And Sam, although a rational man, is far from a genius.

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Clutter Avoidance

$P \vdash P \vee Q$

Our limits restrict the resources and times to devote to empirical search, testing and inquiry, as *well as to the inference worth carrying out*.

From "It is raining in Tilburg" to "It is raining in Tilburg or Lily is in Amsterdam" is a valid inference. In fact, there are infinitely many such trivial consequences (p, $p \vee q$, $p \wedge p$, $p \wedge (q \wedge q)$, $p \rightarrow p$, $p \vee q \vee r$, etc.), but these will just "clutter the mind".

Also, if one "looses" the origination of this disjunctive belief, one may be mislead to think that there is a special reason to believe Lily is in Amsterdam or there is a special connection between rain in Tilburg and Lily being in Amsterdam.

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Discovering a Contradiction

Sally believes A, B, C and has just come to realize that $A, B, C \vdash D$. Unfortunately, she also believes for very good reasons that D is false. So she now has reason to stop believing A, B or C, rather than a reason to believe D.

Reasoning May Lead to Revising

Modus Ponens: $P, P \rightarrow Q \vdash Q$

Suppose that Ann believes that if she will attend Yale, then she will become an atheist. She also believes that she will attend Yale.

She concludes that she will become an atheist.

But although MP gives Ann a reason to believe the conclusion, it does not decide that she will believe it. Instead of believing the conclusion, she may decide to drop her belief in the conditional.

Reasoning

"Reasoning is not the conscious rehearsal of argument; it is a process in which antecedent beliefs and intentions are minimally modified, by addition and subtraction, in the interests of explanatory coherence and the satisfaction of intrinsic desires." (G. Harman, pg. 56, "Practical Reasoning")

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Epistemic Closure

Epistemic Closure EC: If i knows that P and i knows that P implies Q, then i knows that Q.

(1) The animal I am looking at is a zebra.

(2) If the animal I am looking at is a zebra, then it is not a mule cleverly disguised to look like a zebra.

(3) The animal I am looking at is not a mule cleverly disguised to look like a zebra.

S. Luper. *The Epistemic Closure Principle*. Stanford Encyclopedia of Philosophy: `http://plato.stanford.edu/entries/closure-epistemic/`.

Transfer of Warrant

(1) The Smiths are making an extravagant wedding for their daughter.

(2) The Smiths are wealthy.

(3) In making the extravagant wedding, the Smith are not just appearing to be wealthy.

- Suppose that you are arguing about whether the Smiths are really wealthy.
- It *begs the question* to use (2) as a reason to believe (3).
- (1) can only provided evidence for (2) if (3) is presupposed
- The warrant or support that (1) lends to (2) does not *transmit* to the conclusion (3).

C. Wright. *Cogency and Question-Begging: Some Reflections on McKinsey's Paradox and Putnam's Proof*. Philosophical Issues 10 Skepticism, pgs. 140 - 163, 2000.

Dogmatism Paradox

"If I know that h is true, I know that any evidence against h is evidence against something that is true; I know that such evidence is misleading. But I should disregard evidence that I know is misleading. So, once I know that h is true, I am in a position to disregard any future evidence that seems to tell against h."

G. Harman. *Thought*. Princeton University Press, 1973.

1. My car is in the parking lot.
2. If my car is in the parking lot and Doug reports otherwise, then Doug's report is misleading.
3. If Doug reports that my car is not in the parking lot, then his report is misleading.
4. Doug reports that my car is not in the parking lot.
5. Doug's report is misleading.

Dogmatism Paradox

If there is evidence against my knowledge, then that evidence is mistaken or misleading.

$\vdash p \rightarrow [(q \rightarrow \neg p) \rightarrow \neg q]$

If my wife is cheating on me, I would never know.

The Scandal of Deduction

"... in addition to this scandal of induction there is an equally disquieting scandal of deduction. Its urgency can be brought home to each of us by any clever freshman who asks, upon being told that deductive reasoning is tautological or analytical and that logical truths have no empirical content and cannot be used to make factual assertions: in what other sense, then, does deductive reasoning give us new information? Is it not perfectly obvious there is some such sense, for what point would there otherwise be to logic and mathematics?"

J. Hintikka. *Logic, language games and information. Kantian themes in the philosophy of logic*. Oxford: Clarendon Press, 1973.

The Scandal of Deduction

" If that were correct, all that deductive inference could accomplish would be to render explicit knowledge that we already possessed: mathematics would be merely a matter of getting things down on paper, since, as soon as we had acknowledged the truth of the axioms of a mathematical theory, we should thereby know all the theorems. Obviously, this is nonsense: deductive inference has here been justified at the expense of its power to extend our knowledge and hence of any genuine utility."

M. Dummett. *The logical basis of metaphysics*. 1991.

M. D'Agostino and L. Floridi. *The Enduring Scandal of Deduction*. Synthese, 2008.

Classical Logic and Rational Beliefs

- ✓ Cognitive limitations: rationality $\neq$ genius
- ✓ Are logically omniscient agents rational? No.
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- Ordinary language challenges
- Psychology of reasoning

Ordinary Language Challenges

1. John goes drinking and John gets arrested.
2. John gets arrested and John goes drinking.

1. John will order either pasta or steak, but he order pasta.
2. John does not order steak.

1. If you tutor me in logic, I'll pay you 50 EUR.
2. If you don't tutor me, I won't pay you 50 EUR.

Ordinary Language Challenges: Gricean Implicature

> *He [the speaker] has said that p; there is no reason to suppose that he is not observing the maxims, or at least the Cooperative Principle; he could not be doing this unless he thought that q; he knows (and knows that I know that he knows) that I can see the supposition that he thinks that q is required....he intends me to think...that q; and so he has implicated q.*

Cooperative Principle: The speaker intends his contribution to be informative, warranted, relevant and well formed.

H. P. Grice. *Studies in the Way of Words*. Harvard University Press, 1989.

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Wason Selection Task

You are shown a set of four cards placed on a table, each of which has a number on one side and a letter on the other side. Also below is a rule which applies only to the four cards. Your task is to decide which if any of these four cards you *must* turn in order to decide if the rule is true. Don't turn unnecessary cards.

Rule: If there is a vowel on one side, then there is an even number on the other side.

P. C. Wason. *Reasoning about a rule*. Quarterly Journal of Experimental Psychology, 20:273 - 281, 1968.

Rule: If there is a vowel on one side, then there is an even number on the other side.

Rule: If there is a vowel on one side, then there is an even number on the other side.

Wason Selection Taks: Analysis

- Reasoning *to* an interpretation vs. reasoning *from* an interpretation
- How do people interpret *rules* or *if, then* statements?

K. Stenning and M. van Lambalgen. *Human Reasoning and Cognitive Science*. The MIT Press, 2008.

"Common Sense" Reasoning

(1) Bill brought his backpack to class every day of the semester.

So, [probably] (2) Bill will bring it to the next class.

(1.1) Bill's backpack was stolen.

(3) Tweety is a bird

So, (4) Tweety flies.

(3.1) Tweety is a penguin.

Beyond Classical Logic

- Non-classical rules
- Reasoning under uncertainty/"common-sense reasoning"
- Foundational issues
- Common fallacies

Non-Monotinicity

If $A \vdash B$ holds then $A, C \vdash B$ also holds.

Conclusions that are reasonable on the basis of specific information can become unreasonable if further information is added. Given the announced schedule for the course, and your previous experience, and that today is Thursday, it is reasonable to conclude that the course will meet in the evening. However upon learning there is an announcement on the website that class is canceled, then it is reasonable to drop this belief. Further, if it is discovered that there was a mistake on the website, then it is reasonable to believe that there will be class.

$A \rightarrow B \vdash (A \wedge C) \rightarrow B$

'If you put sugar in the coffee, then it will taste good' can be true without 'If you put sugar and gasoline in the coffee, then it will taste good' being true.

Closed-world reasoning

Negation as failure

Suppose you are interested in whether there are any direct flights from Amsterdam to Cleveland, Ohio.

After searching online at a number of relevant sites (Expedia, Orbitz, KLM, etc.), you do not find any. You conclude that there are *no direct flights between Amsterdam and Cleveland*.

Beyond Classical Logic

- ✓ Non-classical rules: non-monotonicity, closed-world reasoning
- Induction/"common-sense reasoning"
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Induction

Enumerative Induction
Given that all observed Fs are Gs, you infer that all Fs are Gs, or at least the next F is a G.

Inference to the best explanation
Holmes infers the best explanation for footprints, the absence of barking, the broken window: 'The butler wears size 10 shoes, is known to the dog, broke the window to make it look like a burglary...'

Scientific hypothetic induction
Scientists infer that Brownian motion is caused by the movement of invisible molecules.

Beyond Classical Logic

- ✓ Non-classical rules: non-monotonicity, closed-world reasoning
- ✓ Induction/"common-sense reasoning": default logic, non-monotonic logic, inductive logic, defeasible reasoning, Bayesian inference, reasoning under uncertainty, etc.
- Foundational issues
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Hume: Does positive inductive evidence support rational beliefs?

In the past, Fs have been followed by Gs (and never by non-Gs)

So, the present case of an F will be followed by a G
In the past, Fs have been followed by Gs (and never by non-Gs)

UN: Nature is uniform (at least in regard to Fs followed by Gs).

So, the present case of an F will be followed by a G
In the past, nature has been uniform (at least in regard to Fs followed by Gs)

The present case is an instance of that uniformity

The Ravens Paradox

(IC) A hypothesis of the form "All As are Bs ($\forall x(A(x) \rightarrow B(x))$)" is confirmed by any positive instance "Aa & Ba".

(EQ) If H and H' are logically equivalent, then if e confirms H, e confirms H'.

H: All ravens are black.

H': All nonblack things are nonravens.

But, then does a red jacket confirm H?

Goodman's New Riddle of Induction

All emeralds examined thus far are green.

This leads us to conclude (by induction) that (H1) all emeralds are green, and every next green emerald discovered strengthens this belief.

Call an emerald **grue** if "it is green before time t and blue after time t."

Suppose that t is some time in the future. Let H2 be "all emeralds are grue".

The data collected thus far seems to confirm H1 as well as H2, but H1 seems to be a "better explanation"...

N. Goodman. *Fact, Fiction and Forecast*. Bobbs-Merrill, 1965.

Probabilities

Maybe we should give up qualitative confirmation for a quantitative approach through probability.

There are a huge number of nonblack things as well as nonravens, the antecedent probability of finding a nonraven among nonblack things is extremely high. Consequently, finding a nonbalck nonraven only slightly increase the probability of "All ravens are black."

e supports h if the probability of h given e and the background information is greater than the probability of h given the background information alone:

$$p(h \mid e\&b) > p(h \mid b).$$

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- ✓ Non-classical rules: non-monotonicity, closed-world reasoning
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- ✓ Foundational issues: Hume, paradox of the ravens, Goodman's new paradox of induction
- ▶ Common fallacies

Conjunction Fallacy

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

1. Linda is a bank teller.
2. Linda is a bank teller and is active in the feminist movement.

Typically a large percentage of people asked say 2 is more probable than 1.

A. Tversky and D. Kahneman. *Extensions versus intuitive reasoning: The conjunciton fallacy in probability judgment*. Psychological Review 90 (4): 293 - 315, 1983.

Base-Rate Fallacy

In a city of 1 million inhabitants there are 100 known terrorists and 999,900 non-terrorists. The base rate probability of one random inhabitant of the city being a terrorist is thus $\frac{100}{1{,}000{,}000}$.

In an attempt to catch the terrorists, the city installs a surveillance camera with automatic facial recognition software. If one of the known terrorists is seen by the camera, the system has a 99% probability of detecting the terrorist and ringing an alarm bell. If the camera sees a non-terrorist, it will only incorrectly trigger the alarm 1% of the time.

Suppose somebody triggers the alarm. What is the chance he/she is really a terrorist?

Common Answer: $p(T|B) = p(B|T) = 99\%$

$p(T|B) = p(B|T)\frac{p(T)}{p(B)} = 0.99(100/1,000,000)/[(0.99 \cdot 100 + 0.01 \cdot 999900)/1,000,000] = 1/102 \approx 0.98\%$

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- ✓ Foundational issues: Hume, paradox of the ravens, Goodman's new paradox of induction
- ✓ Common fallacies: the Linda problem, base-rate fallacy

Conclusions: Rules of Reasoning

- *Normative*: reasoning as it should be, ideally
  Modus Tollens, Bayes Theorem

- *Descriptive*: reasoning as it is actually practiced
  many people do not endorse Modus Tollens or make base rate fallacies

- *Prescriptive*: take into account bounded rationality (computational limitations, storage limitations)
  closed-world reasoning, heuristics

Conclusion: Positions

- Human reasoning is normatively correct. What appears to be incorrect reasoning can be explained by various maneuvers, such as different interpretation of logical terms, etc.
- Actual human performance follows prescriptive rules, but they are not the normative rules because of the heavy demands of normatively correct reasoning
- Actual human reasoning falls short of prescriptive standards, so there is room for improvement by suitable education
- Reasoning rarely happens in real life: we have developed "fast and frugal algorithms" which allow us to take quick decisions which are optimal given constraints of time and energy.

Conclusion

"A partial diagnosis of the situation turns on a misunderstanding which has haunted the discussion of such ideas as "rule of inference".... In games, there are rules and there are rules. There are such rules as serve to define the game, e.g., the rules of chess. I shall call them 'definitory rules'. They tell which moves are possible, or, as it is sometimes put, which moves are admissible. The crucial fact about definitory rules is that they say absolutely nothing about which moves are good, which ones are bad and which ones are better than others. Such questions are handled by rules of another kind. I shall call them 'strategic rules'. They have to be distinguished from definitory rules. Admittedly, the notion of strategy in a given game is possible to define only after the definitory rules have been set up. Only after that has been done can we hope to begin to investigate which strategies are better than others."

J. Hintikka. *Inquiry as Inquiry*. Kluwer Academic Publishers, 1999.

Next Week: Practical Reasoning and Reasons for Action