Rationality

Lecture 13

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Additional topics

- Manipulation ▸ Skip
- Fair division ▸ Skip
- Sen's liberal paradox ▸ Skip

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(Taken from A. Taylor *Social Choice and the Mathematics of Manipulation* who took it from D. Black *A Theory of Committees and Elections* who took it from Dodgson.)

Manipulation: setting the agenda

# voters	1	1	1
	B	A	C
	D	B	A
	C	D	B
	A	C	D

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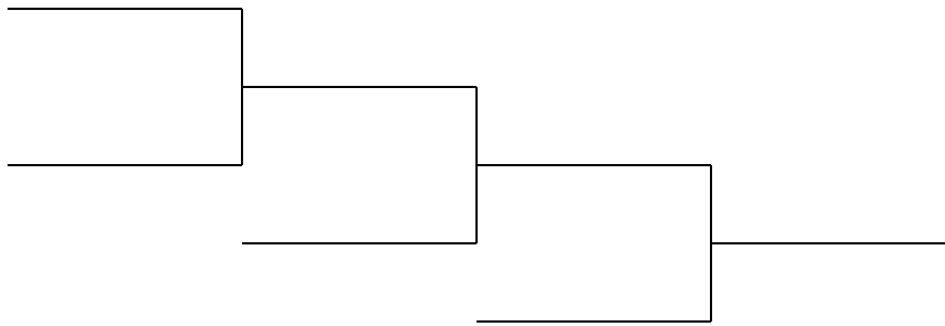

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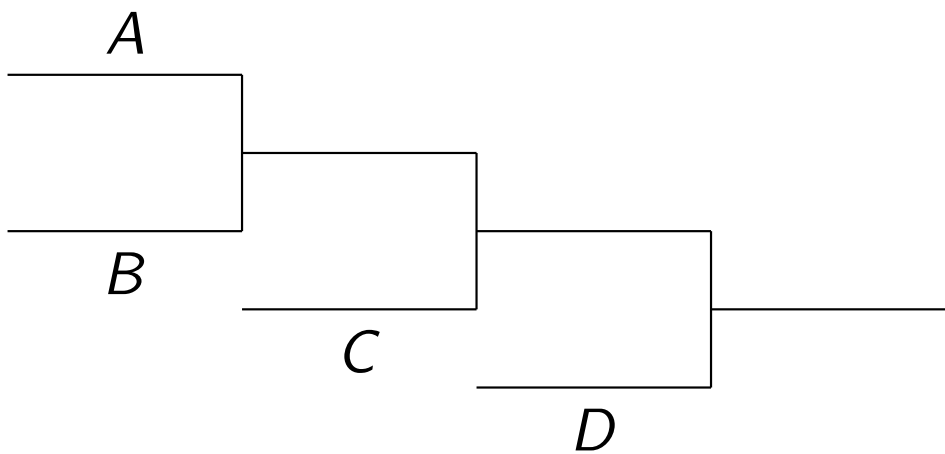

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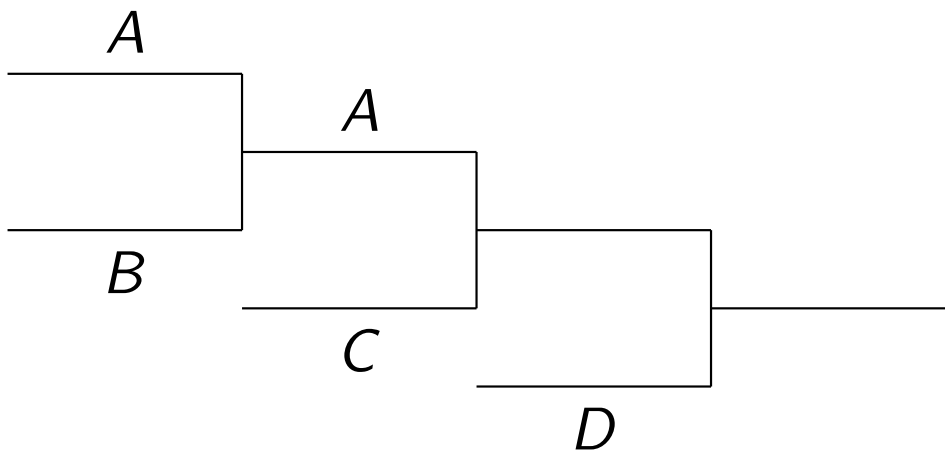

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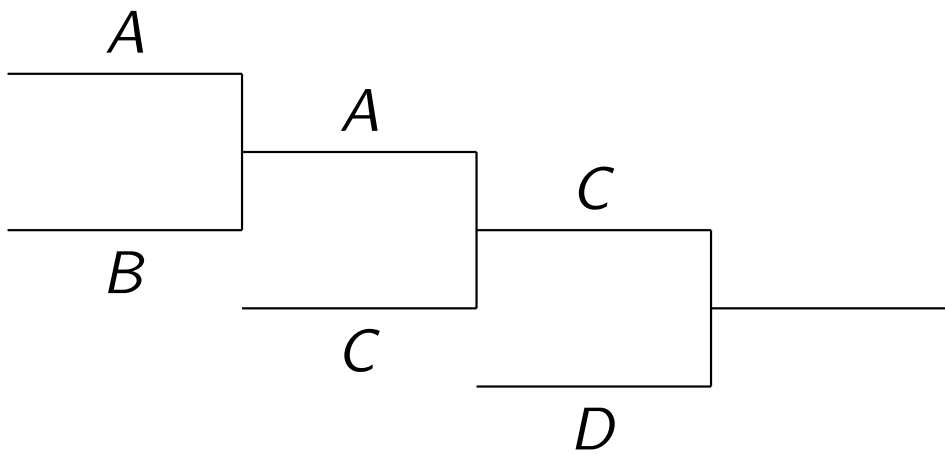

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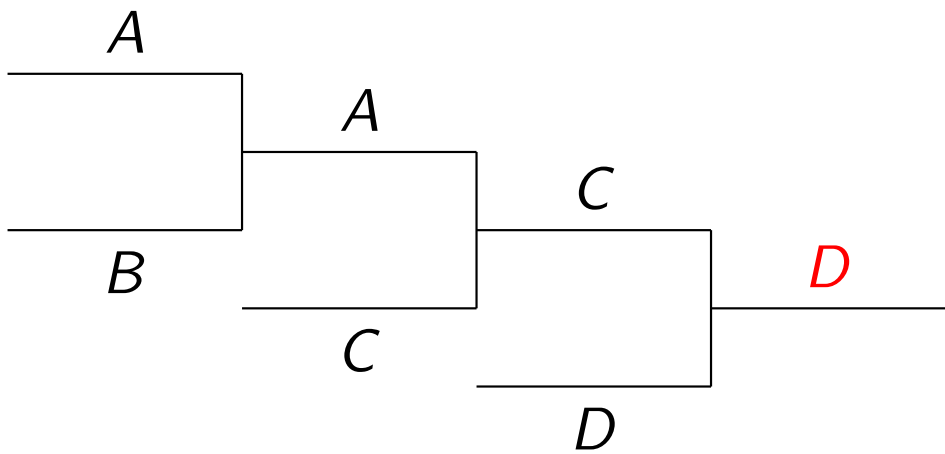

Manipulation: misrepresenting preferences

# voters	3	3	1
	A	B	C
	B	A	A
	C	C	B

Borda Winner: *A*

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Borda Winner: *A*

# voters	3	3	1
	A	B	C
	B	C	A
	C	A	B

Borda Winner: *B*

Strategizing

Gibbard-Satterthwaite Theorem There must be situations where it 'profits' a voter to vote *strategically*, i.e., not according to his or her *actual preference*.

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Gibbard-Satterthwaite Theorem There must be situations where it 'profits' a voter to vote *strategically*, i.e., not according to his or her *actual preference*.

Under suitable conditions,

1. If P denotes the actual preference ordering of voter i,
2. and $\vec{Y}$ denotes the profile consisting of the preference orderings of all the other voters,
3. and F the aggregation rule,

Then the theorem says that there must exist P, Y, P' such that $F(P', Y) >_P F(P, Y)$.

A. Gibbard. *Manipulation of Voting Schemes: A General Result*. Econometrica, 1973.

M. Satterthwaite. *Strategy-Proofness and Arrow's Conditions*. Journal of Economic Theory (1975).

Additional topics

- ✓ Manipulation ▸ Skip
- ▸ Fair division ▸ Skip
- ▸ Sen's liberal paradox ▸ Skip

Fair Division

S. Brams, P. Edelman and P. Fishburn. *Paradoxes of Fair Division*. Journal of Philosophy, **98:6**, pgs. 300-314.

J. Robertson and W. Webb. *Cake-Cutting Algorithms: Be Fair if You Can*. A.K. Peters, 1998.

S. Brams and A. Taylor. *Fair Division: From cake-cutting to dispute resolution*. Cambridge University Press, 1998.

S. Brams and A. Taylor. *The Win-Win Solution*. W. W. Norton & Company, 2000.

Fairness Conditions

- **Proportional:** (for two players) each player receives at least 50% of their valuation.

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- **Equitable:** each player values its allocation the same *according to its own valuation function*.
- **Efficiency:** there is no other division better for everybody, or better for some players and not worse for the others

Envy-Freeness and Efficiency

Ann:	1	$\succ$	2	$\succ$	3	$\succ$	4	$\succ$	5	$\succ$	6
Bob:	4	$\succ$	3	$\succ$	2	$\succ$	1	$\succ$	5	$\succ$	6
Cath:	5	$\succ$	1	$\succ$	2	$\succ$	6	$\succ$	3	$\succ$	4

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Ann: $\{1, 3\}$
Bob: $\{2, 4\}$
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Adjusted Winner

Adjusted winner (*AW*) is an algorithm for dividing n divisible goods among two people (invented by Steven Brams and Alan Taylor).

For more information see

- *Fair Division: From cake-cutting to dispute resolution* by Brams and Taylor, 1998
- *The Win-Win Solution* by Brams and Taylor, 2000
- `www.nyu.edu/projects/adjustedwinner`

Item	Ann	Bob
A		
B		
C		

Suppose Ann and Bob are dividing three goods (A, B, C)

Item	Ann	Bob
A	5	4
B	65	46
C	30	50
Total	100	100

Suppose Ann and Bob are dividing three goods (A, B, C)

Point Assignment: Both Ann and Bob distribute 100 points among the three items

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Item	Ann	Bob
A	5	0
B	65	0
C	0	50
Total	70	50

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Winner Take All: The person who assigned the most points is given that good

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Find the item whose ratio is closes to 1: $65/46 \geq 5/4 \geq 1 \geq 30/50$

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Still not equal, so give (some of) B to Bob: $65p = 100 - 46p$ yielding $p = \frac{100}{111} = 0.901$

Item	Ann	Bob
A	5	4
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C	30	50
Total	100	100

Item	Ann	Bob
A	0	4
B	58.56	4.56
C	0	50
Total	58.56	58.56

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Easy Observations

- For two-party disputes, proportionality and envy-freeness are equivalent.
- *AW* only produces equitable allocations (equitability is essentially built in to the procedure).
- *AW* produces allocations σ that in which at most one good is split.

Adjusted Winner is Fair

Theorem (Brams and Taylor) *AW produces allocations that are efficient, equitable and envy-free (with respect to the announced valuations)*

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However, while honesty may not always be the best policy it is the only **safe** *one, i.e., the only one which will guarantee 50%.*

Strategizing

Item	Ann	Bob
Matisse	75	25
Picasso	25	75

Ann will get the Matisse and Bob will get the Picasso and each gets 75 of his or her points.

Strategizing: Example

Suppose Ann knows Bob's preferences, but Bob does not know Ann's.

Item	Ann	Bob
M	75	25
P	25	75

Item	Ann	Bob
M	26	25
P	74	75

So Ann will get M plus a portion of P.

According to Ann's announced allocation, she receives 50 points

According to Ann's actual allocation, she receives
$75 + 0.33 * 25 = 83.33$ points.

Strategizing: Example

Suppose *both* players know each other's preferences but neither knows that the other knows their own preference.

Item	Ann	Bob
M	75	25
P	25	75

Item	Ann	Bob
M	26	74
P	74	26

Each will get 74 of his or her announced points, but each one is really getting only 25 of his or her *true* points.

Strategizing: Example

Suppose *both* players know each other's preferences. Moreover, Ann knows that Bob knows her preference and Bob doesn't know that Ann knows.

Item	Ann	Bob
M	26	74
P	74	26

Item	Ann	Bob
M	73	74
P	27	26

What happens as the level of knowledge increases?

Additional topics

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l: Lewd reads the book;
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$l \rightarrow p$: If Lewd reads the book, then so does Prude.

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Prude desires to not read the book, and that Lewd not read it either, but in case Lewd does read the book, Prude wants to read the book to be informed about the dangerous material Lewd has read.

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	l	p	$l \rightarrow p$

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	l	p	$l \rightarrow p$
Lewd	True	True	True

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2. Unanimous desires of all individuals must be respected.

So, society must be inconsistent!

Conclusions

Given a group of people faced with some decision, how should a central authority combine the individual opinions so as to best reflect the "will of the group"?

- Majority rule is the "best" for two candidates (May's Theorem), but it does not generalize (Condorcet Paradox)
- Many different procedures (Plurality, Plurality with runoff, Borda Count, Approval)
- Failure of monotonicity (multi-stage elections, no show paradox)
- Different normative constraints on group decision methods are in conflict (Arrow's Theorem)

Some Literature

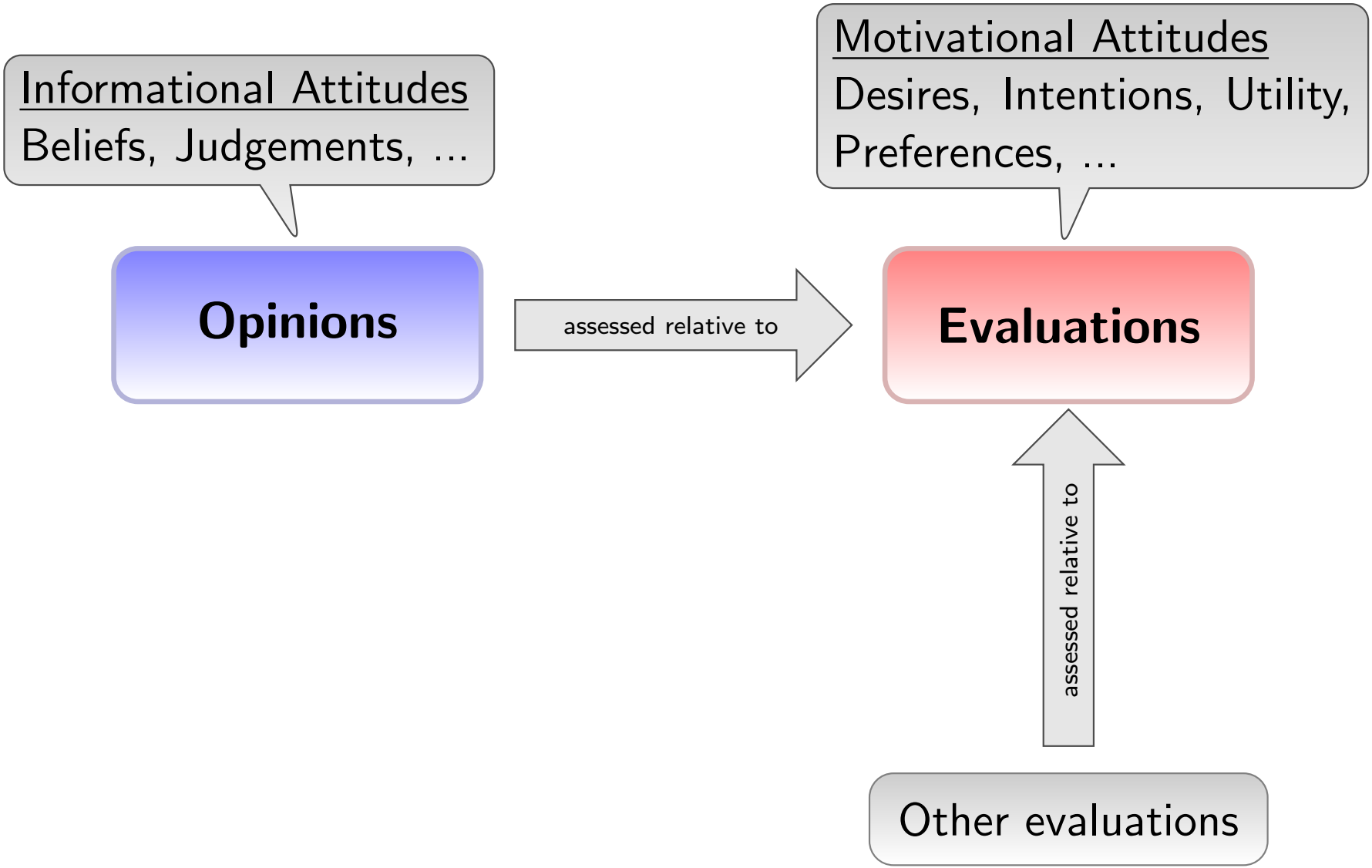
S. Brams. *Mathematics and Democracy*. Princeton University Press, 2008.

EP. *Voting Methods*. Article prepared for SEP.

W. Poundstone. *Gaming the Vote*. Hill and Wang, 2008.

D. Saari. *Decisions and Elections; Explaining the Unexpected*. Cambridge University Press, 2001.

Concluding Remarks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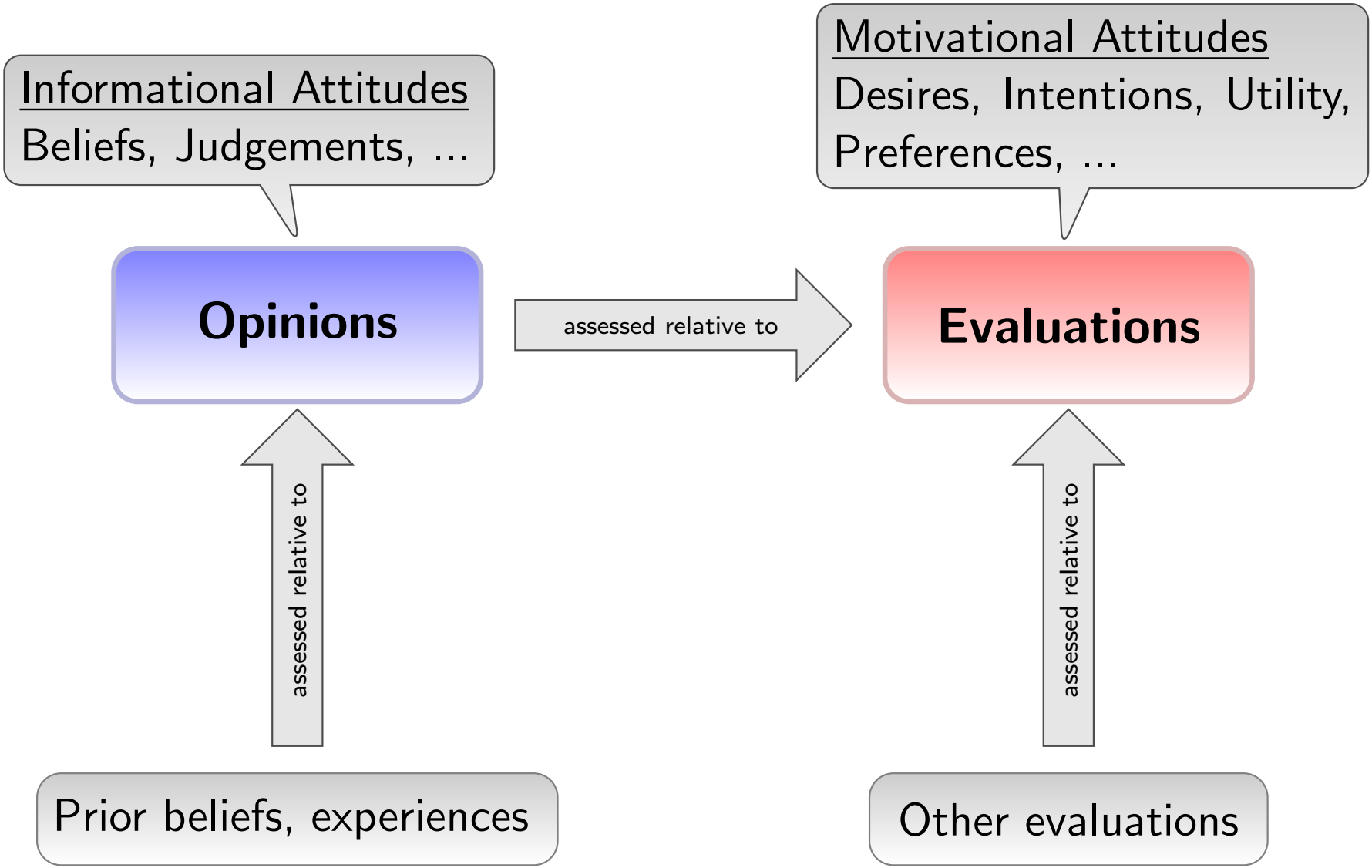
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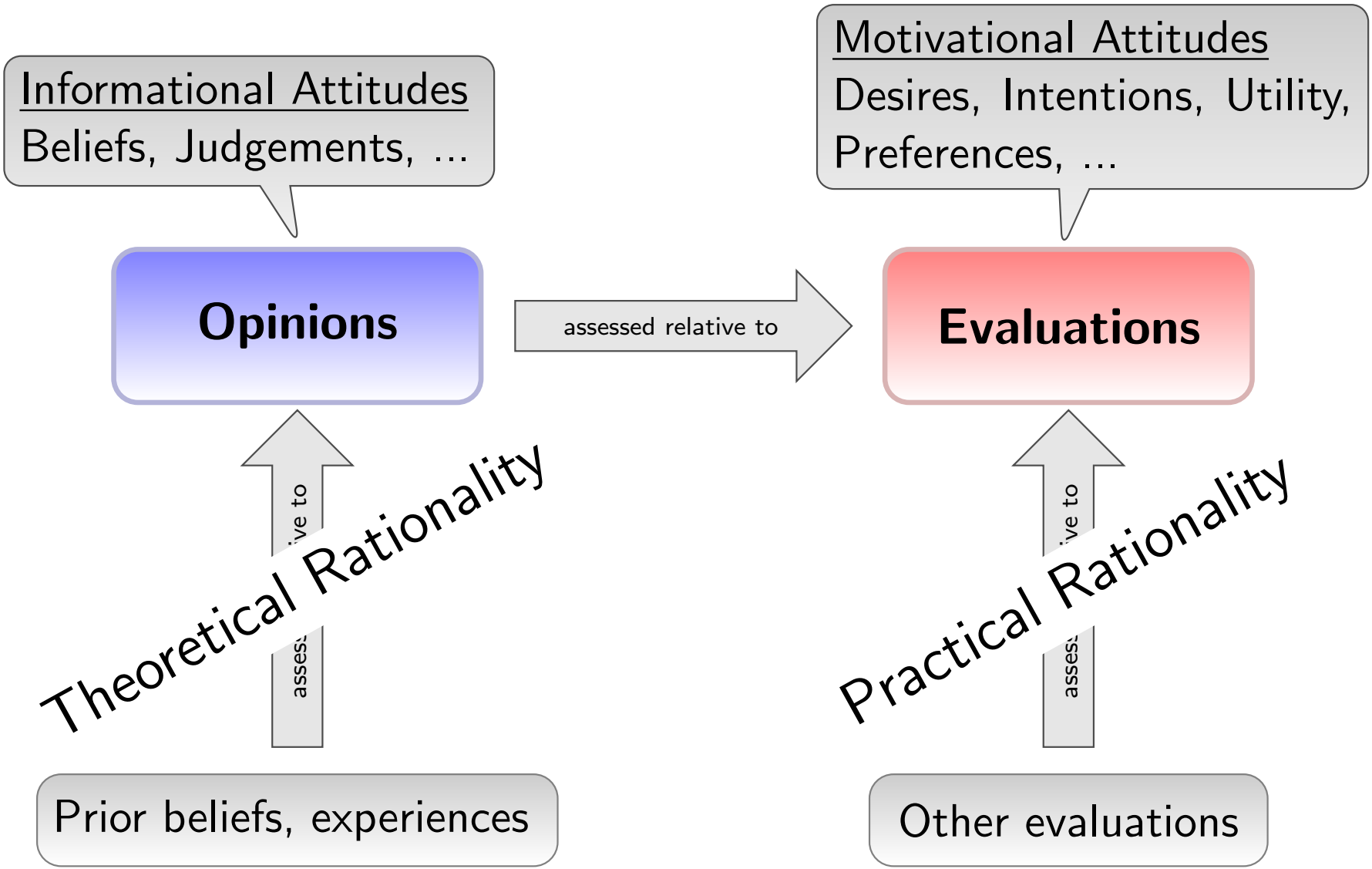
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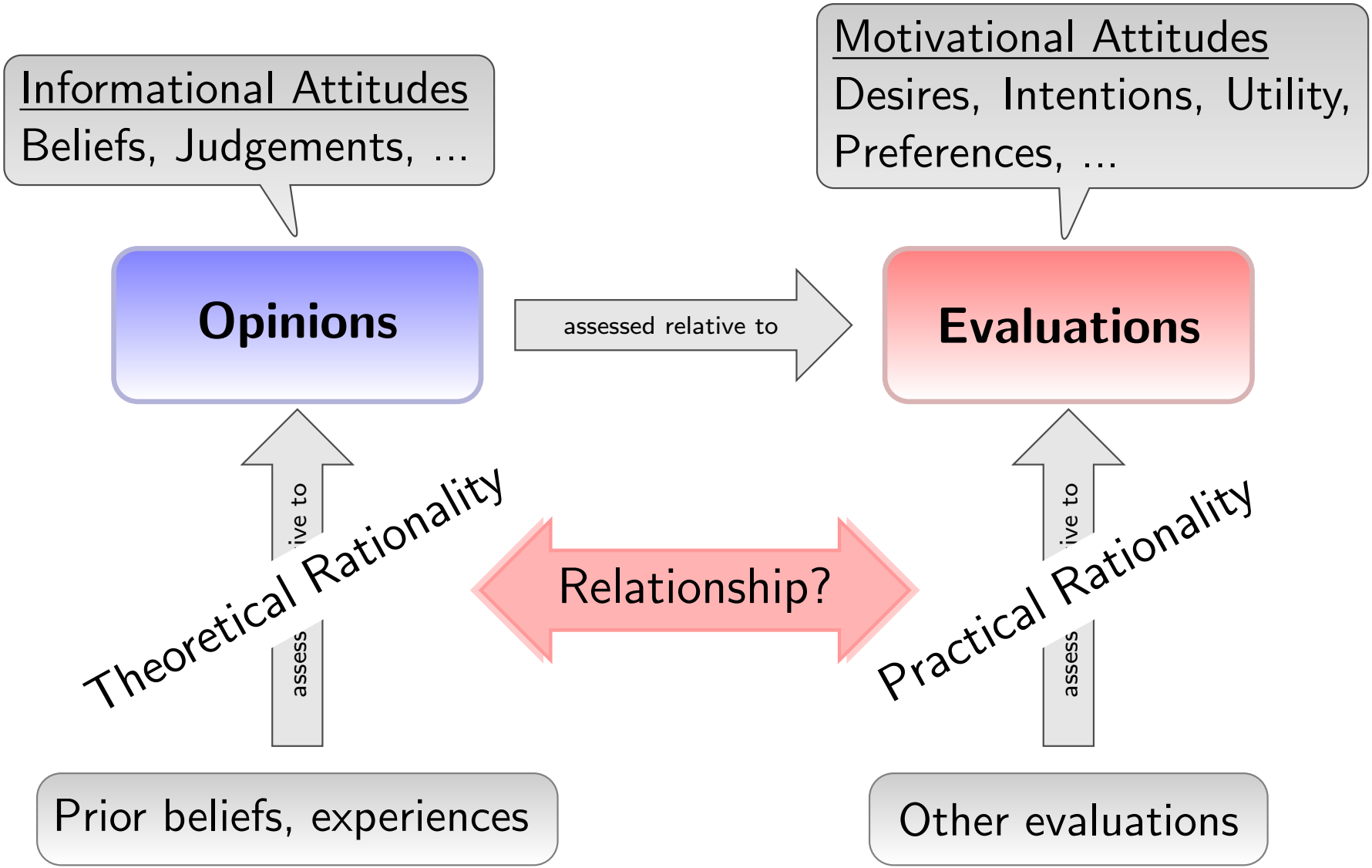
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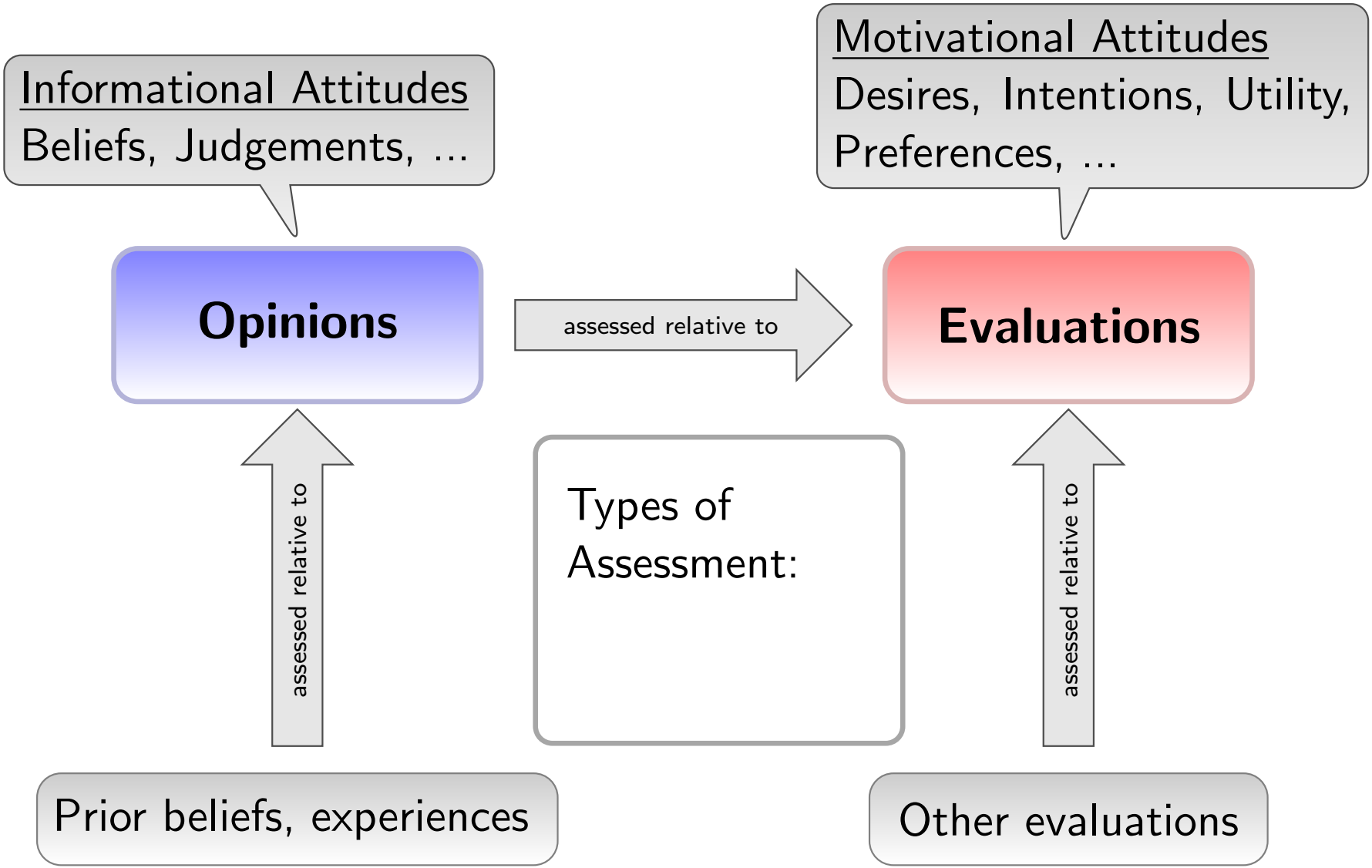
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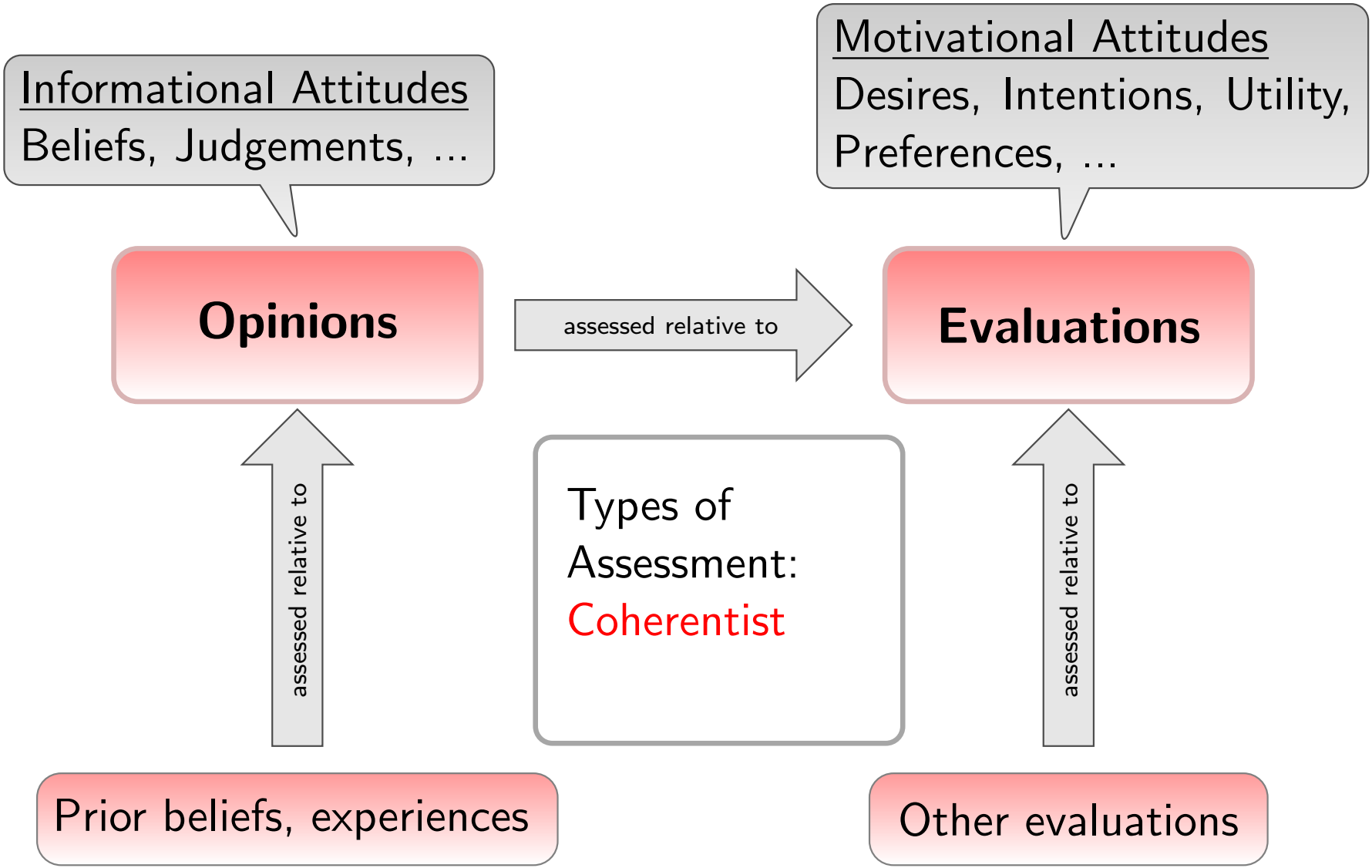
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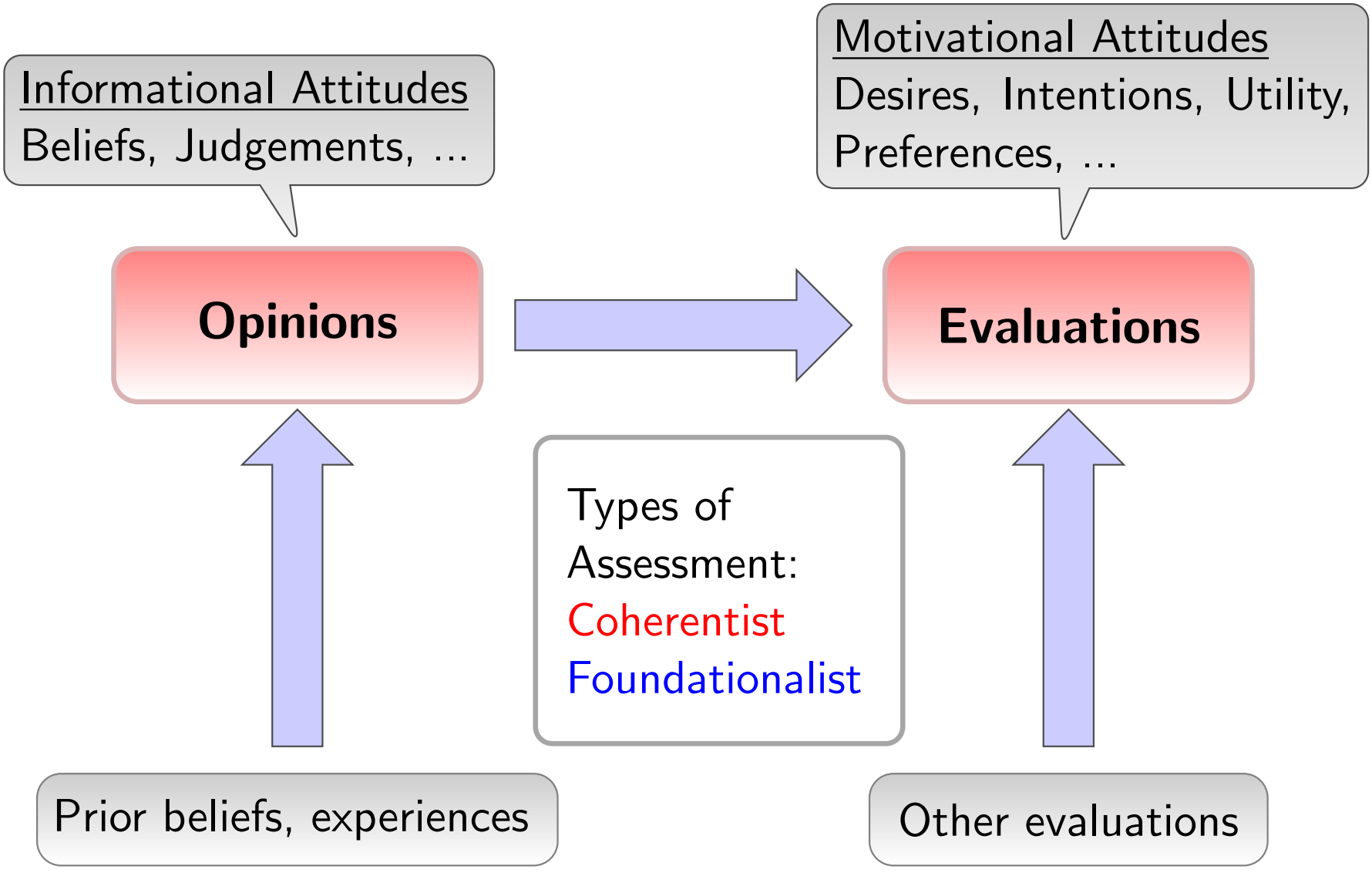
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Group Rationality
What does it mean for a *group* to be *rational*?

Philosophy of Normativity

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Various logics, scientific methodology, heuristics, probability, decision theory all have claims to *normative status* here, but what does *normativity* mean here:

1. What does it mean for someone to be *rationally required* to be in a particular state of mind?
2. *Why be rational*?

Methodological Issues

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- axiomatic method

Normative vs. Description Theories

Methodological Issues

Interdisciplinary: Logic, Philosophy (Epistemology, Philosophy of Action, Meta-Ethics), Economics (Rational Choice Theory, Game Theory, Social Choice Theory), Psychology and Cognitive Science

Formal Philosophy:

- make use of ideas and results from other areas,
- build formal models of reasoning, decision making and social interaction (which can be rigorously analyzed and even implemented),
- axiomatic method

Normative vs. Description Theories: How can/should we incorporate *empirical data* into our *normative* theory of rationality? (reflective equilibrium)

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R. Aumann. *Irrationality in Game Theory*. in: *Aumann's Collected Papers, Volume 1*, Chapter 35, 1992.

Thank you! Send me your final paper by **June 15, 2011**.