# Rationality: Short Essay Questions

Tilburg University, 2010-2011

*Please put your answers in my pigeon hole before* **March 17, 2011** *(you may also email, but make sure you put "Rationality midterm" in the subject line). Make sure you properly cite any sources you use when answering these questions.*

1. Write a short essay (2-3 pages) explaining in your own words the difference between the internalist and externalist views of practical reasons. Make sure you discuss the following example of B. Williams:

   > "The agent believes that this stuff is gin, when in fact it is petrol. He wants a gin and tonic. Has he reason, or a reason, to mix this stuff with tonic and drink it?"

   In particular, how does Williams maintain an internalist position and argue that mixing the petrol with tonic is *irrational*?

For the remaining questions, please provide *short* answers (maximum of a paragraph, but 1-2 sentences is acceptable for some of the questions).

2. Consider the following lotteries.

| | 1/3 | 1/3 | 1/3 |
|---|---|---|---|
| $L_1$ | €50 | €50 | €50 |
| $L_2$ | €100 | €50 | €0 |
| $L_3$ | €50 | €0 | €50 |
| $L_4$ | €100 | €0 | €0 |

   Suppose that you prefer $L_1$ to $L_2$ and $L_4$ to $L_3$. Show that your preferences are incompatible with the principle of maximizing expected utility, no matter what your utility of money happens to be.

3. Consider the following decision matrix:

| | State 1 | State 2 |
|---|---|---|
| Option 1 | €100 | €0 |
| Option 2 | €0 | €100 |

Suppose that you are indifferent between Option 1 and Option 2. What can you conclude about your subjective probability for the two states? What if you prefer Option 1 over Option 2?

Suppose that the €100 outcome is replaced with a black box with unknown contents,a nd that you feel indifferent between the two options. What can you conclude about your subjective probability of the two states?

4. Joyce sketched a proof of the Dutch Book Theorem his article *Bayesianism* in the Handbook of Rationality (we discussed this proof during the 5th and 6th lectures). In particular, he explained that if your fair prices do not follow the laws of probability, then one can construct a set of swaps that will guarantee that the you loose money. For this question, you will extend this proof to other properties of probabilities.

   One law of probability which we have used is that if $X \subseteq Y$ then $P(X) \leq P(Y)$. As I discussed on the first lecture, this can be derived from the other axioms of probability. But, we can also argue directly that a rational agent's graded beliefs must conform to this property. Suppose that $X \subseteq Y$ and the agent has the following fair prices:

   - $f = 0.3$ for the wager $W_X = [\,1 \text{ if } X, 0 \text{ else}\,]$; and
   - $f = 0.2$ for the wager $W_Y = [\,1 \text{ if } Y, 0 \text{ else}\,]$

   Explain how to make "Dutch Book" against the agent with these fair prices (*hint*: consider the sets $\mathcal{W}_1 = \{0.1, W_Y\}$ and $\mathcal{W}_2 = \{W_X\}$. Will the agent swap $\mathcal{W}_1$ for $\mathcal{W}_2$? If so, is this a good thing to do?)

   Provide a similar argument to show that the fair price assigned to a contradiction (i.e., to a logical impossibility: an event that is never true) should be 0.

5. Similar to the Allais Paradox, Daniel Ellsberg came up with a choice situation where the "rational" choice seems to contradict standard expected utility theory.

   D. Ellsberg (1961). Risk, Ambiguity, and the Savage Axioms *Quarterly Journal of Economics*, **75 (4)**, pgs. 643 - 669.

   Here is one situation analyzed in this paper. There is one urn with 300 balls: 100 of the balls are red ($R$) and the rest are either blue ($B$) or yellow ($Y$). Consider the following two choice situations:

   **<u>Situation 1</u>**

   $l_1$ Win \$100 if a ball drawn from the urn is $R$ and nothing otherwise

   $l_2$ Win \$100 if a ball drawn from the urn is $B$ and nothing otherwise

**Situation 2**

$l_3$ Win \$100 if a ball drawn from the urn is $R$ or $Y$ and nothing otherwise

$l_4$ Win \$100 if a ball drawn from the urn is $B$ or $Y$ and nothing otherwise

Most people have the following preference: $l_1 \succ l_2$ and $l_4 \succ l_3$.

Answer the following questions. Please keep your answers concise and be sure to cite any additional sources you use.

(a) Explain the rationale behind these preferences. (I.e., why might these preferences be "rational"?)

(b) Why do these preferences contradict standard expected utility calculations?

(c) Do the Ellsberg Paradox and the Allais Paradox highlight the same phenomena? Explain your answer.