Tutorial: Some Questions on Logic and Probability

Tilburg University, 2010-2011

Spend some time trying to answer the following questions. The goal here is to give you a chance to think critically about types of problems which we have and will discuss in this course. Dominik will go over the answers to the questions after 30-40 minutes.

1. **The Birthday Paradox**: What is the probability of two or more people out of a group of n do have the same birthday? This is not a paradox but a result that people often find puzzling. Of course, if there are 367 people in the room, then there *must* be two people that share the same birthday. What is surprising is that if there are only 23 people in the room, then there is about a 50% chance that two people have the same birthday and with only 75 people the probability goes up to 99.9% that two people have the same birthday (why?).

2. **Problems from *Rational Choice* by I. Gilboa**

   (a) Explain what is wrong with the claim, Most good chess players are Russian; therefore a Russian is likely to be a good chess player.

   (b) Comment on the claim, Some of the greatest achievements in economics are due to people who studied mathematics. Therefore, all economists had better study mathematics first.

   (c) 5. Trying to understand why people confuse $P(A \mid B)$ and $P(B \mid A)$, it is useful to see that qualitatively, if A makes B more likely, it will also be true that B will make A more likely.

      i. Show that for any two events A, B

      $P(A \mid B) > P(A \mid B^C)$
      iff
      $P(A \mid B) > P(A) > P(A \mid B^C)$
      iff
      $P(B \mid A) > P(B \mid A^C)$
      iff
      $P(B \mid A) > P(B) > P(B \mid A^C)$

      where A^C is the complement of A. (Assume that all probabilities involved are positive, so that all the conditional probabilities are well defined.)

      ii. If the proportion of Russians among the good chess players is higher than their proportion overall in the population, what can be said?

3. **The Surprise Examination Paradox**: This paradox that has been widely discussed by logicians and philosophers (there still is no general agreement about its "solution").

   > A teacher announces in class that an examination will be held on some day during the following week, and moreover that the examination will be a surprise. The students argue that a surprise exam cannot occur. For suppose the exam were on the last day of the week. Then on the previous night, the students would be able to

predict that the exam would occur on the following day, and the exam would not be a surprise. So it is impossible for a surprise exam to occur on the last day. But then a surprise exam cannot occur on the penultimate day, either, for in that case the students, knowing that the last day is an impossible day for a surprise exam, would be able to predict on the night before the exam that the exam would occur on the following day. Similarly, the students argue that a surprise exam cannot occur on any other day of the week either. Confident in this conclusion, they are of course totally surprised when the exam occurs (on Wednesday, say). The announcement is vindicated after all. Where did the students' reasoning go wrong?

4. **Monty Hall Puzzle**: The original formulation (from *Ask Marilyn* in *Parade Magazine*) is

> Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say number 1, and the host, who knows what's behind the doors, opens another door, say number 3, which has a goat. He says to you, "Do you want to pick door number 2?" Is it to your advantage to switch your choice of doors?

5. **The Two-Envelop Paradox**: This paradox is similar to the Monty Hall Puzzle, though much more difficult. See this link

   `http://www.maa.org/devlin/devlin_0708_04.html`

   for a nice discussion of the paradox and a proposed solution.

> You are presented with two indistinguishable envelopes containing some money. You are further informed that one of the envelopes contains twice as much money as the other. You may select any one of the envelopes and you will receive the money in the selected envelope. When you have selected one of the envelopes at random but not yet opened it, you get the opportunity to take the other envelope instead. Should you switch to the other envelope? (There is a convincing argument that you should switch, but then ask yourself, *should you switch again*?)