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MAXIMIZATION AND THE ACT OF CHOICE[1]

BY AMARTYA SEN

The act of choosing can have particular relevance in maximizing behavior for at least two distinct reasons: (1) *process significance* (preferences may be sensitive to the choice process, including the identity of the chooser), and (2) *decisional inescapability* (choices may have to be made whether or not the judgemental process has been completed). The general approach of maximizing behavior can—appropriately formulated—accommodate both concerns, but the regularities of choice behavior assumed in standard models of rational choice will need significant modification. These differences have considerable relevance in studies of economic, social, and political behavior.

KEYWORDS: Choice act, games and common knowledge of norms, incomplete preference ranking, maximizing behavior, rational choice, satisficing, axiomatic choice theory.

1. THE ACT OF CHOICE

IN 1638, WHEN PIERRE DE FERMAT sent to René Descartes a communication on extremal values (pointing in particular to the vanishing first derivative), the analytical discipline of maximization was definitively established.[2] Fermat's "principle of least time" in optics was a fine minimization exercise (and correspondingly, one of maximization). It was not, however, a case of *maximizing behavior*, since no volitional choice is involved (we presume) in the use of the minimal-time path by light. In physics and the natural sciences, maximization typically occurs without a deliberate "maximizer." This applies generally to the early uses of maximization or minimization, including those in geometry, going back all the way to "the shortest arc" studied by Greek mathematicians, and other exercises of maximization and minimization considered by the "great geometers" such as Apollonius of Perga.

The formulation of maximizing behavior in economics has often paralleled the modelling of maximization in physics and related disciplines. But maximizing *behavior* differs from nonvolitional *maximization* because of the fundamental relevance of the choice act, which has to be placed in a central position in analyzing maximizing behavior. A person's preferences over *comprehensive* outcomes (including the choice process) have to be distinguished from the conditional preferences over *culmination* outcomes *given* the acts of choice. The

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[2] Fermat's manuscript was circulating in Paris for a few years before it was sent to Descartes, who incidentally was not particularly impressed with it.

responsibility associated with choice can sway our ranking of the narrowly-defined outcomes (such as commodity vectors possessed), and choice functions and preference relations may be parametrically influenced by specific features of the *act* of choice (including the *identity* of the chooser, the *menu* over which choice is being made, and the relation of the particular *act* to behavioral social norms that constrain particular social actions). All these call for substantial analytical attention in formulating the theory of choice behavior.[3]

Also from a practical point of view, differences made by *comprehensive* analysis of outcomes can have very extensive relevance to problems of economic, political, and social behavior whenever the act of choice has significance. Illustrations can be found in problems of labor relations, industrial productivity, business ethics, voting behavior, environment sensitivity, and other fields.

Second, in addition to the significance of the *process* of choice in what is chosen, the importance of the act of choice also lies in its *inescapability* or *urgency*. A chooser, who may have to balance conflicting considerations to arrive at a reflected judgement, may not, in many cases, be able to converge on a complete ordering when the point of decision comes.[4] If there is no escape from choosing, a choice decision will have to be made even with incompleteness in ranking.

The characterization of *maximizing behavior as optimization*, common in much of economic analysis, can run into serious problems in these cases, since no *best* alternative may have been identified for choice. In fact, however, optimization is quite unnecessary for "maximization," which only requires choosing an alternative that is not judged to be worse than any other. This not only corresponds to the commonsense understanding of maximization (viz. not rejecting an alternative that would be better to have than the one chosen), it is also how "maximality" is formally defined in the foundational set-theoretic literature (see, for example, Bourbaki (1939, 1968), Debreu (1959, Chapter 1)).

In Sections 2 and 3, I shall consider the reasoning behind including the choice act in comprehensive analysis of decisions, and the connection between choosing and responsibility. Section 3 deals with the analytical implications of this broadening in terms of "chooser dependence" and "menu dependence" of choices. Section 4 is concerned with the use of norms as well as strategies in rational decisions and games. Section 5 deals with the comparisons and contrasts

[3] This paper is concerned with choice behavior, rather than with *normative choice theory*. However, in so far as choice norms influence actual choice behavior, they enter this investigation. On the connection between the two exercises, see Sen (1987).

[4] Incompleteness can arise from limited information, or from "unresolved" value conflicts (see Sen (1970a, b), Williams (1973), Levi (1986), Putnam (1996)). Levi (1986) takes the latter as the starting point of his far-reaching analysis of "hard choices." See also Blackorby (1975), Fine (1975), Basu (1980, 1983), Levi (1980), Putnam (1996), Walsh (1996).

between optimizing and maximizing choice functions, and the possibility of moving from one to the other. The subject matter of Section 6 is the relation between incorporating concerns about choice acts in the form of self-imposed *choice constraints*, and incorporating them within the *preference relation* itself. There is a concluding section. The proofs of some formal propositions have been relegated to the Appendix.

2. DIRECT INTEREST VERSUS INSTRUMENTAL EXPLANATIONS

An example may help to illustrate the role of "comprehensive" description of choice processes and outcomes, in particular the "chooser dependence" of preference. You arrive at a garden party, and can readily identify the most comfortable chair. You would be delighted if an imperious host were to assign you to that chair. However, if the matter is left to your own choice, you may refuse to rush to it. You select a "less preferred" chair. Are you still a maximizer? Quite possibly you are, since your preference ranking for choice behavior may well be defined over "comprehensive outcomes," including choice processes (in particular, who does the choosing) as well as the outcomes at culmination (the distribution of chairs).[5]

To take another example, you may prefer mangoes to apples, but refuse to pick the last mango from a fruit basket, and yet be very pleased if someone else were to "force" that last mango on you. In these choices, there is no tension at all with the *general* approach of maximizing behavior, but to accommodate preferences of this kind, the choice act has to be internalized *within* the system. This can require reformulation of behavioral axioms for "rational choice" used in economic and political theory (to be explored in Sections 3–6).

The influence of the choice act on preferences, and in particular the dependence of preference on the identity of the chooser, can go with rather different motivations and may have several *alternative* explanations. The comprehensive descriptions may be relevant in quite different ways and for quite distinct reasons.

(i) *Reputation and indirect effects*: The person may expect to profit in the future from having the reputation of being a generally considerate person, and not a vigilant "chair-grabber."

[5] A common reaction to this type of chooser dependence (judging from seminar experience) is to think that the "problem" arises because of a mistaken attempt to define this person's preference in terms of the chair on which she herself gets to sit, and not over the full "vector" of chair allocations (involving others as well). But this is not the source of the variability here. The person may be very happy with a full vector of chair assignments that allocates the most comfortable chair to her, *if* that vector were to be brought about by someone else's choice, but not if it had to be secured through her *own* choice.

(ii) *Social commitment and moral imperatives*: She may not think it morally "right" to grab the most comfortable chair, cutting others out, and such "moral sentiments" could be explicitly followed or only implicitly obeyed.[6]

(iii) *Direct welfare effects*: The person's well-being may be affected directly by the process of choice (for example, by what people think of her—she may not enjoy the looks she gets as she makes a dash for the great chair), and this requires that the reflective utility function (and the person's conception of her self-interest) be defined not just over culmination outcomes (such as final commodity vectors, as in standard consumer theory), but inter alia also over choice processes and their effects.

(iv) *Conventional rule following*: She may be simply following an established rule of "proper behavior" (as the on-going norm), rather than being influenced by direct welfare effects, or by reputation effects, or even by any self-conscious ethics.

The process of choice has rather different roles in these distinct cases, and they may, in fact, occur in various mixed forms.[7] The first line of explanation ("reputation and indirect effects") is most in harmony with the established conventions of standard neoclassical economics. It does not require any basic departure from the ultimate concentration on culmination outcomes (and from rational choice guided only by self-interest). Instrumental analysis links immediate concern with the choice act with the underlying pursuit of preferred culmination outcomes (see Kreps and Wilson (1982)).

In contrast with the first case, in the other three cases, the choice act is *directly* relevant, not just for its indirect effects. However, alternative explanations are possible about how this direct interest comes about—what underlying forces cause it to occur. The recent work on evolutionary game theory has thrown much light on how conventional rule following—explanation (iv) in the above list—may emerge from evolutionary selection.[8] Even though *ultimately* no individual may be directly concerned with the nature of the choice act, concern with the nature of the choice act may be instrumentally important in social rules of behavior that survive. This type of reasoning can be contrasted with behav-

[6] Both Immanuel Kant (1788) and Adam Smith (1790) emphasized the importance of "moral sentiments" and their significance in rational choice. Adam Smith also discussed extensively how various moral values (including "generosity" and "public spirit") can alter our choice behavior, even though self-interest may be adequately explanatory in the special case of explaining *mutually profitable exchange* (such as the trade between the consumer, on the one hand, and the butcher, the brewer and the baker, on the other, in the often-quoted passage in *The Wealth of Nations* (Smith (1776)). In a common interpretation of Smith (see, for example, Stigler (1981)), Smith's general claims about behavioral diversities are largely ignored, by concentrating exclusively on his particular point about the profitability of exchange, thereby radically distorting Smith's choice theory (I have tried to discuss this issue in Sen (1987)).

[7] See Sen (1987), Sacco and Zamagni (1993), Zamagni (1993, 1995), Walsh (1996).

[8] Different types of linkage between behavioral rules and strategic rationality have been explored in this rapidly expanding literature; see Axelrod (1984), Kreps, Milgrom, Roberts, and Wilson (1982), Fudenberg and Maskin (1986, 1990), Fudenberg and Tirole (1992), Binmore (1994), Weibull (1995), among other contributions.

ioral rules being deliberately chosen by an individual through an ethical examination of how one "should" act (thereby combining explanations (2) and (4)). Consciously reflective—rather than evolutionarily selected—use of ethical rule-following was most famously explored by Immanuel Kant (1788).[9] That approach has been pursued in different forms in modern ethical writings as well, varying from Rawls' (1971) characterization of "comprehensive" goals and Harsanyi's (1976) analysis of ethical preference and social behavior, to the sociological exploration of the complex values that influence people's conduct.[10]

I would make four brief comments on these two alternative lines of explanation. First, they need not be *just* "alternatives." Even if we deliberately choose behavioral norms on ethical (or social) grounds, their long-run survival can scarcely be completely independent of their impact on each other and of the evolutionary processes that must come into play. On the other side, in studying evolutionary processes, there is no need to confine attention only to preferences that ultimately relate exclusively to *culmination* outcomes. Evolutionary studies of rules when people *also* attach intrinsic—not just instrumental—value to acts and conduct can be important to understand society.[11]

Second, evolutionary processes may not only influence the *rules* of conduct that we may consciously follow, but also our psychological *preferences* about the actions involved. The literature on endogeneity of preference can be fruitfully linked with evolutionary theories.[12] The same can be said about the survival of ethical norms as well. Paying reflective ethical attention to behavior neither nullifies, nor is nullified by, the importance of evolutionary forces.[13]

Third, even if it *were* the case that—"ultimately"—everything were determined by "basic" preferences exclusively over *culmination outcomes*, it would still be interesting and important to see how the *derived* preferences ("nonbasic" but functionally important) actually work in relation to the choice act. The analytical and mathematical aspects of these choice functions would still deserve examination. Thus, the analysis pursued in this paper can have interest at different levels of investigation—instrumental as well as basic.

[9] Kant founded his deontological ethics on "rationality," but his interpretation of rationality departed from the conscious pursuit exclusively of self-interest. Because of the narrowing of the concept of rationality in parts of modern economics (which tends to classify as "irrational" any behavior that is not—directly or indirectly—justifiable in terms of the person's own self-interest), Kant's idea of reflective rationality has become rather difficult for some to stomach. It has also led to the demand, in Binmore's (1994) words, for the "DeKanting" of ethics, which he applies, with agreeable (if somewhat confusing) cheer, even to "DeKanting Rawls" (pp. 7–86).

[10] For a variety of perspectives on broader influences, see Nagel (1970), Sen (1973a, b), Scitovsky (1986), Frank (1988), Anderson (1993), Baigent (1994), Lewin (1996), Walsh (1996); also the collections in Hahn and Hollis (1979), Elster (1986), Mansbridge (1990), and Zamagni (1995). On behavioral analysis linked with Rawlsian theory, see also Scanlon (1982) and Laden (1991).

[11] This would apply to many cases of economic, political, and social behavior empirically investigated recently (see Sections 3–5).

[12] In an important research initiative, led jointly by Herb Gintis and Paul Romer (sponsored by the MacArthur Foundation), this is one of the central areas of current investigation.

[13] See Sen (1987), Sacco and Zamagni (1993), and Zamagni (1995).

Fourth, I shall also argue that sometimes even the understanding of games and strategies can be enhanced by allowing broader formulations of preferences and of rules of behavior, and of the common knowledge of norms (a simple example is considered in Section 4). In following such games, it is important to take note of the influence of the nature of the choice act on strategies, no matter what view we take about the "ultimate" origin of that influence.

3. RESPONSIBILITY, CHOOSER DEPENDENCE AND MENU DEPENDENCE

The direct importance of the choice act typically relates to the idea of responsibility. Our attitudes towards responsibility may or may not be mediated through our personal well-being.[14] We may enjoy exercising responsibility; or not enjoy it at all but still feel the duty to act responsibly; or—as in the garden-chair example—we may find the responsibility of choice a constraint and a burden.

To take a very different type of case from the garden-chair example, the act of voting in an election may be very important for a person because of the significance of political participation. This has to be distinguished from whatever may be added by a person's vote to the likelihood of the preferred candidate's chances of winning (the addition could be negligible when the electorate is large). It is possible that the voter may enjoy participation, or that she may act under some "deontic" obligation to participate whether or not she enjoys it. So long as she attaches importance to the participatory act of voting, the analysis of the rationality of voting must take note of that concern, whether that concern arises from anticipated enjoyment, or from a sense of duty (or of course, both). In either case, it can be argued that the well-known literature on "why do rational people vote" may have tended to neglect an important concern underlying voting behavior, viz. *the choice act of voting*. There may, in fact, be no puzzle whatever as to why people vote even when the likelihood of influencing the voting outcome is minuscule.

Similarly, in understanding "work ethics," it may be inadequate to confine attention to the simple fact that work may be a strain, or that work can be a pleasurable activity, or even that a worker may take a familial interest in the consequential fortunes of the firm (apparently an important part of Japanese

[14] While the concentration of this paper is not on the substantive content of our objectives, I have discussed elsewhere (Sen (1973b, 1977b)) the limitations imposed by taking an overly narrow view of human motivation (see also Frey (1992)). I shall not pursue that debate further in this essay. There is a different debate—one on "consequentialism"—with which the subject of this essay also indirectly relates. The idea of judging all choice variables by their—and *only* by their—consequences is called "consequentialism." Consequentialism in a fairly restrictive form is, in fact, simply taken for granted in much of traditional economics. But its basic soundness has been disputed in many philosophical writings (see, for example, Williams (1973) and Nozick (1974)). This issue will not be further pursued here, but I have elsewhere defended "consequential evaluation" in a *broad* form: (i) by including actions performed *within* the relevant consequences, and (ii) by admitting "positional" perspectives in evaluating consequences (Sen (1982b, 1983)). The use of consequential evaluation in this paper is in this broad form. See also Hammond (1985, 1986), Binmore (1994), Moulin (1995), Walsh (1996).

work ethics).[15] The importance of participation itself may be closely related to work ethics, and different interpretations of participation may contribute to the explanation of varying work ethics in different countries and cultures. The importance of participation can be quite crucial also in the operation of "environmental values," which is one of the reasons why the market analogy is often quite deceptive in assessing "existence values" of what people try actively to preserve.[16]

Sometimes the connection between preference and choice acts may be rather subtle and complex, and turn on the exact nature of the actions involved. For example, in the context of work ethics, there may be a substantial difference between (i) actively choosing to "shirk" work obligations, and (ii) passively complying with a general atmosphere of work laxity. The latter may happen much more readily than the former, and the exact nature of the choice act can be very important in this difference. In fact, "herd behavior" not only has epistemic aspects of learning from others' choices (or being deluded by them, on which see Banerjee (1992)), but can also be linked with the possibility that joining a "herd" makes the choice act less assertive and perspicuous. The diminished use of forceful and aggressive volition in (ii) may make it much harder to resist than (i). Such differences may be of great importance in practice, even though they may be difficult to formalize completely.

Some types of influences of choice acts are more easy to formalize than others, and these include: (i) chooser dependence, and (ii) menu dependence. Consider the preference relation P_i of person i as being conditional on the chooser j and the set S from which the choice is being made: $P_i^{j,S}$. Chooser dependence and menu dependence relate to the parametric variability of P_i with j and S respectively.[17]

Consider chooser dependence first—already introduced in the motivational discussion. To return to one of the earlier examples, in choosing between alternative allocations of fruits from a set $S = \{m^1, a^1, a^2\}$ of one mango and two apples for two persons i and k, person i who prefers mangoes may like the allocation m^1 that gives the mango to him (and an apple to k), over the allocation a^1 whereby i gets an apple (say, a^1), *so long as* the choice is made by someone else j:

$$(3.1) \qquad m^1 P_i^{j,S} a^1,$$

[15] On different interpretations of Japanese work ethics, see Morishima (1982, 1995), Dore (1987), Ikegami (1995). For related issues in economic analyses, taking extensive note of institutional and behavioral features, see Aoki (1989) and Suzumura (1995).

[16] A "social choice" approach (going beyond the market analogy underlying procedures of "contingent valuation") can be helpful in incorporating the value of participation (see Sen (1995b)) in environment-sensitive choices of actions.

[17] The variations here relate to the "positionality" of the observer (see Sen (1982b) on the influence of positionality), and in particular on the "position" of being the chooser, over a given menu.1973

and yet prefer to go for the opposite if he himself has to do the choosing:

(3.2) $a^1 P_i^{i,S} m^1$.

Along with this chooser dependence, there is a related feature of menu dependence, particularly in the case of self-choice. If the set of available options is expanded from S to T containing two mangoes and two apples, person i himself may have no difficulty in choosing a mango, since that still leaves the next person with a choice over the two fruits. On the other hand, menu-dependence of preference is precisely what is ruled out by such assumptions as the weak axiom of revealed preference (WARP) proposed by Paul Samuelson (1938), not to mention Houthakker's (1950) strong axiom of revealed preference (SARP). Indeed, even weaker conditions than WARP, such as Properties α and τ (basic contraction and expansion consistency), which are necessary and sufficient for binariness of choice functions over finite sets (see Sen (1971)), much used in general choice theory as well as social choice theory, are violated by such choices.[18]

How are these basic conditions of intermenu consistency violated by the concerns we are examining? Consider the same example again. While an apple a^1 is what person i may pick if he is choosing from S (as given by (3.2)), he may sensibly go for one of the mangoes (say, m^1) from the enlarged set $T = \{m^1, m^2, a^1, a^2\}$:

(3.3) $m^1 P_i^{i,T} a^1$.

The combination of (3.2) and (3.3) violates Property α as well as WARP and SARP, and it can be easily shown with further examples that this type of menu dependence can lead to the violation of the other standard consistency conditions.[19] Menu dependence—when true—may be quite a momentous characteristic of choice functions.[20]

[18] WARP demands that if an alternative x is picked from some set S, and y (contained in S) is rejected, then y must not be chosen and x rejected, from some other set T to which they both belong. Property α demands that if some x is chosen from a set T and is contained in a subset S of T, then x must be chosen from the subset S. Property τ demands that if x is chosen from each of a class of sets, then it must be chosen from their union. Analyses of these and related choice conditions can be found in Hansson (1968a, 1968b), Sen (1970a, 1971, 1982a), Herzberger (1973), Plott (1973), Parks (1976), Aizerman and Malishevski (1981), Suzumura (1976, 1983), Deb (1983), Moulin (1985), Sugden (1985), Levi (1986), Kreps (1988), Heap et al. (1992), Aizerman and Aleskerov (1995), Baigent (1995).

[19] For example, to check that Property τ will be violated, we can note that apple a^1 may be picked over mango m^1 when the choice is over $\{m^1, a^1, a^2\}$, and also apple a^1 over mango m^2 in the choice over $\{m^2, a^1, a^2\}$, and yet the person could, consistently with his priorities, choose mango m^1 or m^2, and *not* an apple, when the choice is over the foursome $\{m^1, m^2, a^1, a^2\}$, the union of the two previous sets.

[20] My experience in presenting this paper in seminars has alerted me to the possibility that some readers will seek explanation of the alleged "inconsistency" in the influence of "framing" (in line with Kahneman and Tversky's (1984) important findings). But these two problems are quite distinct. The influence of "framing" arises when essentially the *same* decision is presented in different ways, whereas what we are considering here is a *real* variation of the decision problem, when a change of the menu from which a choice is to be made makes a material difference. There is, in fact, no inconsistency here, only menu dependence of preference rankings (see Sen (1993)).

The above discussion concentrates only on one kind of reason for menu dependent preferences (related to the direct relevance of the choice act), but there can be other reasons for such dependence (on this see Sen (1993)). One connection may come from the value we place on our autonomy and freedom of decisions.[21] We may value not merely the alternative we eventually choose, but also the set over which we can exercise choice. In valuing the "autonomy" of a person, it is not adequate to be concerned only with whether she receives what she would choose if she had the opportunity to choose; it is also important that she actually gets to choose herself.[22]

Also, when our knowledge is limited, the menu may have epistemic importance, and we may "learn" what is going on from the menu we face. For example, if invited to tea (t) by an acquaintance you might accept the invitation rather than going home (O), that is, pick t from the choice over $\{t, O\}$, and yet turn the invitation down if the acquaintance, whom you do not know very well, offers you a *wider* menu of either having tea with him, or some heroin and cocaine (h), that is, you may pick O, rejecting t, from the larger set $\{t, h, O\}$. The expansion of the menu offered by this acquaintance may tell you something about the kind of person he is, and this could affect your decision even to have tea with him (see Sen (1993)).

A different type of example of epistemic use of menus can be found in using one's own menu to judge the opportunities that *others* would have to undertake similar behavior. In explaining "corrupt" behavior in business and politics in Italy, a frequent excuse given has been: "I was not alone in doing it." A person may resist seizing a unique opportunity of breaking an implicit moral code, and yet be willing enough to break that code if there are many such opportunities, on the indirect reasoning that the departures may be expected to become more "usual."[23] Similarly, a unique opportunity of "crossing the picket line" may be rejected by someone, who may nevertheless not hesitate to do that crossing if he expects others to do the same. If there is only one opportunity x_1 of crossing a picket line, a person may refrain from grabbing that (knowing that she would be alone in this), and yet she may choose that very opportunity x_1 if there are other opportunities x_2, etc. (expecting others to take them).

[21] The importance of autonomy and the freedom to choose is central to ethics and is of great potential relevance to welfare economics (even though standard welfare economics tends frequently to eschew this consideration). On this issue, see Sen (1970a, 1983, 1991, 1992b), Nozick (1974), Suppes (1987), Gärdenfors (1981), Sugden (1981, 1986, 1993), Roemer (1982, 1996), Suzumura (1983), Hammond (1985), Cohen (1990), Pattanaik and Xu (1990), Schokkaert and Van Ootegem (1990), Steiner (1990), Gaertner, Pattanaik, and Suzumura (1992), Heap et al. (1992), Foster (1993), Nussbaum and Sen (1993), van Hees (1994), Arrow (1995), Van Parijs (1995), Puppe (1996), among others.

[22] We can, for example, consider an authoritarian system of allocation that fully *mimics* what a decentralized system with autonomy of choice would achieve in terms of commodity productions, distributions, and consumptions. Even if such an authoritarian social alternative did exist, it need not be judged to be just as good as a system that allows the individuals to choose, because the exercise of the freedom to choose can be itself important.

[23] On these and related issues, see Camera dei deputati (1993).

Another type of epistemic relevance of the menu is illustrated by a "moderate" voter who tends to choose a middle-of-the-way candidate among the ones offered for choice, for example, some "median" alternative according to some politically perspicuous ranking (such as "relative conservatism"). The range of options offered in the menu may give the person a "reading" of the real policy options in the country at that time, and the menu-dependent choice of a "moderate" candidate may, thus, reflect that epistemic reading.[24]

It may, again, be tempting to think that the violation of the standard "consistency conditions" (such as WARP) can be eliminated by some suitable redefinition, for example by defining an alternative in terms of choosing a fruit from a set. The alternative m^1/S, taking m^1 from set S, can be seen as a different alternative from m^1/T, taking m^1 from set T. But that would make all inter-menu conditions, such as WARP, SARP, α, τ, etc., vacuous, since these have cutting power only when "the same" alternative can be picked from two *different* sets—precisely what is ruled out by this recharacterization. Similarly, if we try to apply conditions like WARP, SARP, α, etc., to alternatives defined as complete allocations of commodities for everyone in the community, these conditions have severely reduced discriminating power, because of the tendency of each option to become a unique alternative. Much would depend on the exact circumstances. (In fact, in the next section, in discussing Frisch's choice problem, I consider a case in which the route of redefining the alternatives in terms of complete allocations *does* work rather well, up to a point.) Not surprisingly, Samuelson (1938) and others employed their choice consistency conditions, in general, by defining an "alternative" for the choice of a person to be his or her own commodity basket (independently of the overall menu *and* of the allocations to everyone else in the community). It is in this form that these conditions have been used, with much force and profit, both in consumer theory and also to obtain results in general equilibrium theory (see, for example, Samuelson (1947), Debreu (1959), Arrow and Hahn (1971)).

The kinds of influences considered here suggest the need for limiting the domain of applicability of such conditions. But we should also consider a different type of argument which says that while menu dependence may occur and may be important for some problems (such as "social choice" judgments), an individual chooser need not really worry about it, since it is not relevant to her decisions. It could be argued that menu dependence cannot affect the form of maximizing behavior for an individual, since the individual does not get to choose the menu from which she can select an alternative. Menu dependence, in this view, may be true, but irrelevant for the individual's choice problem, since the person always faces a choice over a *given* menu, rather than having to choose *between* menus.

[24] Kolm (1994) has noted that the choice of the "median" violates some consistency conditions, and Gaertner and Xu (1995) have provided extensive explorations of such behavior, and an axiomatic derivation of the choice of the median alternative. See also Luce and Raiffa (1957) for other examples of epistemic use of the menu offered.

This line of argument is faulty for two distinct reasons. First, we do have occasion to make choices that affect our own future choices (or future menus), and indeed the literature on "preference for flexibility" (see Koopmans (1964) and Kreps (1979)) has extensively considered just such choices. We do not live in a world of a "one-shot choice." Kreps (1979, 1988) has presented illuminating analyses of preference for flexibility in choosing between future menus.[25] Such concerns may be important in *strategic* choice in many games as well, and an example of this will be considered presently (Section 4).

Second, the issue is not just whether the chooser herself has to "do something" about menu dependence, but whether in the study of choice behavior the possibility of menu dependence has to be included. It is the behavioral scientist who has to consider how a person's choices vary with alterations of the menu, and in particular whether a canonical binary relation of preference can be used to predict choices of that person over different menus. The point is that even when the option set (or the menu) S is given, the nature of the menu can influence the ranking of the alternatives x in S, and this relationship is of immediate relevance in understanding and predicting choice behavior.[26]

Menu-*independence* as a formal characteristic of preference can be defined in terms of R^S in the following way.

Menu-independent preference: There exists a binary relation R^X defined over the universal set X such that for all $S \subseteq X$, R^S is exactly the "restriction" of R^X over that S:

$$R^S = R^X|^S. \tag{3.4}$$

The condition of menu-*independence* is a standard assumption—typically made implicitly—in mainstream utility theory and choice theory. In Bourbaki's language, R^S is simply "induced by" an overall ordering R^X, and R^X is an "extension" of R^S on X (Bourbaki (1968, p. 136)). This relationship is implicitly

[25] Indeed, Kreps's analysis is quite definitive for the case in which the overriding concern is with outcomes only, but when the evaluation of outcomes must take note of uncertainty of one's own future tastes. That analysis can be extended to incorporate the importance one may attach to the freedom to choose and its responsibilities. In my Kenneth Arrow Lectures given at Stanford in 1991, to be published (eventually), an attempt is made to integrate the two perspectives of (i) valuing the option-value of *outcomes*, and (ii) valuing the *process* of choice, including being free to choose. See also Sen (1985a, 1991), Suppes (1987), Pattanaik and Xu (1990), Foster (1993), Arrow (1995), Puppe (1996), among other analyses.

[26] In considering the importance of freedom, it must also be noted that sometimes the chooser may react forcefully to the nature of the menu itself. If, for example, we decide that our freedom of choice is being wilfully curtailed by some "authority" (e.g., by preventing us from reading newspapers it does not approve of), we may react by making choices in the "contrary" direction (e.g., not read the authority's favored newspaper, even if we would have had no objection to reading it otherwise).

presupposed when a utility function $U(x)$ is defined just over the culmination outcome x, as is the standard practice (see, for example, Hicks (1939), Samuelson (1947), Debreu (1959), Arrow and Hahn (1971), Becker (1976)).[27]

In what follows, I shall consider choice functions based on optimization, that is, choosing an element from the optimal set $B(S, R)$ (that is, choosing a "best" element) from each menu set S, according to a weak preference relation R (interpreted as "preferred or indifferent to"), which ranks the set of available alternatives X of which each "menu" S is a nonempty subset.[28]

$$(3.5) \qquad B(S, R) = [x | x \in S \ \& \text{ for all } y \in S\colon xRy].$$

While (3.4) defines *menu-independent preference*, taking preference to be the primitive, there is an analytically different problem of characterizing a *menu-independent choice function*. For this it is convenient to define the "revealed preference" relation R_c^S of a choice function $C(S)$ over a given menu S. Although the revealed preference relation R_c is standardly defined without restricting the observation of choice to one particular set S only (see, for example, Samuelson (1938), Arrow (1959)), it is of course possible to consider the revealed preference R_c^S for a *specific* menu S.

Menu-specific revealed preference: For any x, y in X, and any $S \subseteq X$,

$$(3.6) \qquad xR_c^S y \Leftrightarrow [x \in C(S) \ \& \ y \in S].$$

Obviously, there would tend to be much incompleteness in the relation R_c^S for any given S, since any two *unchosen* alternatives in S would not be ranked *vis-à-vis* each other; we must take note of this elementary fact in using R_c^S.

Menu independence of choice can now be defined in terms of there being a canonical, menu-independent R_o, not varying over option sets, in terms of which we can explain the choices over every menu.

[27] When a preference is menu-dependent, a variation of the menu would mistakenly appear to be a *change* of preference. Even though Becker himself has tended to abstract from menu-dependent preference relations, the above observation is generally in line with Gary Becker's (1976) important diagnosis that many cases of *apparent* preference change are nothing of the kind and arise from inadequate characterization of preference.

[28] For a finite set of alternatives (presumed throughout this paper), it is required that R be *complete, acyclic, and reflexive* for there to be a nonempty $B(S, R)$ for every subset S (see Sen (1970a, Lemma $1*1$)). These conditions, especially completeness, can be relaxed if we use "maximization" rather than optimization (see Section 6).

Menu-independent choice function: There exists a binary relation R_o over X such that for all $S \subseteq X$:

(3.7.1) for all x, y in S: $xR_c^S y$ entails $xR_o y$;

(3.7.2) $C(S) = B(S, R_o)$.

How does menu-independence of *choice* relate to menu-independence of *preference*? If preference is defined simply as "revealed preference," there is obviously no gap between the two, given the constructive form of (3.7.1) and (3.7.2). But this is trivial, since "revealed preference" is only a reflection of choice itself, and gives no real role to conscious use of preference. To consider a nontrivial problem, consider a person who makes conscious optimizing decisions on the basis of a potentially menu-dependent preference R^S, and the choice function that results from it is given by $C(S) = B(S, R^S)$. The menu-independence of R^S and that of $C(S)$ would not then necessarily coincide.

However, the following relation will hold, denoting R^X as in (3.4) and R_o as in (3.7.1) and (3.7.2), when those respective conditions are satisfied. We take both R^X and R_o to be complete, acyclic, and reflexive rankings (CARR for short).

THEOREM 3.1: *Menu-independence of preference entails menu-independence of the generated choice function, but menu-independence of a choice function need not entail menu-independence of the preference that generated this choice function.*

A proof of this proposition can be found in the Appendix.

It can also be seen that menu-independence of the choice function is not really different from the binariness of the choice function. Binariness of a choice function is the condition that guarantees that for every set S that is chosen $C(S)$ is exactly what would be chosen if the best elements of S were picked using the ranking given by the revealed preference relation R_c of the choice function as a whole (see Sen (1971), Herzberger (1973), Suzumura (1983)).

Weak revealed preference: For any x, y in X:

(3.8) $xR_c y \Leftrightarrow$ [for some S: $x \in C(S)$ & $y \in S$].

Binariness of a choice function: A choice function is binary if and only if, for all S:

(3.9) $C(S) = B(S, R_c)$.

Now we present an equivalence result (for a proof see the Appendix).

THEOREM 3.2: *A choice function is binary if and only if it is menu-independent.*

This permits us to get the following result, in view of the known property that a complete choice function over a finite set X (defined over all nonempty subsets) is binary if and only if it satisfies Properties α and τ (see Sen (1971)).[29]

THEOREM 3.3: *A complete choice function over a finite set X has a menu-independent revealed preference if and only if it satisfies Properties α and τ.*

In fact, binariness can be intuitively well understood as a condition of menu-independent maximization. The choice over any *given menu* can, of course, be rather trivially shown to be based on *optimization* according to a preference relation that incorporates the revealed preference *over that menu.* What menu-independence does is to assert that some grand binary relation R_c can "take over" all the different menu-specific weak revealed preferences and still work to generate that entire choice function and be exactly generated by it.

4. FIDUCIARY RESPONSIBILITY, NORMS, AND STRATEGIC NOBILITY

The responsibility associated with the choice act can take many different forms. It can be particularly "heavy" when a person has to act on behalf of others, in a fiduciary capacity. In so far as choosing over the lives of others can be avoided, many may well prefer that. There is nothing particularly "irrational" or "contrary" about such a preference to shun particular choice acts (affecting the lives of others), but nevertheless its operation can go against standard formulations of axioms of rational choice, in the presence of uncertainty.

The general point about the relevance of fiduciary choice roles can be illustrated by a case that was discussed (in Sen (1985b)) in the context of reviewing rationality under uncertainty. In a remote rural area of China, Dr. Chang has one unit of medicine, but faces two children who are both fatally ill; either one can possibly be saved by that unit of medicine, but not both. Dr. Chang believes that the medicine, if given to sick child A, will save her life with a slightly higher probability than it would save the life of B if given to him (say, 91% probability of cure, according to standard medical statistics, for A, and 90% for B). If Dr. Chang has to give the medicine to one or the other (with certainty), he might well prefer to give it to A, since A has a somewhat better chance of recovery. And yet he might prefer most not to have to take a decision

[29] Similar correspondences can be established with related results in personal and social choice, for example: Hansson (1968a, 1968b), Chipman, Hurwicz, Richter, and Sonnenschein (1971), Pattanaik (1971), Fishburn (1973), Herzberger (1973), Plott (1973), Brown (1974), Kanger (1975), Blair, Bordes, Kelly, and Suzumura (1976), Blau and Deb (1977), Sen (1977a), Aizerman and Malishevski (1981), Blair and Pollak (1982), Deb (1983), Pattanaik and Salles (1983), Kelsey (1984), Moulin (1985), Schwartz (1986), Blau and Brown (1989), Heaps et al. (1992), Aizerman and Aleskerov (1995), among other contributions.

that would deny the medicine for survival to one of the two children. Dr. Chang can opt for a probabilistic mechanism (with or without slightly favoring A in the fixing of probabilities) either because he finds that denying the medicine outright to B is unfair or unjust (given that B too has an excellent chance of recovery with the medicine), or because he has a simple desire to avoid "playing God" (in deciding who might live and who should die). In either case, it is the *choice act* of giving the medicine definitely to one and denying it to the other that Dr. Chang may be shunning.

Such choice behavior would violate the "sure thing principle" and the framework of expected utility theory, which would demand that if giving the medicine to A is preferred to giving it to B, then giving it to A must be preferred to any lottery over the two.[30] The violation of the "expected utility" axioms can be prevented by redefining the options in more *comprehensive* terms, through the inclusion of choice acts and processes; for example, the outcome in which A gets the medicine *through* a lottery need not be seen as the same result as one in which A is simply *given* the medicine by Dr. Chang. But this would be at the cost of making the expected utility axioms trivially fulfilled (or non-violated), robbing the theory of much of its operational content. And yet Dr. Chang could be *maximizing* an objective function, which is easy to articulate, that happens to be sensitive to the unattractiveness of having to make some particular types of choices and to take the associated responsibilities. The general issue, once again, is the accommodation of the salience of the choice act in the process of decision making.

Fiduciary responsibility can influence choice behavior not only through the preference to avoid it when possible (as in Dr. Chang's case), but also through the nature of the choices made when that responsibility is seized. Ragnar Frisch (1971a), in whose memory this Frisch Memorial Lecture is being given, discussed the far-reaching impact of responsibility when one is trusted with acting for others.[31] He illustrated his point with an example.

> Assume that my wife and I have had dinner alone as we usually do. For dessert two cakes have been purchased. They are very different, but both are very fine cakes and expensive —according to our standard. My wife hands me the tray and suggests that I help myself. What shall I do? By looking up my own total utility function I find that I very much would like to devour one particular one of the two cakes. I will propound that this introspective observation is *completely irrelevant* for the choice problem I face. The really relevant problem is: which one of the two cakes does my wife prefer? If I knew that the case would

[30] See also the discussion in Sen (1985b) of other types of cases in which the axioms of expected utility can be sensibly violated; also see Machina (1981) and Anand (1993). The point here is not so much to argue against "expected utility" theory *in general*; I know of no other theory which does, on the whole, quite so well in a wide variety of circumstances. It is more a question of knowing what its limitations are, and *why* they arise.

[31] Frisch (1971a, 1971b) was particularly concerned with policy decisions for the society taken by experts, and in that context with the "cooperation between politicians and econometricians."

be easy. I would say "yes please" and take the *other* cake, the one that is her second priority.[32]

It is important, in this context, to note that Frisch's characterization of the problem is not one of maximization of a compound personal utility function that *incorporates* his altruism towards others (as in, say, Becker (1976)). Rather, the other person's well-being remains a separate concern, of which note has to be taken *over and above* the extent to which it enters what Frisch calls "my own total utility function."[33]

There is an aspect of Frisch's interesting remark that demands particular attention in the present context. Suppose Frisch's own utility function, as he sees it, places cake x above the other cake y, and he also thinks that his wife too would enjoy x more than y. Frisch argues that he would definitely then choose y, given the choice over a set containing exactly one of each kind of cake. If, on the other hand, there were two of each kind of cake, then presumably Frisch too would choose the cake he likes more, to wit, x, since that would still leave his wife with the option of having her preferred type of cake x. So, at a simple level of ranking cakes, this kind of choice behavior would seem to be *menu-dependent*, and in particular violate basic contraction consistency (Property α).

It can, however, be argued that there is no real violation of menu-independent preferences here, *provided* the outcomes are not characterized only in terms of what Ragnar Frisch himself picked, but through a fuller account of the respective overall outcomes, in particular the consumption of *both* of them. If Frisch chooses the "preferred" cake x from the smaller set of one type of cake each, then he is in effect picking x for himself and y for his wife, whereas from the larger set of more than one cake of each type, when he chooses x, he permits the consumption of the "preferred" cake of type x by each.

In this broader consequential formulation, the two choices of Frisch are rationalizable within one menu-*independent* preference ordering. There is, in principle, a similar option of a broader interpretation in the other cases discussed earlier (such as the garden-chair story) between (i) menu-dependent choice of personal options, and (ii) menu-independent choice of broader consequences. However, in following rules of behavior such as "never pick the most attractive chair" or "never choose the last fruit" (or, for that matter, "never pick

[32] This translation of a passage from Frisch (1971b), his last paper, is by Loav Bjerkholt, and occurs in Bjerkholt (1994), who cites this as an example of Frisch's revisiting his early interest in utility analysis. I was sent an earlier translation of this by Leif Johansen, whose personal communication on this drew my attention to Frisch's rejection of the assumption of self-interested behavior standardly assumed in modern economics (Johansen was commenting on Sen (1973b)). See also Johansen's (1977) own analysis of these issues.

[33] In terms of the distinction presented in Sen (1977b), this is a case which involves "commitment," not just "sympathy." While this substantive distinction is not pursued in this paper, which is more concerned with the formal structure of choice functions (and the particular role of the choice act), it has much interpretative significance in that altruism through sympathy is ultimately self-interested benevolence, whereas doing things for others through commitment may require one to "sacrifice some great and important interest of our own," as Adam Smith put it in distinguishing "generosity" from "sympathy" (see Smith (1790, p. 191)).

a unique opportunity of a corrupt deal," or "never take a solitary chance of crossing a picket line"), the motivating factor need not be any concern about the well-being of others (as it clearly is, in Frisch's own case), but simply following an established rule—or a mode of choice—that is quite menu-*dependent.*[34] Menu dependence may not, thus, be avoidable in all the cases considered earlier through broader characterization, even though in Frisch's case this actually does work well enough.

The important issue in the Frisch example is not menu dependence as such, but *chooser dependence.* There is a particularly pointed aspect of responsibility of fiduciary obligation that is interestingly raised by Frisch's description of his choice problem. Frisch's motivating concern can be interpreted in two rather different ways:

(i) to maximize the choice-related value he attaches to the *joint* outcome (including joint well-being);

(ii) to maximize his wife's well-being when he has the *fiduciary* responsibility for what she gets (but to maximize his own well-being when his wife has all the relevant options).

Frisch's choice behavior can be explained in either way, but it is the latter interpretation that appears to correspond more closely to the way Frisch himself describes the situation. He seems to give complete priority to his wife's interest, in the role in which he is placed (his own enjoyment of the cake is seen as "*completely irrelevant* for the choice problem" he faces). When the responsibility of acting for others makes people give priority to what they are charged to do, the nature of the preference function and the choice behavior will reflect the way the interests of others are put together, which takes us to "social choice theory." The investigation will join here with the rather large literature on regularity conditions in social choice theory, including the uses and violations of such properties as α and τ (critical assessments of the main results can be found in Suzumura (1983) and Sen (1986)).[35]

There is a connection to be further explored here with game theory as well. The influence that roles and acts of choice have on what is chosen can be strategically significant, and one's choice of strategy has to take adequate note of the dependence of people's actual choice on their exact roles. One consequence of this is that sometimes one can serve self-interest better through

[34] The same applies to such behavioral norms as "never pick the largest slice of cake" (irrespective of concern for others); see Sen (1973b, 1993), Baigent and Gaertner (1996).

[35] Kenneth Arrow (1951) had used a framework of classical optimization for individual as well as social choice, in a rather demanding form, including transitivity of social preference. James Buchanan (1954) raised the important question as to whether *any* internal regularity conditions should be imposed on social choice at all, since the society is not like an individual, and went on to ask what role these regularity conditions play in generating Arrow's impossibility theorem. These questions are addressed in Sen (1993, 1995a), and include establishing an extension of Arrow's theorem *without* any internal regularity condition on social choice.

behaving more "nobly" and by handing over the choice to others. This phenomenon may be called "strategic nobility." Such connections can be illustrated with a simple game, to be called the "fruit-passing game."

Take a two-person game of choosing in turn a fruit from a basket containing one mango and two apples (as in set S considered earlier), with "passing" being a permissible option. Players 1 and 2 choose successively until the basket is withdrawn, which happens when each has got a fruit, or when each has passed, or when one has passed after the other has got a fruit. The game is shown in Figure 4.1.

Consider the case in which both players prefer having a mango over an apple, but follow "norms" or "rules" of behavior, related to choice roles, that exclude the picking of the last of any fruit except after the other person has already got a fruit. Had there been no such norm (and no special responsibility of choice role), player 1 would simply grab the solitary mango. If, however, taking such a responsibility is a part of the social norm that player 1 follows, he will not allow himself to do this.

If he selects an apple, then he settles for a suboptimal outcome. Can he do better? If he passes, then he might have another chance, but that may not help (i) if player 2 takes the mango, or even worse, (ii) if player 2 passes also, thereby bringing the game to an end. The latter expectation may not be entertained by player 1 if he knows that player 2 would definitely prefer either fruit to none (while preferring a mango to an apple). The former possibility would be

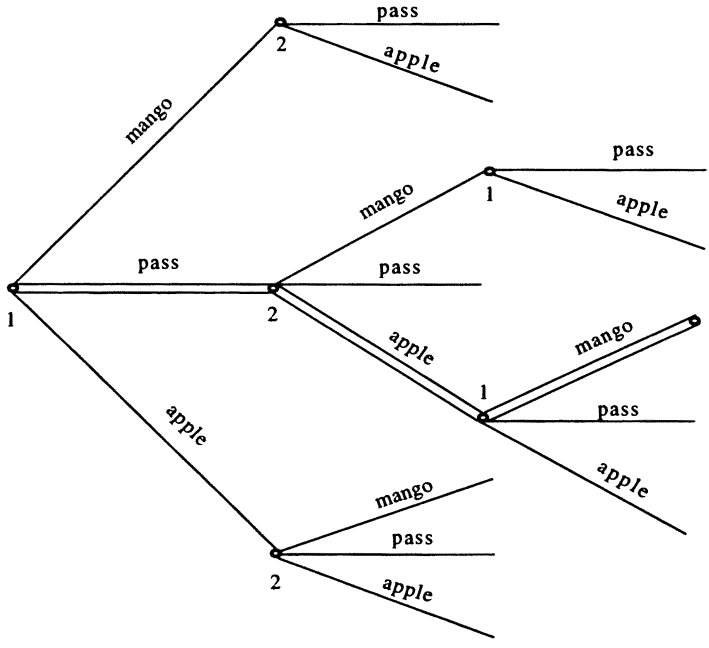

FIGURE 4.1.—Fruit passing game with common knowledge of norms.

eliminated too if it were common knowledge that player 2 would also follow the norm of not choosing the solitary mango until the other has had a fruit. In that case, the game is entirely predictable, with player 1 passing first, followed by player 2 picking an apple, and the mango then going to player 1 in the end. The outcome is represented by the double-lined route.

Strategic issues of this kind, including the use of *common knowledge of norms*, can enrich the formulation of games, and indicate why a person might wish to behave in a more "noble" manner (giving others the choice), despite having self-interested preferences over culmination outcomes. Social interactions of this kind, including strategic nobility, based on the common knowledge of norms, can be fruitfully incorporated in the formulation of games and strategic behavior.[36]

5. MAXIMIZATION AND OPTIMIZATION

The classical framework of optimization used in standard choice theory can be expressed as choosing, among the feasible options, a "best" alternative, as already defined in (3.5).[37] The general discipline of maximization differs from the special case of optimization in taking an alternative as choosable when it is not known to be worse than any other (whether or not it is also seen to be as good as any other). To define the maximal set, we use the asymmetric factor ("strictly preferred to") P of the weak preference relation R. For an element of S to qualify for the maximal set $M(S, R)$, no other alternative in S must be strictly preferred to it:

$$(5.1) \qquad M(S, R) = [x | x \in S \ \& \text{ for no } y \in S\text{: } yPx].$$

The basic contrast between maximization and optimization arises from the possibility that the preference ranking R may be incomplete, that is, there may be a pair of alternatives x and y such that x is not seen (at least, not *yet* seen) as being at least as good as y, and further, y is not seen (at least, not yet seen) as at least as good as x.[38] It is useful to consider the distinction between: *tentative incompleteness*, when some pairs of alternatives are *not yet* ranked

[36] Indeed, even Mrs. Frisch's choice in the example considered earlier ("My wife hands me the tray and suggests that I help myself") can have a strategic explanation. Of course, I am not making the monstrous suggestion that this was indeed the case in the anecdote of familial love recounted by Frisch.

[37] In fact, optimization can be characterized either in terms of a binary relation R ("preference"), or a real-valued function U ("utility"). The relational framework is rather more general, since R need not be an ordering with transitivity (acyclicity will do), whereas a utility function must have (i) ordering properties, and (ii) some additional characteristics (such as some continuity of preference) that guarantee numerical representability (on this see Debreu (1959)). The analysis in the preceding sections of this paper has been based on the relational form, and I shall continue with it.

[38] On properties of incomplete rankings (and the extendability of incomplete orderings to complete orders), see Szpilrajn (1930) and Arrow (1951). See also Sen (1970a), Suzumura (1983), and Levi (1986). Levi approaches the problem of "unresolved conflicts" somewhat differently from that pursued here, in terms of his important notion of "V-admissibility."

(though they may all get ranked with more deliberation or information), and *assertive incompleteness*, when some pair of alternatives is asserted to be "non-rankable."[39] Assertive incompleteness is the claim that the failure of completeness is not provisional—waiting to be resolved with, say, more information, or more penetrating examination. The partial ranking, or the inexhaustive partitioning, may simply not be "completable," and affirming that some x may not be rankable *vis-à-vis* some y may be the right answer in these cases. I shall not further pursue this distinction here, nor presume that any incompleteness must necessarily be tentative.[40]

How does the maximal set relate to the optimal set? I present below five basic propositions on this.[41] The trivial case of a one-element ("unit") set is excluded, and also attention is confined to finite sets S (though there are fairly straightforward extensions to infinite sets). No domain restrictions are imposed on the permissible preference relation R, which could be any binary relation whatsoever, except that it is assumed that R is reflexive (xRx for all x), every alternative is seen to be as good as itself (not an exacting demand, if I am any judge).

THEOREM 5.1: $B(S, R) \subseteq M(S, R)$, *but not generally the converse. The cases when* $B(S, R)$ *and* $M(S, R)$ *differ can be partitioned into two categories*:
Case 1: $B(S, R)$ *is empty while* $M(S, R)$ *is not*; *and*
Case 2: $B(S, R)$ *and* $M(S, R)$ *are both nonempty, and so is* $[M(S, R) - B(S, R)]$.

THEOREM 5.2: $M(S, R)$ *is nonempty for any finite set if and only if* R *is acyclic.*

THEOREM 5.3: $B(S, R) = M(S, R)$ *if either of two following conditions hold*: (I) R *is complete, or* (II) R *is transitive and* $B(S, R)$ *is nonempty.*

THEOREM 5.4: *Every maximizing choice function with respect to a preference relation* R *can be replicated by an optimizing choice function with respect to a devised binary relation* R^+; *that is, there exists a binary relation* R^+ *such that for every* S: $B(S, R^+) = M(S, R)$.

THEOREM 5.5: *Not every optimizing choice function can be replicated by some maximizing choice function*; *that is for some binary relation* R *which generates a class of optimal sets* $B(S, R)$, *there may exist no binary relation* R^+ *such that* $M(S, R^+) = B(S, R)$ *for all* S.

[39] This distinction is discussed in Sen (1992a, pp. 46–49), and Sen (1996). On related matters, see also Sen (1970a), Suzumura (1983), and Levi (1986).

[40] The need to accommodate incompleteness in preference theory has been illuminatingly discussed by Putnam (1986). See also Williams (1973).

[41] These theorems both systematize and extend results established in Sen (1970a, 1971) and Suzumura (1976, 1983).

What are the lessons from all this? Theorem 5.1 tells us that while a best alternative must also be maximal, a maximal alternative need not be best. *Case 1* covers the case in which there are no best alternatives whatever, but a maximal choice can still be made. This is easily seen by considering the situation in which neither xRy nor yRx, so that $B(\{x, y\}, R) = \varnothing$, whereas $M(\{x, y\}, R) = \{x, y\}$.

A classic example of *Case 1* is given by one interpretation of the story of Buridan's ass: the tale of the donkey that dithered so long in deciding which of the two haystacks x or y was better, that it died of starvation z. There are two interpretations of the dilemma of Buridan's ass. The less interesting, but more common, interpretation is that the ass was indifferent between the two haystacks, and could not find any reason to choose one haystack over the other. But since there is no possibility of a loss from choosing either haystack in the case of indifference, there is no deep dilemma here either from the point of view of maximization or of that of optimization. The second—more interesting—interpretation is that the ass could not rank the two haystacks and had an incomplete preference over this pair. It did not, therefore, have any optimal alternative, but both x and y were maximal—neither known to be worse than any of the other alternatives. In fact, since each was also decidedly better for the donkey than its dying of starvation z, the case for a maximal choice is strong. Optimization being impossible here, I suppose we could "sell" the choice act of maximization with two slogans: (i) maximization can save your life, and (ii) only an ass will wait for optimization.[42]

Case 2 is more subtle. Consider a preference ranking that consists exactly of xIy and yIz, with no other pair in the set $S = \{x, y, z\}$ being ranked (where I is the symmetric factor—indifference—of the weak preference relation R). Clearly, $B(S, R) = \{y\}$, and $M(S, R) = \{x, y, z\}$. But the real force of Theorem 5.1 lies in showing that maximization may work even when optimization does not (*Case 1*), with the added lesson that sometimes maximization may permit a wider set of possible choices than optimization would (*Case 2*).

Theorem 5.2 shows the reach of maximization—in particular that it works whenever there is the weak property of acyclicity (neither completeness nor transitivity is needed).[43] How can we go from there to optimization? Obviously, completeness would eliminate the difference between maximization and optimization, and transitivity is not needed for optimization either. But perhaps

[42] However, with nonoptimizing maximization, we have to drop the insistence on the weak axiom of revealed preference (WARP) and other so-called "consistency conditions" such as Properties α and τ.

[43] Acyclicity is the absence of any strict preference cycle of finite length. The central role of this property in rational choice is analyzed in Sen (1970a, Chapter 1*; 1971). In the special case in which R is transitive, it can be shown that the maximal set generated by R is the *union* of the optimal sets generated by all possible complete orderings R^* *compatible* with R (see Banerjee and Pattanaik (1995); see also Levi (1986, Theorem 7, p. 100)). The dual of this proposition has been established by Kotaro Suzumura (unpublished note), showing that the optimal set generated by a complete ordering R^* is exactly the *intersection* of the maximal sets generated by all subrelations R of R^*. Nick Baigent has examined the interpretative aspects of these connections (unpublished note).

more interestingly, if we have transitivity—even without completeness—the maximal and optimal sets would fully coincide, if there is any optimal alternative at all.[44] Theorem 5.3 identifies that the real significance of "nonoptimizing maximization" relates to two types of cases for any given menu of alternatives S:

(i) when there may be no optimal alternative at all, but still a maximal alternative (this can result from *incompleteness*); *or*

(ii) when there is an optimal alternative but not every maximal alternative is optimal (this can happen with *intransitivity*).[45]

The last two of the results deal with the possibility of "mimicking" maximization by optimization and the converse. Theorem 5.4 shows that every maximizing story with $M(S, R)$ can be made into a case of *as if* optimization, for some suitably devised "as if" preference ranking R^+.[46] This is done by converting incompleteness into indifference, in particular by taking xR^+y if and only if *not* yPx, which may be called the "completed extension" of R. This way of proceeding as a constructive device has some substantive interest of its own. A significant case of converting incompleteness into indifference is the "Pareto-extension rule" (discussed in Sen (1969, 1970a)): xRy if y is not Pareto-preferred to x. This incorporates all Pareto relations but makes Pareto noncomparable pairs socially indifferent. It can be shown that this rule satisfies all the conditions invoked in Arrow's (1951) impossibility theorem, for the case of quasi-transitive social preference (that is, for the case in which social strict preference is transitive, but not necessarily weak preference).[47]

The regularity properties of optimization, including that of "binariness," the roles of Properties α and τ, and so on, can be applied, by virtue of Theorem 5.4, to *maximization* in general, operating on the completed extension R^+ of the primitive R (see Sen (1970a, 1986), Suzumura (1976, 1983), Moulin (1985)). This is of much *formal* convenience, but since R^+ is really a figment of our constructive imagination, it is important not to *interpret* it as the person's actual "preference," which continues, of course, to be given by R, not R^+.

[44] To check this, take x from $B(S, R)$ and y from $M(S, R)$, with x distinct from y. Clearly, xRz for all z in S, since x is optimal; but not xPy since y is maximal. So xIy. Hence by transitivity, yRz, for all z in S. So y too is optimal.

[45] These results provide a fuller understanding of the difference between the two possibilities called Case 1 and Case 2 respectively in Theorem 5.1.

[46] This general result was first established by Suzumura (1976).

[47] Theorem V in Sen (1969). It is analytically remarkable that the axiomatic requirements that together generate Arrow's impossibility result are satisfied by the Pareto extension rule, except for weakening the demand of transitivity by quasi-transitivity (that is, except for dropping the transitivity of indifference). Nevertheless, substantively this is an unattractive social decision rule and cannot be seen, in any sense, as a "resolution" of Arrow's impossibility problem (Sen (1969, 1970a)). In fact, this line of weakening led in the 1970's and 1980's to a series of related results about the arbitrariness of the distribution of decisional power, presented by Allan Gibbard, Andreu Mas-Colell, and Hugo Sonnenschein, Charles Plott, Donald Brown, Ashok Guha, Douglas Blair, Georges Bordes, Jerry Kelly, and Kotaro Suzumura, Julian Blau and Rajat Deb, Douglas Blair and Robert Pollak, and David Kelsey, among others (on which see Sen (1986)).

Theorem 5.4 shows that any maximizing framework can be seen formally as an optimizing structure for a suitably devised *as if* preference relation R^+. What about the converse? Is any optimizing framework also an *as if* maximizing one? This question might look redundant since we know already that an optimal alternative is also maximal for a shared preference relation R. But that is not decisive for two distinct reasons. First, there may exist no binary relation R^+ that generates $B(S, R) = M(S, R^+)$ for all S, with *the same* R^+ for all subsets S of the universal set.[48] We return here to the issue of menu-dependence again: there may exist no menu-independent "as if" preference R^+ that mimics the maximal sets of the real preference R. Second, even for any given menu set S, and for every possible devised ranking R^+ (no matter what), the optimal set may tend to be systematically *too small* to match exactly the maximal set. Consider an "unconnected" pair $\{x, y\}$, such that *not* xRy and *not* yRx. Hence $B(\{x, y\}, R)$ must be empty. But we cannot devise any R^+ which would make $M(\{x, y\}, R^+)$ empty, for that would be possible if and only if xP^+y and yP^+x, which is logically contradictory, given the asymmetry, by construction, of the "asymmetric factor" P^+ of R^+.[49]

Since all these results relate to menu-*independent* preferences, questions will naturally be asked, in view of our previous discussion (particularly in Section 3), as to whether these results carry over to menu-dependent preferences as well. The answer is basically in the affirmative, as indicated by Theorem 5.6, but the formal discussion of this extension (and its proof) is left for the Appendix.

THEOREM 5.6: *Even with menu-dependent preferences, Theorems* 5.1–5.5 *hold for some set S separately, for some weak binary relation of preference* R^S *over S, and furthermore, all except Theorem* 5.5 *hold for every possible set S (except for unit sets), for some* R^S.

The set of results Theorem 5.1–5.6 identify the relation between maximal choice and optimal choice. Maximality does have a wider scope, and the difference can be substantial *whether or not* there is a nonempty optimal set. The fact that maximization can be matched by an *as if* optimization exercise

[48] See Suzumura (1983), who considers the respective demands of what he calls "M-rational choice" (corresponding to choosing R-maximal elements for all sets for some binary relation R) and "G-rational choice" (corresponding to choosing R-greatest elements for all sets for some binary relation R). Suzumura demonstrates that, even though every M-rational choice function can be G-rationalized (this corresponds to Theorem 5.4 above), nevertheless a G-rational choice function need not be M-rationalizable (Example 1, Appendix B, in Suzumura (1983, p. 56)).

[49] An alternative way of proving this is to use the example employed to establish Case 2 in Theorem 5.1 (see also Suzumura (1983, p. 56)). Consider, to recapitulate that case, a preference ranking that consists exactly of xIy and yIz, with no other pair in the set $S = \{x, y, z\}$ being ranked. We have here: $B(\{x, y\}, R) = \{x, y\}$; $B(\{y, z\}, R) = \{y, z\}$; $B(\{x, z\}, R) = \emptyset$; $B(\{x, y, z\}, R) = \{y\}$. Even if we leave out the assertion $B(\{x, z\}, R) = \emptyset$, much the same as the case already considered in the text, and concentrate on the *other* assertions regarding optimal sets, there is no R^* that can simultaneously guarantee $M(\{x, y\}, R^*) = \{x, y\}$; $M(\{y, z\}, R^*) = \{y, z\}$; and the much too small $M(\{x, y, z\}, R^*) = \{y\}$.

does not reduce the importance of broadening the focus from optimization to maximization, since R and R^+ may have quite different contents, and the interpretation of what is being optimized changes in the move from R to R^+. In fact, *as if* optimization works with a devised preference relation R^+ precisely by *mirroring* the result of nonoptimizing maximization.[50] Indeed, the possibility of this mirroring suggests that the move to maximization—away from sticking to optimal choice with given preference relations R—is helpful even for those who remain wedded (as many economists seem to be) to the *formal* aspects of optimization. Given the case for seeing "rational choice" as *maximization* (rather than optimization), when the focus is on the choice act of a rational but partly undecided individual (as discussed in Section 1), this is an important set of analytical connections and disjunctions.

The point can be illustrated with the important and influential concept of "satisficing" developed by Herbert Simon (1957, 1982), which has often been seen as nonmaximizing behavior. This is one of the rich concepts that Simon has contributed, and it relates to his larger focus on "bounded rationality."[51] To illustrate, a businessman may strive hard to reach a satisfactory level of profit (say, a million dollars), but accept to settle for a sufficiently high level of profits, without its being necessarily the highest possible (for example, he may find $1 million and $1.01 million both satisficing, given the bounds on his information, ability to calculate, etc.).[52]

The discussion of "satisficing versus maximizing" has been somewhat deflected by the tendency to identify maximization with optimization. The businessman who is willing to settle for $1 million, without continuing to worry about the possibility of raising it to $1.01 million, regards both $1.00 million and $1.01 million as acceptable, but does not necessarily regard the two as "equally good." Denoting the former as x and the latter as y, *in terms of his welfare function*, this businessman i might well place y above x. On the other hand, given his other priorities and the limits of time and organization that influence his *choice behavior*, he is ready to settle for either x or y. That is, in terms of his goal (possibly tentative goal), neither is x placed above y, nor is y ranked above x. Nor is there any decision here to accept the two as "equally good" as goals—only he is ready to settle for either. So in terms of *the goal function* (as opposed to his *welfare function*), there is a "tentative incompleteness" here, and both x and y can be seen as "maximal" in terms of his *operational* goals.

Thus interpreted, satisficing corresponds entirely to maximizing behavior. And yet it does not correspond to optimization (either of the welfare function, or of a goal function, or of course of profits). This is one illustration of the reach of the

[50] The general "mirroring" obtained here has some similarity with the particular relationship between "the Pareto extension rule" and "the strong Pareto quasi-ordering" discussed by Weymark (1984).

[51] General analysis of "bounded rationality," pioneered by Herbert Simon, has transformed, in many ways, our understanding of what it is to be rational in a world of limited epistemic, cognitive, and analytical opportunities.

[52] See also Akerlof and Yellen (1985) on the related idea of "near rationality."

general framework of maximizing behavior explored here. Simon's cogent reasons for "satisficing" need not be seen as anti-maximization arguments. Can "satisficing" also be seen as "as if" optimization? By virtue of Theorem 5.4, a maximization exercise can, of course, be *formally* seen as an *as if* optimization exercise for a completed extension R^+. But as was discussed earlier, while there is an isomorphism here, the formal use of an "as if" preference is *interpretatively* quite different. Thus the substantive gap between satisficing and optimizing remains (closable only in a *purely formal* way), whereas the gap between satisficing and maximizing is both formally and substantively absent.

6. PREFERENCE AND SELF-IMPOSED CONSTRAINTS

In the discussion so far, the influence of the process of choice, and in particular of the menu, has been considered interchangeably (i) through the *preference ranking* (incorporating concerns about choice acts *within* the preference ranking), and (ii) through *self-imposed choice constraints*, excluding some options from "permissible" conduct (we leaned towards this latter way in the formulation of the "fruit-passing game"). They are not, of course, formally equivalent, and it is useful to consider how they may relate. We must also examine the nature of self-imposed constraints as parts of "norms" of behavior or "rules" of choice.

The practice of enjoining rules of conduct that go beyond the pursuit of specified goals has a long tradition. As Adam Smith (1790) had noted, our behavioral choices often reflect "general rules" that "actions" of a particular sort "are to be avoided" (p. 159). To represent this formally, we can consider a different structure from choosing a maximal element, according to a comprehensive preference ranking (incorporating inter alia the importance of choice acts), from the given feasible set S (allowed by externally given constraints). Instead, the person may first restrict the choice options further by taking a "permissible" subset $K(S)$, reflecting *self-imposed* constraints, and then seek the maximal elements $M(K(S), R)$ in $K(S)$. The "permissibility function" K identifies the permissible subset $K(S)$ of each option set (or menu) S.

How different an approach is the use of such a permissibility function in comparison with incorporating our concerns fully in the preference ranking itself? The formal features of the difference can be more readily disposed of than its substantive relevance. Consider a person with a preference R over the universal set X; I am taking this R to be menu-independent, but the argument to be presented would hold a fortiori if the preference were menu-*dependent*. When it comes to choosing from a specified menu S (determined only by externally-given limits, but *no* self-imposed constraints), the person aims at identifying the maximal elements $M(S, R)$ of S with respect to R. The effect of a self-imposed constraint that specifies a permissible set $K(S)$ to which she deliberately confines her selection is to make her pick a maximal element,

according to R, of $K(S)$ rather than of S:

$$C(S) = M(K(S), R). \tag{6.1}$$

Can the route of self-imposed choice constraint be represented as maximization with an *as if* preference relation R_*? The answer to this question turns on the issue of *menu dependence*, as the following results immediately establish (for proofs, see the Appendix).

THEOREM 6.1: *For any permissibility function K and any S, there exists an "as if" preference R_*^S such that*:

$$M(S, R_*^S) = M(K(S), R) = C(S). \tag{6.2}$$

THEOREM 6.2: *For any reflexive R, there is some permissibility function K, such that there is no menu-independent R_* that induces*

$$M(S, R_*) = M(K(S), R), \quad \textit{for all } S. \tag{6.3}$$

It is, thus, clear that while the approach of "as if" preferences can take on the role of "mimicking" the use of self-imposed choice constraints, the indexation S in R_*^S is necessary for this to work (a menu-independent "as if" preference R_* would not do). Thus, the route of self-imposed constraints $K(S)$ has a close *formal* correspondence with that of maximization according to *menu-dependent preferences*. Indeed, the different examples of menu-dependence discussed in the previous sections can be interpreted either in terms of (i) menu-dependent preferences R^S (with or without any self-imposed choice constraint), or (ii) self-imposed choice constraints $K(S)$ (with or without a menu-dependent basic preference).

Despite this formal isomorphism with menu-dependent preferences, the procedure of self-imposition of choice constraints can make a real difference in substance. The *as if* preference R_*^S is, of course, a devised construction and need not have any intuitive plausibility *seen as preference*. A morally exacting choice constraint can lead to an outcome that the person does not, in any sense, "desire," but which simply mimics the effect of his self-restraining constraint.

To illustrate, there has been a good deal of discussion recently on the alleged tendency of many Japanese workers to work extraordinarily hard, and the idea of "karoshi" (death through overwork) has been discussed in that context (see, for example, Morishima (1995)). The tendency to do one's "duty" to the point of severely damaging one's health (whether or not leading literally to "death") is easier to explain as the consequence of adhering to a deontological obligation rather than as an outcome that is actually "preferred" by the hapless worker. Social psychology can be important here. The *as if* preference works well enough formally, but the sociology of the phenomenon calls for something more than the establishment of formal equivalences.

This issue is close to Adam Smith's general point that many behavioral regularities can be explained better by understanding people's attitude to

actions, rather than their valuation of final outcomes.[53] Similarly, Immanuel Kant gave a central position in social ethics to a class of restrictions on actions, which formed a part of what he saw as the "categorical imperative," as elucidated by the following remark in the *Groundwork*: "There is... but one categorical imperative, namely this: Act only on that maxim whereby thou canst at the same time will that it should become a universal law" (translated by Abbott (1889, p. 38)). The form of the imperative, which is crucial to Kant's reasoning, is the need to impose on oneself some constraint on how one can *act*.[54]

While the focus of Smith's and Kant's reasoning is normative rather than descriptive, the two are closely linked in their analyses, since both understood actual behavior to be partly based on norms.[55] Their behavioral analysis included seeing the process of actual choice through $K(S)$, and not just through an "everything considered" grand preference ranking R^S. Nor is the force of Smith's or Kant's claims regarding self-imposed "action constraints" reduced by the *formal equivalence*, given by Theorem 6.1, since the role of the devised R^S_* is entirely representational.

7. CONCLUDING REMARKS

In this paper I have tried to examine the role of the choice act in *maximizing behavior*, which has to be distinguished from maximization *without* volitional choice by a maximizer, as, for example, in standard models of physics (Section 1). The *process* of choice can be an important concern (Sections 3, 4, 6), and so can be the *necessity* of choice even when the alternatives are not fully ordered and the conflicting considerations not fully resolved (Section 5). The analysis shows how the maximizing framework can adequately accommodate *both* issues, once its axiomatic structure is correspondingly adjusted.

[53] As Smith (1790) put it, many of the rule-governed choices involving self-sacrifice are "not so much founded upon [their] utility," but reflect primarily "the great, the noble, and the exalted property of such actions" (p. 192).

[54] Kantian analysis is not grounded on the strategic rationality of conduct, or on the idea that if one follows the maxim (or generally behaves well towards others), then others are more likely, for one reason or other, to reciprocate. Indeed, elaborating his argument—by commenting on a rather simpler connection than the ones recently investigated in evolutionary game theory—Kant argued that "everyone knows very well that if [a person] secretly permits himself to deceive, it does not follow that everyone else will do so, or that if, unnoticed by others, he [is] lacking in compassion, it does not mean that everyone else will immediately take the same attitude toward him" (Kant (1788), *Critique of Practical Reason*, in Beck's 1956 translation; see also Herman (1990, p. 243)). Rather, Kant's claim was that a person has a reasoned moral obligation to follow such a maxim no matter what others do. See also Smith (1790, III.4).

[55] Smith also emphasized the connection between consciously moral motivation and the use of good moral conduct as general behavioral norms accepted in the society: "Many men behave very decently, and through the whole of their lives avoid any considerable degree of blame, who yet, perhaps, never felt the sentiment upon the propriety of which we found our approbation of their conduct, but acted merely from a regard to what they saw were the established rules of behavior" (Smith (1790, p. 162)). On related matters, see also Sacco and Zamagni (1993).

Some of the findings can be briefly identified. First, one aspect of volitional choice is the possibility that choice acts may have to be undertaken with substantial incompleteness in judgements (arising from instrumental or valuational reasons). While this is problematic for the framework of classical optimization standardly used in economics, there is no great difficulty in systematically accommodating such incompleteness in a framework of maximizing behavior and to study its regularity properties as distinct from those of optimization (Section 5). Exploration of the relationship between maximization and optimization (characterized in Theorems 5.1–5.6) shows exactly how they relate and where the gaps are. The difference between maximizing and optimizing can be formally closed, in one direction (from maximization to optimization, not vice versa), through an "as if" preference, but a substantive interpretative difference remains even here. The directional asymmetry lends further support (in addition to the larger reach of maximization) to the case for taking maximization to be the mainstay for rational choice functions.

Simon's formulation of "satisficing" behavior, connected with his important idea of bounded rationality, can be accommodated *within* a general maximizing framework, eliminating the tension between satisficing and maximizing (but the tension with optimization remains, except in terms of the formal device of an "as if" preference).

Second, the process of choice—and in particular the act of choice—can make substantial difference to what is chosen. While the differences can take various complex and subtle forms (Sections 2–4 and 6), there is a particular necessity to take note of (i) chooser dependence, and (ii) menu dependence, of preference, even judged from a particular person's perspective. The parametric preference relation $R_i^{j,S}$ of person i can reasonably rank the same elements x and y differently depending on who (j) is making the choice (in particular whether it is the person i herself: $i=j$), and the menu S from which the choice of x or y is being considered (Section 3). This is analytically important for understanding the nature of rational choice and maximizing behavior (it militates, in particular, against many widely-used "consistency conditions" that ignore these parametric variations). It is also practically important in explaining a variety of behavioral regularities in economic, political, and social affairs—from variations in work discipline and in economic corruption to the operation of social norms and of voting behavior (Sections 2–4 and 6).

Third, it is necessary to distinguish between menu-independence of preferences and menu-independence of choice functions, since there is, in general, no one-to-one correspondence between preference relations and choice functions. While menu-independence of preference entails menu-independence of the generated choice function, menu-independence of a choice function need not entail menu-independence of the preference that generated that choice function, as shown by Theorem 3.1. The connection between binariness and menu-independence can also be identified, and it is in fact convenient to see binariness of choice as a condition of menu independence (Section 3).

Fourth, the role of the choice act can be particularly significant in decisions made on behalf of others—a feature of economic policy-making on which Ragnar Frisch himself had put much emphasis. The presence of fiduciary responsibility calls for some reformulation of the standard axioms of choice theory because of the role of the choice acts. This also has implications for the formulation of games and strategic concerns, as the "fruit-passing game" illustrates (Section 4). The role of behavioral norms in general, and of the *common knowledge of norms* in particular, can be quite important for understanding strategic actions (including "strategic nobility") and the corresponding game outcomes.

Finally, the accountability and obligation to others may take the form of self-imposed *choice constraints* (as formulated by Immanuel Kant and Adam Smith) rather than being incorporated *within* reflective preferences in the *binary* form. This is not a major *technical* gulf, unless we insist on preferences being menu-*independent* (as is standardly assumed in traditional theory of preference and choice). The operation of self-imposed choice constraints can be readily represented through devised "as if" binary preferences in a menu-*dependent* format (Theorem 6.1), but not in general through menu-*independent* "as if" preferences (Theorem 6.2). However, irrespective of formal representability, the tangible differences made by the use of choice constraints can be materially important for the psychology of choice as well as the substantive nature of economic, political, and social behavior.

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APPENDIX

This Appendix establishes some results presented in the text without proof.

THEOREM 3.1: *Menu-independence of preference entails menu-independence of the generated choice function, but menu-independence of a choice function need not entail menu-independence of the preference that generated this choice function.*

PROOF: Suppose preference is menu-independent with an R^X which "induces" (in Bourbaki's sense) R^S for each S. It follows immediately that for $R_o = R^X$, (3.7.1) and (3.7.2) will be satisfied, so that the choice function is menu-independent.[56]

To check why the converse does not work, suppose with a menu-independent choice function, we get a binary relation R_o that would be menu-independent had it been a preference relation. But it is possible for a *menu-dependent* reflective preference relation R^S to generate exactly the *same* choice function as a menu-independent binary relation R_o. A simple example establishes this. Consider a definitely menu-dependent reflective preference relation R^S defined over $T = \{x, y, z\}$ and its

[56] Note, however, that the asymmetric strict factor xP^Sy need not entail the corresponding asymmetric xP_oy.

subsets, such that: $xI^{\{x,y\}}y$; $yP^{\{y,z\}}z$; $zP^{\{x,z\}}x$; yP^Tx; yP^Tz. Maximization according to this reflective preference relation will yield the following choices: $C(\{x,y\}) = \{x,y\}$; $C(\{y,z\}) = \{y\}$; $C(\{x,z\}) = \{z\}$; $C(T) = \{y\}$. This is a menu-independent choice function, and will correspond to the complete, acyclic and reflexive relation R_o given by: xI_oy; yP_oz; zP_ox. Indeed, this R_o is Samuelson's "revealed preference" relation for this choice function (even though Samuelson's "weak axiom of revealed preference" is violated). But menu-independent R_o is not congruent with menu-dependent R^S (even though the two generate the same choice function). So a menu-independent choice function $C(S, R^S)$ may be generated by a menu-dependent preference relation R^S.[57]

THEOREM 3.2: *A choice function is binary if and only if it is menu-independent.*

PROOF: Binariness of choice follows immediately from (3.7.2), with the revealed preference relation $R_c = R_o$. To check the converse, (3.7.2) is directly entailed for $R_c = R_o$. Now note that xR_c^Sy entails xR_cy, and that entails xR_oy for $R_c = R_o$, so that (3.7.1) also holds.

Turning now to Theorems 5.1–5.5, in establishing them we need only to refer to the analytical arguments presented in the text, which extend the formal demonstrations in Sen (1970a, 1971) and Suzumura (1976, 1983). However, Theorem 5.6 has not yet been addressed.

THEOREM 5.6: *Even with menu-dependent preferences, Theorems* 5.1–5.5 *hold for some set S separately, for some weak binary relation of preference R^S over S, and furthermore, all except Theorem* 5.5 *hold for every possible set S (except for unit sets), for some R^S.*

The extensions are, in fact, trivial for Theorems 5.1–5.3, since they are, in any case, concerned with any one set S at a time and one ranking R defined over S. The restriction of Theorem 5.6 for any given S is actually a *weaker* result than Theorem 5.4. It establishes that for every maximizing choice function with respect to a preference relation R, there is an optimizing choice function with respect to a devised binary relation R^+, which yields $B(S, R^+) = M(S, R)$ for all subsets S of X. Clearly, then, such an R^+ must exist for *any* given S.

So we are really left with extending the impossibility result (Theorem 5.5) for *some* given S. Since the proof of Theorem 5.5 was given for a case in which $B(S, R)$ and $M(S, R^+)$ were considered over a pair $S = \{x, y\}$, without invoking the choice over any subset of S, it will do for Theorem 5.6 also. To check this: with *not* xR^Sy and *not* yR^Sx, $B(S, R^S)$ must be empty, but $M(S, R^+)$ cannot be, since P^+ must be asymmetric.

THEOREM 6.1: *For any permissibility function K and any S, there exists an as if preference R_*^S such that*:

$$M(S, R_*^S) = M(K(S), R) = C(S). \tag{6.2}$$

PROOF: This is immediately established by the following construction:

(i) for all $x \in K(S)$ and all $y \in [S - K(S)]$: xP_*^Sy; and

(ii) for all $x, y \in K(S)$: $xR_*^Sy \Leftrightarrow xRy$.

The elements of $[S - K(S)]$ can be ranked in any arbitrary order *vis-à-vis* each other in R_*^S. It is readily seen that this construction will induce the result identified in (6.2), given (6.1).[58]

[57] There is another way in which R_o can differ from the reflective preference relation which generated the choice function. R_o can be the "completed extension" R^+ of an *incomplete* reflective preference relation R.

[58] It is worth noting that while R_*^S is entirely "constructed" for the purpose of getting (6.2), it has an *observational* counterpart in that it incorporates the "revealed preference" that would be observed *if* the observer ignores the chooser's self-constraint $K(S)$ and takes her to be choosing over the whole of S (as, in an important sense, she clearly is doing).

THEOREM 6.2: *For any reflexive R, there is some permissibility function K, such that there is no menu-independent R_* that induces*:

$$M(S, R_*) = M(K(S), R), \quad \text{for all } S. \tag{6.3}$$

PROOF: Consider a permissibility function K such that $K(\{x, y, z\}) = \{y\}$, and $K(\{y, z\}) = \{z\}$. If, contrary to the hypothesis, there is such an R_*, then clearly we need zP_*y to ensure $M(\{y, z\}, R_*) = M(K(\{y, z\}), R) = \{z\}$. But this contradicts $M(\{x, y, z\}, R_*) = M(K(\{x, y, z\}), R) = \{y\}$.[59]

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[59] Had we started with a menu-*dependent* R^S itself, rather than a menu-*independent* R, the amended Theorem 6.2 would, of course, hold *a fortiori*.

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