Rationality
Lecture 6

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The Dutch Book Argument

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Anyone whose beliefs violate the laws of probability is *practically irrational*.

F. P. Ramsey. *Truth and Probability*. 1931.

B. de Finetti. *La prévision: Ses lois logiques, ses sources subjectives*. 1937.

Alan Hájek. *Dutch Book Arguments*. Oxford Handbook of Rational and Social Choice, 2008.

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3. **The EU-Thesis** A practically rational agent will estimate that an act best satisfies her desires iff that act maximizes her subjective expected utility
4. **Dutch Book Theorem**. An agent who tries to maximize her subjective expected utility using beliefs that violate the laws for probability will freely preform an act that is sure to leave her worse off than some alternative act would in all circumstances.

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If she is indifferent between $63,81$ EUR and
[100 EUR if it rains, 0 EUR otherwise], then she believes to degree 0.6381 that it will rain.

Dutch Book

An agent will swap an (set of) wagers with the (sum of) their fair prices.

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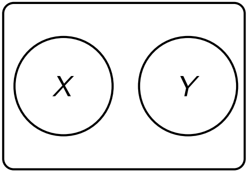

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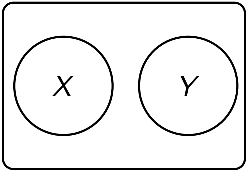

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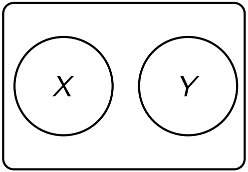

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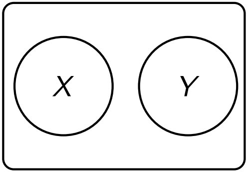

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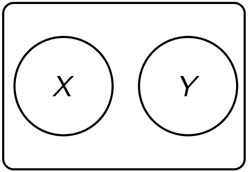

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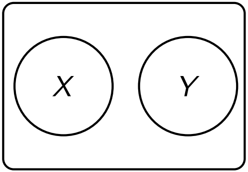

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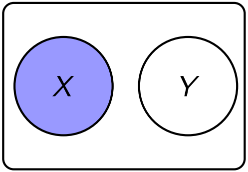

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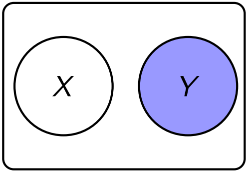

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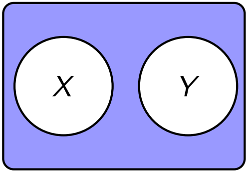

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  - If neither X nor Y is true
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allow agents to have incomplete or imprecise preferences

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justify probabilistic coherence and EU simultaneously

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Converse Dutch Book Theorem. If your degrees of beliefs (fair betting prices) satisfy the laws of probability, then there does not exist a Dutch Book consisting of bets at those prices.

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Expected Utility:

$$Exp_{P,u}(\alpha) = \sum_{w \in W} P(w) \times u(\alpha, w)$$

Small Worlds

States: {the sixth egg is good, the sixth egg is rotten}
Consequences { 6-egg omelet, no omelet and five good eggs destroyed, 6-egg omelet and a saucer to wash....}
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	Good Egg	Rotten Egg
Break into bowl	6-egg omelet	No Omelet and five good eggs destroyed
Break into saucer	6-egg omelet and a saucer to wash	5-egg omelet and a saucer to wash
Throw away	5-egg omelet and one good egg destroyed	5-egg omelet

Representation

EU-coherence: There must be at least on probability P defined on states and one utility function for consequences that **represent** the agent's preferences in the sense that, for any acts α and β, she strictly (weakly) prefers α to β only if $Exp_{P,u}(\alpha)$ is greater (as great as) $Exp_{P,u}(\beta)$.

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5. **Savage's P4** If the agent prefers [O_1 if X, O_2 else] to [O_1 if Y, O_2 else] when O_1 is more desirable than O_2, then she will also prefer [O_1^* if X, O_2^* else] to [O_1^* if Y, O_2^* else] for any other outcomes such that O_1^* is more desirable than O_2^*.

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Representation Theorem

If an agent satisfies all of the above postulates (including some technical ones not discussed), then the agent acts *as if* she is maximizing an expected utility.

These axioms (along with a few others) guarantee the existence of a unique probability P and utility u, unique up to the arbitrary choice of a unit and zero-point, whose associated expectation represents the agent's preferences.

Defining Beliefs from Preferences

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If O_1 is preferred to O_2 then the agent *has a good reason* for preferring [O_1 if X, O_2 else] to [O_1 if Y, O_2 else] exactly if she is more confident in X than in Y.

Are the Axioms Requirements of Practical Rationality?

I. Gilboa. *Questions in Decision Theory*. Annual Reviews in Economics, 2010.

Epistemic vs. Practical Rationality

"...The fact (if it is a fact) that one will be bound to lose money unless one's degrees of belief [obey the laws of probability] just isn't relevant to the truth of those beliefs" (Kennedy and Chihara, 1979)

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There is nothing more to the rationality of beliefs than their propensity to produce practically rational actions.

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  P2=$\{[O_1 \text{ if } Y, \text{ else } O_2], [O_1 \text{ if } Z, \text{ else } O_2]\}$
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  W1=$[O_1 \text{ if } Y \vee Z, \text{ else } O_2]$
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  Suppose $P1 \succeq P2$ but $W_1 \succeq W_2$.

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  P2=$\{[O_1 \text{ if } Y, \text{ else } O_2], [O_1 \text{ if } Z, \text{ else } O_2]\}$
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  W2=$[O_1 \text{ if } X \vee Z, \text{ else } O_2]$
  Suppose $P1 \succeq P2$ but $W_1 \succeq W_2$. This is inconsistent as they are simply rephrasing of the same event!

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- Joyce: relate probability consistency to *accuracy* of graded beliefs.

Next Week: Decision-Theoretic Paradoxes, Changing Beliefs