# Rationality

## Lecture 5

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*It is irrational to hold inconsistent beliefs at time $t$.*

**Diachronic**: Rationality also involves the capacity that takes an agent from one mental state to another (either explicitly or implicitly through reasoning)

*If $S$ believes $p$ and believes $q$ at time $t$ then $S$ should (may/will) believe $p \wedge q$ at time $t' > t$.*

# Preface Paradox

D. Makinson. *The Paradox of the Preface*. Analysis, 25, 205 - 207, 1965.

I. Douven and J. Uffink. *The Preface Paradox Revisited*. Erkenntnis, 59, 389 - 420, 2003.

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$B_A(\neg(s_1 \wedge s_2 \wedge \cdots \wedge s_n))$

But $\{s_1, \ldots, s_n, \neg(s_1 \wedge \cdots \wedge s_n)\}$ is logically inconsistent.

# Preface Paradox

A philosopher who asserts "all of my present philosophical positions are correct" would be regarded as rash and over-confident

A philosopher who asserts "at least some of my present philosophical beliefs will turn out to be incorrect" is simply being sensible and honest.

# Preface Paradox

1. each belief from the set $\{s_1, \ldots, s_n, s_{n+1}\}$ is rational
2. the set $\{s_1, \ldots, s_n, s_{n+1}\}$ of beliefs is rational.

1. does not necessarily imply 2.

# Preface Paradox: The Problem

"The author of the book is being rational even though inconsistent. More than this: he is being rational even though he believes each of a certain collection of statements, which *he knows* are logically incompatible....this appears to present a living and everyday example of a situation which philosophers have commonly dismissed as absurd; that it is sometimes rational to hold incompatible beliefs."

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Eric Schwitzgebel. *Belief*. In The Stanford Encyclopedia of Philosophy.

Franz Huber. *Formal Theories of Belief*. In The Stanford Encyclopedia of Philosophy.

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D. Christensen. *Putting Logic in its Place*. Oxford University Press.

# Lottery Paradox

H. Kyburg. *Probability and the Logic of Rational Belief*. Wesleyan University Press, 1961.

I. Douven and T. Williamson. *Generalizing the Lottery Paradox*. British Journal of the Philosophy of Science, 57, 755 - 779, 2006.

G. Wheeler. *A Review of the Lottery Paradox*. Probability and Inference: Essays in honor of Henry E. Kyburg, Jr., College Publications, 2007.

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For each lottery ticket $t_i$ ($i = 1, \ldots, 1000000$), the agent believes that $t_i$ will loose $B_A(\neg$'$t_i$ will win')

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But, this is a fair lottery, so at least one ticket is *guaranteed* to win!

# The Lottery Paradox

Kyburg: The following are inconsistent,

1. It is rational to accept a proposition that is very likely true,
2. It is not rational to accept a propositional that you are aware is inconsistent
3. It is rational to accept a proposition $P$ and it is rational to accept another proposition $P'$ then it is rational to accept $P \wedge P'$

# Are Beliefs Probabilities?

J. Joyce. *Bayesianism*. in [HR].

# Level of Confidence

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1. How do we make sense of decision making?
2. Evidence comes in a wide variety of types and strengths, and beliefs should be proportional to this evidence.

# Graded Conditional Beliefs

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Define confidence in terms of conditional confidence: $C(\cdot) = C_W(\cdot)$

$C(X) = 1$ indicate **complete certainty in** $X$ and $C(X) = 0$ indicates certainty that the proposition is false.

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For example,

- She is more confident in $X$ than in $Y$
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*Facts about a person's opinions are given by properties that all elements of Con share.*

# Thesis of Graded Belief

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1. Any adequate epistemology must recognize that opinions come in varying gradations of strength.

2. A person's graded beliefs can be represented using a set *Con* of confidence measures.

3. Facts about her beliefs correspond to properties shared by all elements of *Con*.

# Reminder: Probability

A probability measure assigns to propositions an element of $[0, 1]$ such that

**Normalization** $P(W) = 1$

**Additivity** $P(X \vee Y) = P(X) + P(Y)$ (also the countable version)

Conditional probability measure assigns to pairs of propositions an element of $[0, 1]$ such that

**Probability** $P(\cdot \mid Y)$ is a probability measure for all $Y$

**Conditional Normalization** $P(Y \mid Y) = 1$

**Conditioning** $P(X \mid Y \wedge Z) \cdot P(Y \mid Z) = P(X \wedge Y \mid Z)$

# Reminder: Probability

**Logical Consequence**: If $X$ entails $Y$, then $P(X) \subseteq P(Y)$

**Bayes' Theorem**: $P(X \mid Y) = P(Y \mid X)\frac{P(X)}{P(Y)}$

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I.e., There is a $C \in Con$ such that $C_Y(X) = P(X \mid Y)$ for some (conditional) probability measure $P$.

What is the rationale for this?

# The Dutch Book Argument

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Anyone whose beliefs violate the laws of probability is *practically irrational*.

F. P. Ramsey. *Truth and Probability*. 1931.

B. de Finetti. *La prévision: Ses lois logiques, ses sources subjectives*. 1937.

Alan Hájek. *Dutch Book Arguments*. Oxford Handbook of Rational and Social Choice, 2008.

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3. **The EU-Thesis** A practically rational agent will estimate that an act best satisfies her desires iff that act maximizes her subjective expected utility
4. **Dutch Book Theorem**. An agent who tries to maximize her subjective expected utility using beliefs that violate the laws of probability will freely preform an act that is sure to leave her worse off than some alternative act would in all circumstances.

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Emotions, habits, etc. do *cause actions*, BUT the DBA does not rely on the belief-desire psychology as a causal theory of action.

Assumption 2 makes no claim abut what causes actions....it says that what *makes* an act rational is that it bears the right relationship to the actor's beliefs and desires.

# The EU-Thesis

**Expected Utility**: Given an agent's beliefs and desires, the **expected utility** of an **action** leading to a set of outcomes $Out$ is:

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Simplifying assumptions:

1. the agent desires *only* money
2. her desire for money does not vary with changes in her fortune
3. she is not averse to risk or uncertainty

# Betting Behavior

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A **wager**: $W_X = [\ a \text{ if } X,\ b \text{ otherwise}]$: "you get $a$ EUR if $X$ is true and $b$ EUR otherwise.
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The EU-thesis entails that the agent's level of confidence in $X$ will be revealed by the monetary value she puts on $W_X$.

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If she is indifferent between $63,81$ EUR and
[100 EUR if it rains, 0 EUR otherwise], then she believes to degree 0.6381 that it will rain.

# Dutch Book

An agent will swap an (set of) wagers with the (sum of) their fair prices.

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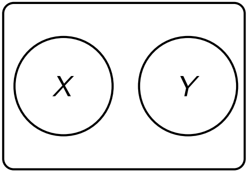

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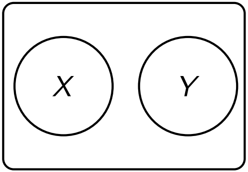

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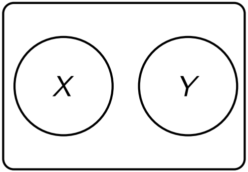

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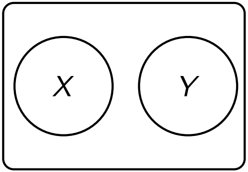

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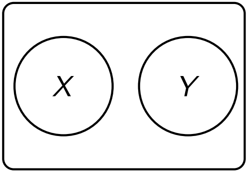

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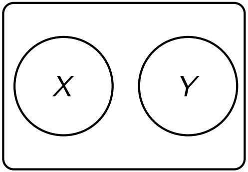
Suppose that $X$ and $Y$ are logically incompatible ($X \cap Y = \emptyset$)

Fair price: $f = 0.25$ for $W_X = [1 \text{ if } X, 0 \text{ else}]$
Fair price: $f = 0.25$ for $W_Y = [1 \text{ if } Y, 0 \text{ else}]$
Fair price: $f = 0.6$ for $W_{X \vee Y} = [1 \text{ if } X \vee Y, 0 \text{ else}]$

Consider $\mathcal{W}_1 = \{0.6, W_X, W_Y\}$ and $\mathcal{W}_2 = \{0.5, W_{X \vee Y}\}$

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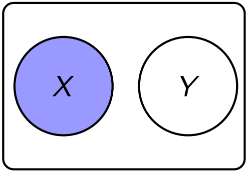


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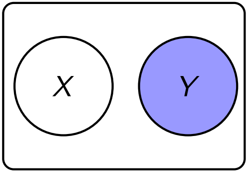


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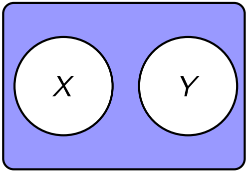


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    payoff($\mathcal{W}_1$) $= 1.6 >$ payoff($\mathcal{W}_2$)$= 1.5$
  - If neither $X$ nor $Y$ is true
    payoff($\mathcal{W}_1$) $= 0.6 >$ payoff($\mathcal{W}_2$)$=0.5$

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*allow agents to have incomplete or imprecise preferences*

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*justify probabilistic coherence and EU simultaneously*

Next Week: Savage's Representation Theorem