Rationality

Lecture 15

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December 10, 2010
Shared cooperative activity
What is a team?

Any group?
What is a team?

Any group?

- Surely not. But interesting phenomena at this level already.
What is a team?

Any group?

- Surely not.

Then a group with:

i A certain *(hierarchical)* structure?

ii Whose members *identify with the group* (c.f. Gold 2005)?
   - Information about who’s in and who’s out.
   - Reasoning and acting as group members.

iii Team- or group objectives/aims/preferences?


v Common knowledge (beliefs?) of (i-iv)?

Note: None of these are necessary conditions!
What is a team?

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► Surely not.

Then a group with:

i A certain (hierarchical) structure?

ii Whose members identify with the group (c.f. Gold 2005)?

iii Team- or group objectives/aims/preferences?
  • Shared by the members?


v Common knowledge (beliefs?) of (i-iv)?

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Then a group with:

i A certain (hierarchical) structure?

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• Shared intentions.
• Sanctions for lapsing?
• Shared praise[blame] for success[failure]?
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Note: *None of these are necessary conditions!*
What is a team?

Acting as a team (at least) involves:

- Adopting the team’s preferences. (Preference transformation).
- Team-reasoning (Agency Transformation).
What is a team?

   - Information about who’s in and who’s out.
   - Reasoning as group members.
   - Shared goal.
     - Group preference / utilities.

2. Shared commitments.
   - Shared intentions.
   - Sanctions for lapsing?
   - Shared praise[blame] for success[failure]?

3. Common knowledge (beliefs?) of the above?
What is a team?

1. **Group identification.**
   - Information about who’s in and who’s out.
   - Reasoning as group members.
   - Shared goal.
     - Group preference / utilities.

2. **Shared commitments.**
   - Shared intentions.
   - Sanctions for lapsing?
   - Shared praise[blame] for success[failure]?

3. **Common knowledge (beliefs?)** of the above?
Intentions: Recap

Motivational attitudes which:

- Are relatively stable.
- Are conduct-controlling, i.e. commit to action.
- Constraint further practical reasoning.

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Intentions anchor inter-temporal and interpersonal coordination.

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Commitments and Intentions

Key Philosophical Work:


Commitments and Intentions

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Intentions and Teamwork:


Shared Intentions

A The Intention part:

1. Me:
   1.1 I intend that we J.
   1.2 I intend that we J in accordance with and because of meshing subplans of (1.1) and (2.1).

2. You:
   2.1 You intend that we J.
   2.2 You intend that we J in accordance with and because of meshing subplans of (1.1) and (2.1).

3. Additional requirements:
   3.1 The intentions in (1) and in (2) are not coerced by the other participant.
   3.2 The intentions in (1) and (2) are minimally cooperatively stable.

B The epistemic part:

1. It is common knowledge between us that (A).

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   - Reasoning as group members.
   - Shared goal.
     - Group Decision Making

2. Shared commitments.
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   - Sanctions for lapsing?
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Main Question

Given a group of people faced with some decision, how should a central authority combine the individual opinions so as to best reflect the “will of the group”?
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Typical Examples:
- Electing government officials
- Department meetings
- Deciding where to go to dinner with friends
- ....
What properties do we want?

- **Pareto Optimality**: If outcome $a$ is unanimously preferred to outcome $b$, then $b$ should not be the social choice.

- **Anonymity**: The names of the voters do not matter (if two voters change votes, then the outcome is unaffected).

- **Neutrality**: The names of the candidates, or options, do not matter (if two candidate are exchanged in every ranking, then the outcome changes accordingly).

- **Monotonicity**: Moving up in the rankings is always better.
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Majority Rules

If there are only two options, then majority voting is the “best” procedure.
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Generalizing May’s Theorem

In May’s Theorem, the agents are making a single binary choice between two alternatives. What about more general situations?
Generalizing May’s Theorem

In May’s Theorem, the agents are making a single binary choice between two alternatives. What about more general situations?

- Agents choose between between more than two alternatives.
- There are multiple interconnected propositions on which simultaneous decisions are to be made.
Group Rationality Constraints

- Defining a group’s preferences and beliefs:
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  - Even if all the agents in a group have rational preferences, the group’s preference may not be rational.
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  - Arrow’s Theorem
  - Sen’s Liberal Paradox
  - Puzzles of Fair Division
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Different normative constraints on group decision making are in conflict.

- Arrow’s Theorem
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- Puzzles of Fair Division

Many proposed group decision methods (voting methods) with very little agreement about how to compare them.
Condorcet Paradox

Even if all the agents in a group have rational preferences, the groups preference may not be rational.
Condorcet Paradox

Suppose that there are three agents have preferences over their options \( \{a, b, c\} \).
Condorcet Paradox

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Ann
\[
\begin{align*}
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  b \\
  c
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Condorcet Paradox

Suppose that there are three agents have preferences over their options \{a, b, c\}.

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What about the group’s preference?
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What about the group’s preference?

- Does the group prefer \( a \) over \( b \) (\( a \succ b \))?
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What about the group’s preference?

- Does the group prefer \( a \) over \( b \) \( (a \succ b) \)? Yes
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Suppose that there are three agents have preferences over their options \{a, b, c\}.

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What about the group’s preference?

- Does the group prefer a over b (a ⪰ b)? Yes
- Does the group prefer b over c (b ⪰ c)?
### Condorcet Paradox

Suppose that there are three agents have preferences over their options $\{a, b, c\}$.

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- Does the group prefer \( b \) over \( c \) (\( b \succ c \))? Yes
- Does the group prefer \( a \) over \( c \) (\( a \succ c \))? No
The Logic of Group Decisions

Even if all the agents in a group have rational beliefs, the groups beliefs may not be rational.
The Logic of Group Decisions

**Fundamental Problem:** groups are inconsistent!
The Logic of Group Decisions: The Doctrinal “Paradox”  
(Kornhauser and Sager 1993)

\( p \): a valid contract was in place  
\( q \): there was a breach of contract  
\( r \): the court is required to find the defendant liable.

\[ (p \land q) \iff r \]

|  |  |  |  |  |
|---|---|---|---|
| 1 | yes | yes | yes | yes |
| 2 | yes | no  | yes | no  |
| 3 | no  | yes | yes | no  |
The Logic of Group Decisions: The Doctrinal “Paradox” (Kornhauser and Sager 1993)

Should we accept $r$?

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<tr>
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<th>$p$</th>
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<td>1</td>
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Should we accept $r$? No, a simple majority votes no.

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Should we accept \( r \)? Yes, a majority votes yes for \( p \) and \( q \) and   
\( (p \land q) \leftrightarrow r \) is a legal doctrine.

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Discursive Dilemma

\( a: \) “Carbon dioxide emissions are above the threshold \( x \)”
Discursive Dilemma

\(a: \text{“Carbon dioxide emissions are above the threshold } x\text{”}\)

\(a \rightarrow b: \text{“If carbon dioxide emissions are above the threshold } x,\text{ then there will be global warming”}\)
Discursive Dilemma

\( a \): “Carbon dioxide emissions are above the threshold \( x \)”

\( a \rightarrow b \): “If carbon dioxide emissions are above the threshold \( x \), then there will be global warming”

\( b \): “There will be global warming”
**Discursive Dilemma**

\[ a: \text{“Carbon dioxide emissions are above the threshold \( x \)”} \]

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**Conclusion**: Groups are inconsistent, difference between 'premise-based' and 'conclusion-based' decision making, ...
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Conclusion: Groups are inconsistent, difference between ‘premise-based’ and ‘conclusion-based’ decision making, ...
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Majority
Discursive Dilemma

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\[
\begin{array}{|c|c|c|}
\hline
 & a & a \rightarrow b \\
\hline
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\hline
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\hline
3 & False & True \\
\hline
\text{Majority} & True & True \\
\hline
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Conclusion: Groups are inconsistent, difference between ‘premise-based’ and ‘conclusion-based’ decision making, ...
Many Variants!

See
http://personal.lse.ac.uk/LIST/doctrinalparadox.htm
for many generalizations!
Group Rationality Constraints

▶ Defining a group’s preferences and beliefs:
  • Even if all the agents in a group have rational preferences, the group’s preference may not be rational.
  • Even if all the agents in a group have rational beliefs, the group’s beliefs may not be rational.

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  • Arrow’s Theorem
  • Sen’s Liberal Paradox
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Arrow’s Theorem


Also, see


Arrow’s Theorem

Let $X$ be a finite set of preferences with \textit{at least three elements}. Assume each agent has a transitive and complete preference over $X$ (ties are allowed).
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Let $X$ be a finite set of preferences with \textit{at least three elements}. Assume each agent has a transitive and complete preference over $X$ (ties are allowed).

\begin{itemize}
  \item Let $P_i \subseteq X \times X$ be a “rational” preference ordering for each individual voter ($xP_iy$ means that agent $i$ weakly prefers $x$ over $y$. Each $P_i$ is assumed to be (for example) reflexive, transitive and connected.)
  \item An \textbf{social welfare function} maps an ordering for each agent to a “social ordering” ($F$ is a function from the voters’ preferences to a preference, so $F(P_1, \ldots, P_n)$ is an ordering over $X$.)
  \item Notation: write $\vec{P}$ for the tuple $(P_1, P_2, \ldots, P_n)$.
\end{itemize}
Unanimity

If each agent ranks $x$ above $y$, then so does the social welfare function
Unanimity

If each agent ranks $x$ above $y$, then so does the social welfare function.

If for each $i \in \mathcal{A}$, $x P_i y$ then $x F(\vec{P}) y$
Universal Domain

Voter’s are free to choose any preference they want.
Universal Domain

Voter’s are free to choose any preference they want.

$F$ is a total function.
Independence of Irrelevant Alternatives

The social relative ranking (higher, lower, or indifferent) of two alternatives \(x\) and \(y\) depends only on the relative rankings of \(x\) and \(y\) for each individual.
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If for each $i \in \mathcal{A}$, $xP_i y$ iff $xP_i' y$, then $xF(\vec{P}) y$ iff $xF(\vec{P}') y$. 
Dictatorship

There is an individual $d \in A$ such that the society strictly prefers $x$ over $y$ whenever $d$ strictly prefers $x$ over $y$.
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There is an individual $d \in \mathcal{A}$ such that the society strictly prefers $x$ over $y$ whenever $d$ strictly prefers $x$ over $y$.

There is a $d \in \mathcal{A}$ such that $xF(\vec{P})y$ whenever $xP_dy$. 
Arrow’s Theorem

**Theorem** (Arrow, 1951) Any social welfare function that satisfies universal domain, independence of irrelevant alternatives and unanimity is a dictatorship.
Sen’s Liberal Paradox


Two members of a small society Lewd and Prude each have a personal copy of *Lady Chatterley’s Lover*, consider
Sen’s Liberal Paradox

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\( l \): Lewd reads the book;
\( p \): Prude reads the book;
\( l \rightarrow p \): If Lewd reads the book, then so does Prude.
Sen’s Liberal Paradox

Lewd desires to read the book, and if he reads it, then so does Prude (Lewd enjoys the thought of Prude’s moral outlook being corrupted).

Prude desires to not read the book, and that Lewd not read it either, but in case Lewd does read the book, Prude wants to read the book to be informed about the dangerous material Lewd has read.
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1. Society assigns to each individual the liberal right to determine the collective desire on those propositions that concern only the individual’s private sphere (e.g., Lewd’s case, Prude’s case).

2. Unanimous desires of all individuals must be respected.

So, society must be inconsistent!
Sen’s Liberal Paradox

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Fairness Conditions

- **Proportional:** (for two players) each player receives at least 50% of their valuation.
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- **Efficiency:** there is no other division better for everybody, or better for some players and not worse for the others.
Fair Division of Indivisible Goods

Players cannot compensate each other with side payments

All players have positive values for every item

A player prefers a set $S$ to different set $T$ if

• $S$ has as many elements as $T$
• for every item in $t \in T - S$ there is a distinct item $s \in S - T$ that the player prefers to $t$. 

Eric Pacuit: Rationality (Lecture 15)
Fair Division of Indivisible Goods

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Envy-Freeness and Efficiency

A unique envy-free division may be inefficient

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\begin{align*}
A &: 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \\
B &: 4 \quad 3 \quad 2 \quad 1 \quad 5 \quad 6 \\
C &: 5 \quad 1 \quad 2 \quad 6 \quad 3 \quad 4 \\
A &: \{1, 3\} \\
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This is the unique envy-free outcome.
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The division \((12, 34, 56)\) pareto-dominates the above division
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\[ A : 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \]
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However, \((12, 34, 56)\) is not (necessarily) envy-free
Envy-Freeness and Efficiency

*A unique envy-free division may be inefficient*

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There is no other division, including an efficient one, that guarantees envy-freeness.
Envy-Freeness and Efficiency

There may be no envy-free division, even when all players have different preference rankings
Envy-Freeness and Efficiency

*There may be no envy-free division, even when all players have different preference rankings*

Trivial if all players have the same preference.
Envy-Freeness and Efficiency

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\[
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A : & \quad 1 \quad 2 \quad 3 \\
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Three divisions are efficient: (1, 3, 2), (2, 1, 3) and (3, 1, 2). However, none of them are envy-free.
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In fact, there is no envy-free division.
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Next: Voting Theory and Conclusions