Rationality: Short Essay Questions

Tilburg University, 2010-2011

Please put your answers in my pigeon hole before **November 4th, 2010**. *Make sure you properly cite any sources you use when answering these questions.*

1. We started off this course with the question: What does it mean to be *rational* or *reasonable* as opposed to *irrational* or *unreasonable*? We have seen that there are many aspects to this question and have studied a number of different "rationality constraints" (looking at both practical and theoretical rationality). Choose one of the issues or paradoxes we have discussed and explain why it is important for an answer to this question. (This is a rather open-ended question. What I am looking for here is an essay of no more than 2-3 pages focusing on one particular issue or paradox that we have discussed which you feel is most important to understand what it means to be *rational*. Note, I do not want a summary of all of the material we have discussed, but rather a concise discussion of one particular issue or paradox. You should explain what is important about the particular issue or paradox for a theory of human rationality, explain in your own words the analysis offered in the relevant literature or in the lecture and what is good or bad about this analysis)

2. Joyce sketched a proof of the Dutch Book Theorem his article *Bayesianism* in the Handbook of Rationality (we discussed this proof during the 5th and 6th lectures). In particular, he explained that if your fair prices do not follow the laws of probability, then one can construct a set of swaps that will guarantee that the you loose money. For this question, you will extend this proof to other properties of probabilities.

   One law of probability which we have used is that if $X \subseteq Y$ then $P(X) \leq P(Y)$. As I discussed on the first lecture, this can be derived from the other axioms of probability. But, we can also argue directly that a rational agent's graded beliefs must conform to this property. Suppose that $X \subseteq Y$ and the agent has the following fair prices:

   - $f = 0.3$ for the wager $W_X = [\,1 \text{ if } X, 0 \text{ else}\,]$; and
   - $f = 0.2$ for the wager $W_Y = [\,1 \text{ if } Y, 0 \text{ else}\,]$

   Explain how to make "Dutch Book" against an against with these fair prices (*hint*: consider the sets $\mathcal{W}_1 = \{0.1, W_Y\}$ and $\mathcal{W}_2 = \{W_X\}$. Will the agent swap $\mathcal{W}_1$ for $\mathcal{W}_2$? If so, is this a good thing to do?)

   Provide a similar argument to show that the fair price assigned to the set of all states (i.e., to a tautology: an event that always is true) should not be less than 1.

3. Similar to the Allais Paradox, Daniel Ellsberg came up with a choice situation where the "rational" choice seems to contradict standard expected utility theory.

   D. Ellsberg (1961). Risk, Ambiguity, and the Savage Axioms *Quarterly Journal of Economics*, **75 (4)**, pgs. 643 - 669.

   Here is one situation analyzed in this paper. There is one urn with 300 balls: 100 of the balls are red (R) and the rest are either blue (B) or yellow (Y). Consider the following two choice situations:

   Situation 1

   l_1 Win \$100 if a ball drawn from the urn is R and nothing otherwise

   l_2 Win \$100 if a ball drawn from the urn is B and nothing otherwise

   Situation 2

   l_3 Win \$100 if a ball drawn from the urn is R or Y and nothing otherwise

   l_4 Win \$100 if a ball drawn from the urn is B or Y and nothing otherwise

   Most people have the following preference: $l_1 \succ l_2$ and $l_4 \succ l_3$.

   Answer the following questions. Please keep your answers concise and be sure to cite any additional sources you use.

   (a) Explain the rationale behind these preferences. (I.e., why might these preferences be "rational"?)

   (b) Why do these preferences contradict standard expected utility calculations?

   (c) Do the Ellsberg Paradox and the Allais Paradox highlight the same phenomena? Explain your answer.