Logic and Artificial Intelligence

Lecture 26

Eric Pacuit

Currently Visiting the Center for Formal Epistemology, CMU

Center for Logic and Philosophy of Science
Tilburg University

ai.stanford.edu/∼epacuit
e.j.pacuit@uvt.nl

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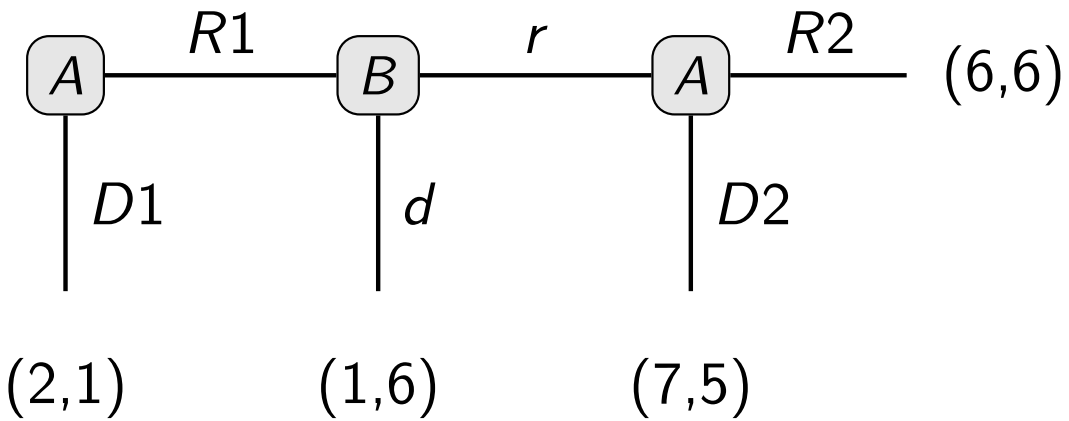
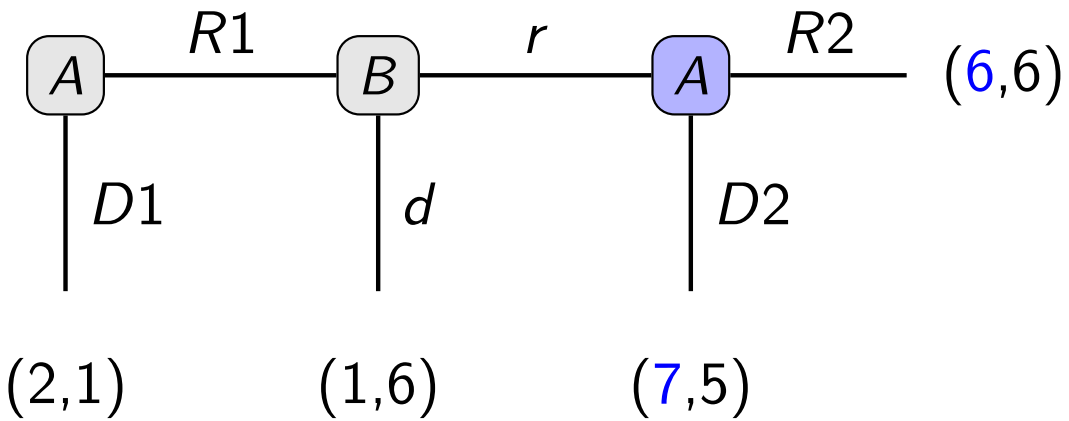
BI Puzzle



BI Puzzle



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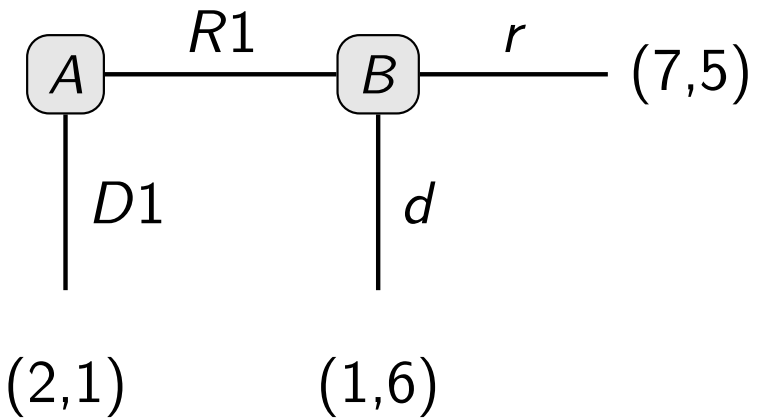

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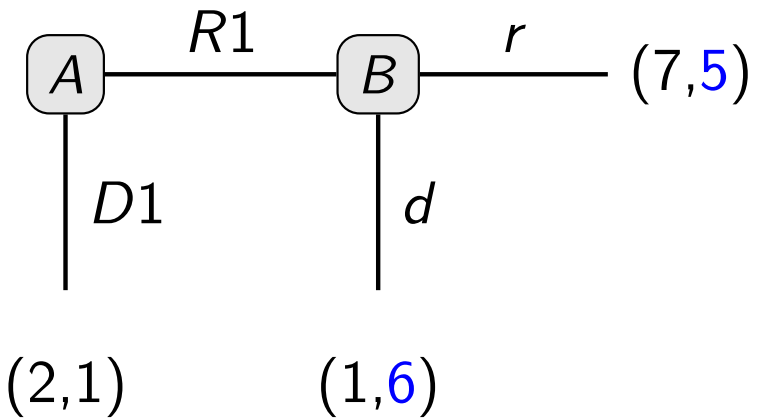

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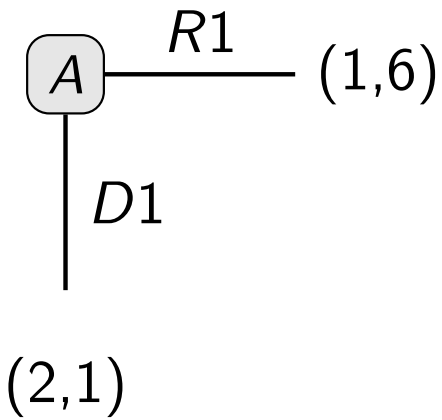

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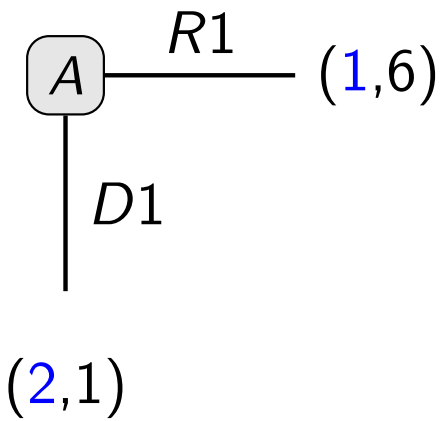

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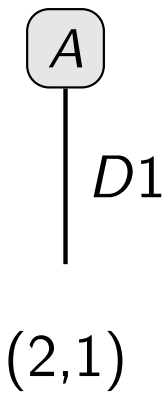
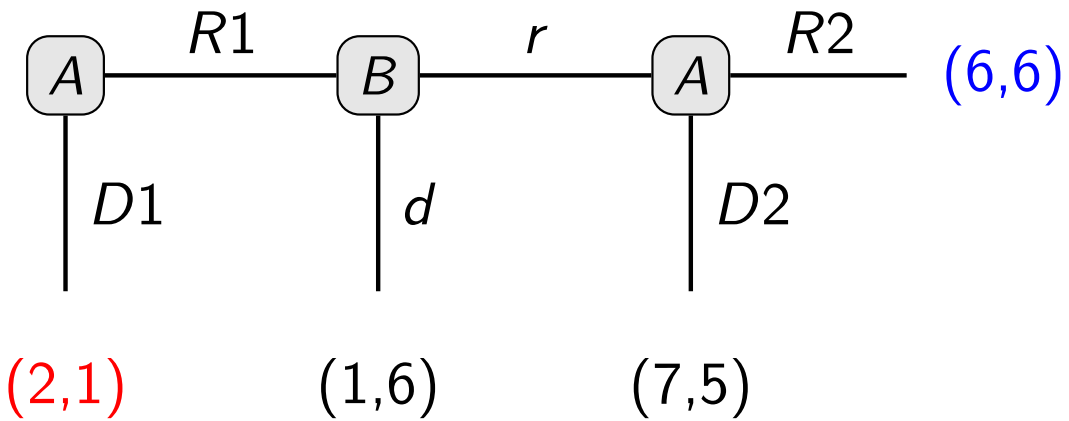


But what if...



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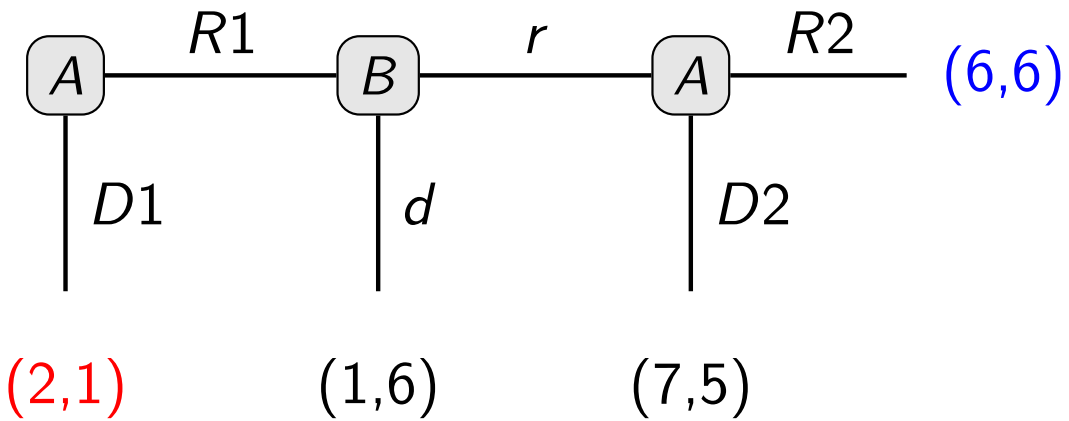

"On the one hand, Under common knowledge of rationality, A *must* go out on the first move. On the other hand, the backward induction argument for this is based on what the players *would* do if A stayed in. But, if she did stay in, then common knowledge of rationality is violated, so the argument that she will go out no longer has a basis."

R. Aumann. *Backwards induction and common knowledge of rationality*. Games and Economic Behavior, 8, pgs. 6 - 19, 1995.

R. Stalnaker. *Knowledge, belief and counterfactual reasoning in games*. Economics and Philosophy, 12, pgs. 133 - 163, 1996.

J. Halpern. *Substantive Rationality and Backward Induction*. Games and Economic Behavior, 37, pp. 425-435, 1998.

Models of Extensive Games

$\mathcal{M}(\Gamma) = \langle W, \sim_i, f, \mathbf{s} \rangle$ where

(A1) If $w \sim_i w'$ then $\mathbf{s}_i(w) = \mathbf{s}_i(w')$.

(F1) v is reached in $f(w, v)$ (i.e., v is on the path determined by $\mathbf{s}(f(w, v))$)

(F2) If v is reached in w, then $f(w, v) = w$

(F3) $\mathbf{s}(f(w, v))$ and $\mathbf{s}(w)$ agree on the subtree of Γ below v

(F4) For all players i and vertices v, if $w' \in [f(w, v)]_i$ then there exists a state $w'' \in [w]_i$ such that $\mathbf{s}(w')$ and $\mathbf{s}(w'')$ agree on the subtree of Γ below v.

Rationality

i **is rational at** v **in** w: for all strategies $s_i \neq \mathbf{s}_i(w)$, $h_i^v(\mathbf{s}(w')) \geq h_i^v((\mathbf{s}_{-i}(w'), s_i))$ for some $w' \in [w]_i$:

$$\bigwedge_{v \in \Gamma_i} \bigwedge_{t_i \in Strat_i(\Gamma)} \neg K_i[h_i^v(s; t_i) > h_i^v(s)]$$

A-Rat: i is rational at vertex v in w for every vertex $v \in \Gamma_i$

S-Rat: i is rational at vertex v is $f(w, v)$ fo every vertex $v \in \Gamma_i$

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Theorem (Halpern). If Γ is a non-degenerate game of perfect information, then for every extended model of Γ in which the selection function satisfies F1-F4, we have $C(\text{S-Rat}) \subseteq BI$.

J. Halpern. *Substantive Rationality and Backward Induction*. Games and Economic Behavior, 37, pp. 425-435, 1998.

Revising beliefs during play:

"Although it is common knowledge that Ann would play across if v_3 were reached, if Ann were to play across at v_1, Bob would consider it possible that Ann would play down at v_3"

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"the rationality of choices in a game depends not only on what players believe, but also on their policies for revising their beliefs" (p. 31)

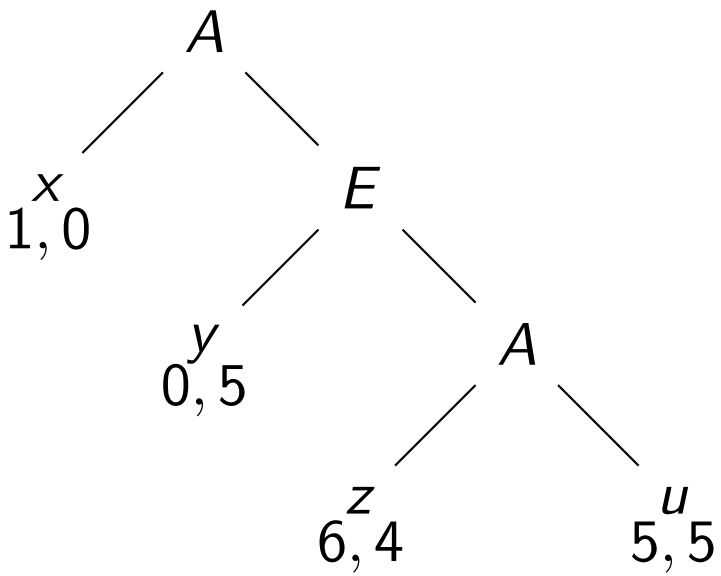
R. Stalnaker. *Belief revision in games: Forward and backward induction*. Mathematical Social Sciences, 36, pgs. 31 - 56, 1998.

"Off-line learning of rationality"

Where do the models satisfying common knowledge/belief of rationality come from?

J. van Benthem. *Rational dynamics and epistemic logic in games*. International Journal of Game Theory Review, 9(1), pgs. 13 - 45, 2007.

“Off-line learning of rationality”



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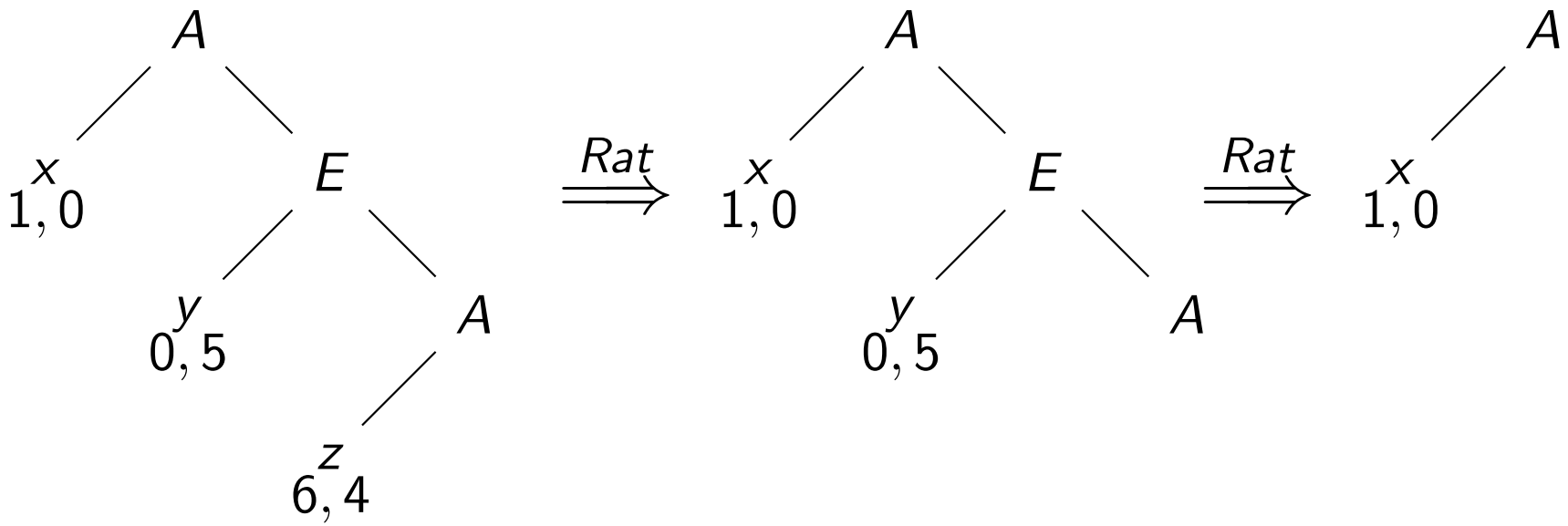

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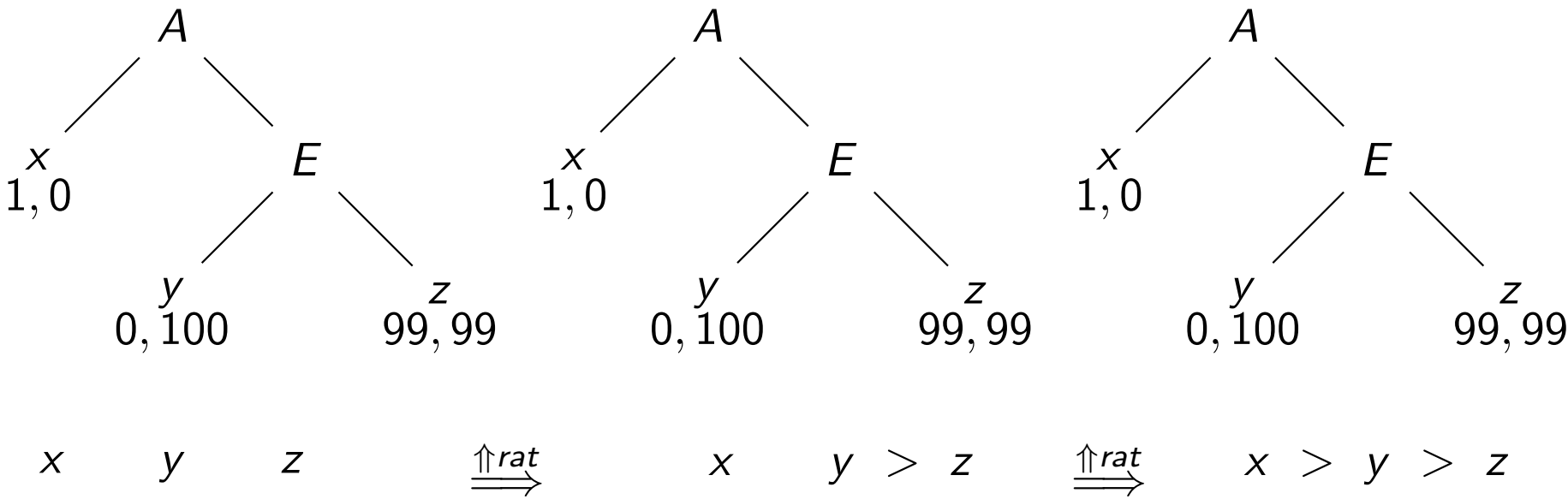

The Dynamics of Rational Play

A. Baltag, S. Smets and J. Zvesper. *Keep 'hoping' for rationality: a solution to the backward induction paradox*. Synthese, 169, pgs. 301 - 333, 2009.

Hard vs. Soft Information in a Game

The structure of the game and past moves are ‘hard information: *irrevocably known*

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Players' 'knowledge' of other players' rationality and 'knowledge' of her own future moves at nodes not yet reached are not of the same degree of certainty.

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What belief revision policy leads to BI?

Dynamic Rationality: The event R that all players are *rational* changes during the play of the game.

Players are assumed to be "incurably optimistic" about the rationality of their opponents.

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Dynamic Rationality: The event R that all players are *rational* changes during the play of the game.

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Theorem (Baltag, Smets and Zvesper). Common knowledge of the game structure, of open future and *common stable belief* in dynamic rationality implies common belief in the backward induction outcome.

$$Ck(Struct_G \wedge F_G \wedge [\,!\,]CbRat) \rightarrow Cb(BI_G)$$

Concluding remarks

We are interested in reasoning about rational (and not-so rational) agents engaged in some form of social interaction.

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- Philosophy (social epistemology, philosophy of action)
- Game Theory
- Social Choice Theory
- AI (multiagent systems)

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What is a "rational agent"? What *are we modeling?*

- has consistent preferences (complete, transitive)
- (acts as if she) maximizes expected utility
- reacts to observations
- revises beliefs when learning a *surprising* piece of information
- understands higher-order information
- plans for the future
- asks questions
- ????

We are interested in reasoning about rational (and not-so rational) agents engaged in some form of social interaction.

- playing a (card) game
- having a conversation
- executing a *social procedure* (voting, making a group decision)
- ....

Goal: incorporate/extend existing game-theoretic/social choice analyses

We are interested in reasoning about rational (and not-so rational) agents engaged in some form of social interaction.

There is a jungle of logical frameworks!

- logics of informational attitudes (knowledge, beliefs, certainty)
- logics of action & agency
- temporal logics/dynamic logics
- logics of motivational attitudes (preferences, intentions)
- deontic logics

(*Not to mention various game-theoretic/social choice models and logical languages for reasoning about them*)

We are interested in reasoning about rational (and not-so rational) agents engaged in some form of social interaction.

- How can we compare different logical frameworks addressing similar aspects of rational agency and social interaction?
- How should we combine logical systems which address different aspects of social interaction towards the goal of a comprehensive (formal) theory of rational agency?
- How does a logical analysis contribute to the broader discussion of rational agency and social interaction within philosophy and the social sciences?

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Refine and test our intuitions: provide many answers to the question *what is a rational agent*? Explore how different answers *fit together*.

Conclusions

We are interested in reasoning about rational (and not-so rational) agents engaged in some form of *social* situations.

What do the logical frameworks contribute to the discussion on rational agency?

Merge with *Game Theory*/*Social Choice Theory*

- From a *Theory of Games* to a *Theory of Players*

J. van Benthem, EP and O. Roy. *A Theory of Play: A Logical Perspective on Games and Interaction*. Games, 2011.

- (Epistemic) foundations of game theory (rational-choice as a parameter)

Ingredients of a Logical Analysis of Rational Agency

$\Rightarrow$ informational attitudes (eg., knowledge, belief, certainty)

$\Rightarrow$ time, actions and ability

$\Rightarrow$ evaluative/motivational attitudes (eg., preferences)

$\Rightarrow$ pro-attitudes (eg., intentions)

$\Rightarrow$ group notions (eg., common knowledge and coalitional ability)

$\Rightarrow$ normative attitudes (eg., obligations, reasons)

Thank you!

Final Exam: Tuesday, December 13th, 5:30 PM - 8:30 PM in PH 125C