Logic and Artificial Intelligence

Lecture 16

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AGM Postulates

AGM 1: $K * \varphi$ is deductively closed

AGM 2: $\varphi \in K * \varphi$

AGM 3: $K * \varphi \subseteq Cn(K \cup \{\varphi\})$

AGM 4: If $\neg\varphi \notin K$ then $K * \varphi = Cn(K \cup \{\varphi\})$

AGM 5: $K * \varphi$ is inconsistent only if φ is inconsistent

AGM 6: If φ and ψ are logically equivalent then $K * \varphi = K * \psi$

AGM 7: $K * (\varphi \wedge \psi) \subseteq Cn(K * \varphi \cup \{\psi\})$

AGM 8 if $\neg\psi \notin K * \varphi$ then $Cn(K * \varphi \cup \{\psi\}) \subseteq K * (\varphi \wedge \psi)$

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Revising by $\neg p$ $(K * \neg p)$ vs. Updating by $\neg p$ $(K \diamond \neg p)$

H. Katsuno and A. O. Mendelzon. *Propositional knowledge base revision and minimal change*. Artificial Intelligence, 52, pp. 263 - 294 (1991).

KM Postulates

KM 1: $K \diamond \varphi = Cn(K \diamond \varphi)$

KM 2: $\varphi \in K \diamond \varphi$

KM 3: If $\varphi \in K$ then $K \diamond \varphi = K$

KM 4: $K \diamond \varphi$ is inconsistent iff φ is inconsistent

KM 5: If φ and ψ are logically equivalent then $K \diamond \varphi = K \diamond \psi$

KM 6: $K \diamond (\varphi \wedge \psi) \subseteq Cn(K \diamond \varphi \cup \{\psi\})$

KM 7: If $\psi \in K \diamond \varphi$ and $\varphi \in K \diamond \psi$ then $K \diamond \varphi = K \diamond \psi$

KM 8: If K is complete then $K \diamond (\varphi \wedge \psi) \subseteq K \diamond \varphi \cap K \diamond \psi$

KM 9: $K \diamond \varphi = \bigcap_{M \in Comp(K)} M \diamond \varphi$, where $Comp(K)$ is the class of all complete theories containing K.

Updating and Revising

$$K \diamond \varphi = \bigcap_{M \in Comp(K)} M * \varphi$$

H. Katsuno and A. O. Mendelzon. *On the difference between updating a knowledge base and revising it*. *Belief Revision*, P. Gärdenfors (ed.), pp 182 - 203 (1992).

Non-monotonic logic: What *should*/*do* I believe?

Classical consequence relation: $\varphi \vdash \psi$: ψ follows from φ using the rules of logic (there is a derivation of ψ using propositional logic and φ)

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Doxastic reading: after coming to believe/accept φ, the agent believes/accepts ψ.

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Failure on monotonicity: B: Tweety is a bird; F: Tweety flies; P: Tweety is a penguin

$B \vdash F$ but $B, P \nvdash F$.

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$\varphi \mid\!\sim \psi$ "If φ then *typically* (*mostly*, etc.) ψ"

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Typical of belief revision: $\psi \in K * \varphi$, but $\psi \notin K * (\varphi \wedge \alpha)$

$\varphi \mid\!\sim \psi$ "If φ then *typically* (*mostly*, etc.) ψ"

$\varphi \mid\!\sim \psi$ iff $\psi \in K * \varphi$.

Nonmonotonic Reasoning

Left logical equivalence: If $\vdash \varphi \leftrightarrow \psi$ and $\varphi \mid\!\sim \alpha$ then $\psi \mid\!\sim \alpha$

Right weakening: If $\vdash \alpha \rightarrow \beta$ and $\varphi \mid\!\sim \alpha$ then $\varphi \mid\!\sim \beta$

And: If $\varphi \mid\!\sim \alpha$ and $\varphi \mid\!\sim \beta$ then $\varphi \mid\!\sim (\alpha \wedge \beta)$

Or: If $\varphi \mid\!\sim \alpha$ and $\psi \mid\!\sim \alpha$ then $(\varphi \vee \psi) \mid\!\sim \alpha$

Monotonicity

Monotonicity: $\varphi \mid\sim \alpha$ then $\varphi \wedge \psi \mid\sim \alpha$

C: coffee in the cup, T: the liquid tastes good; O: oil is in the cup

$C \mid\sim T$ but $C \wedge O \not\mid\sim T$
But note that $O \not\mid\sim T$

Cautious Monotonicity: If $\varphi \mid\sim \alpha$ and $\varphi \mid\sim \beta$ then $\varphi \wedge \alpha \mid\sim \beta$

Rational Monotonicity: If $\varphi \mid\sim \alpha$ and $\varphi \not\mid\sim \neg\beta$, then $\varphi \wedge \beta \mid\sim \alpha$

Cautious Monotonicity

Reflexivity: If $\varphi \in \Gamma$ then $\Gamma \mid\sim \varphi$

Cut: If $\Gamma \mid\sim \varphi$ and $\Gamma, \varphi \mid\sim \psi$ then $\Gamma \mid\sim \psi$

Cautious Monotonicity: If $\Gamma \mid\sim \alpha$ and $\Gamma \mid\sim \beta$ then $\Gamma, \alpha \mid\sim \beta$

Any "well-behaved" inference rule should satisfy the above three properties"

D. Gabbay. *Theoretical foundations for nonmonotonic reasoning in expert systems*. in K. Apt (ed.), Logics and Models of Concurrent Systems, Berlin and New York: Springer Verlag, pp. 439 - 459, 1985.

Rational Monotonicity, I

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Consider the three composers: Verdi, Bizet, and Satie, and suppose that we initially accept (correctly but defeasibly) that Verdi is Italian $I(v)$, while Bizet and Satie are French $(F(b) \wedge F(s))$.

Rational Monotonicity, II

Suppose now that we are told by a reliable (but not infallible!) source of information that that Verdi and Bizet are compatriots ($C(v, b)$). This leads us no longer to endorse either the proposition that Verdi is Italian (because he could be French), or that Bizet is French (because he could be Italian); but we would still draw the defeasible consequence that Satie is French, since nothing that we have learned conflicts with it.

$$C(v, b) \mid\sim F(s)$$

Rational Monotonicity, III

Now consider the proposition $C(v, s)$ that Verdi and Satie are compatriots. Before learning that $C(v, b)$ we would be inclined to reject the proposition $C(v, s)$ because we accept $I(v)$ and $F(s)$, but after learning that Verdi and Bizet are compatriots, we can no longer endorse $I(v)$, and therefore no longer reject $C(v, s)$.

$$C(v, b) \not\vdash \neg C(v, s)$$

Rational Monotonicity, IV

However, if we added $C(v, s)$ to our stock of beliefs, we would lose the inference to $F(s)$: in the context of $C(v, b)$, the proposition $C(v, s)$ is equivalent to the statement that all three composers have the same nationality. This leads us to suspend our assent to the proposition $F(s)$.

$$C(v, b) \wedge C(v, s) \not\sim F(s)$$

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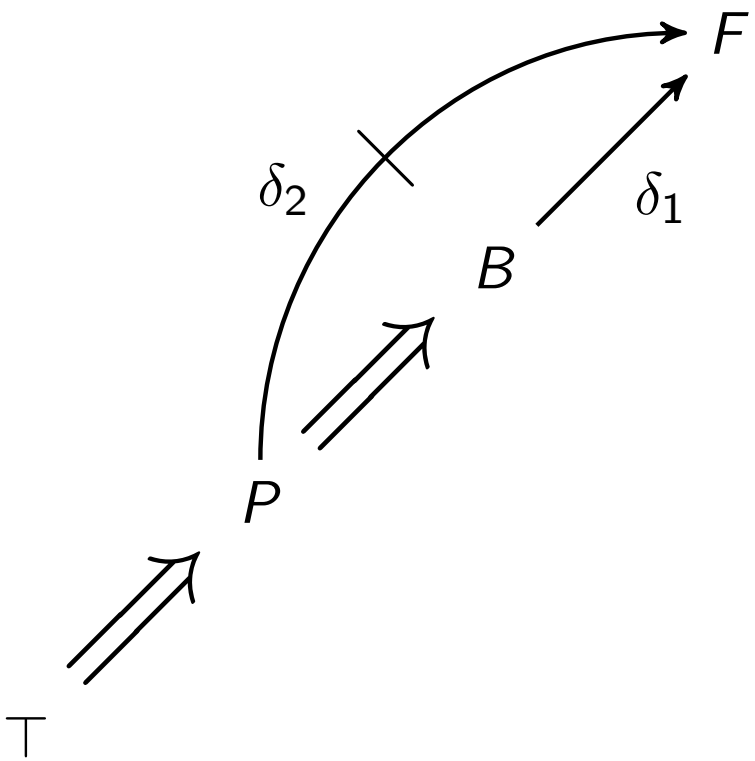
$$C(v, b) \wedge C(v, s) \not\sim F(s)$$

Rational Monotonicity: If $\varphi \mid\sim \alpha$ and $\varphi \not\sim \neg\beta$, then $\varphi \wedge \beta \mid\sim \alpha$

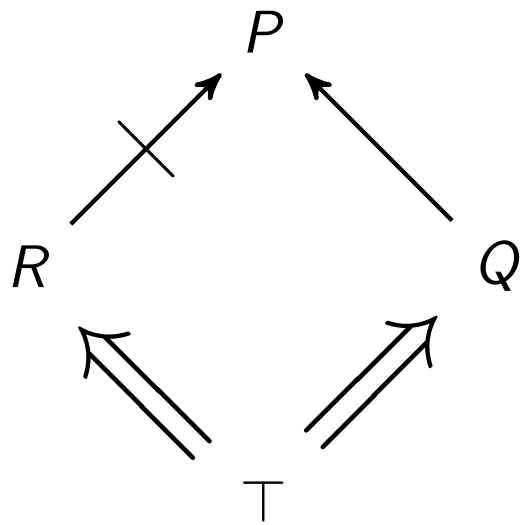
$C(v, b) \mid\sim F(s)$ and $C(v, b) \not\sim \neg C(v, s)$ but
$C(v, b) \wedge C(v, s) \not\sim F(s)$

A separate issue from the formal properties of a non-monotonic consequence relation, although one that is strictly intertwined with it, is the issue of how **conflicts** between potential defeasible conclusions are to be handled.

Tweety Triangle

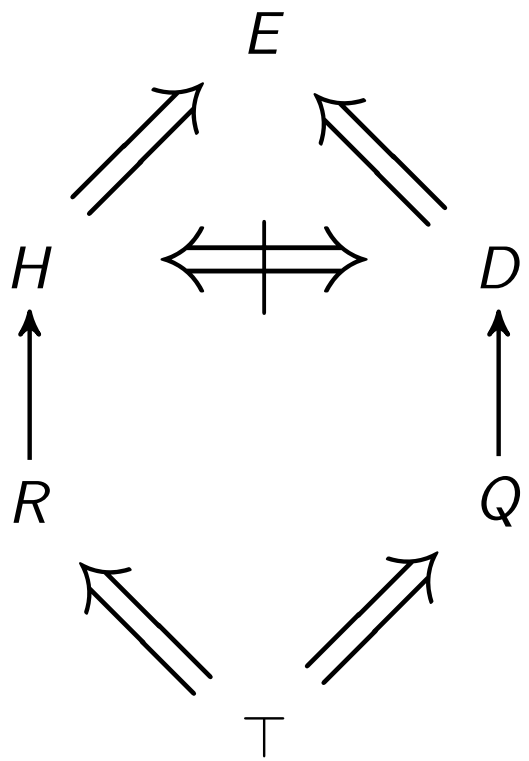


Nixon Diamond

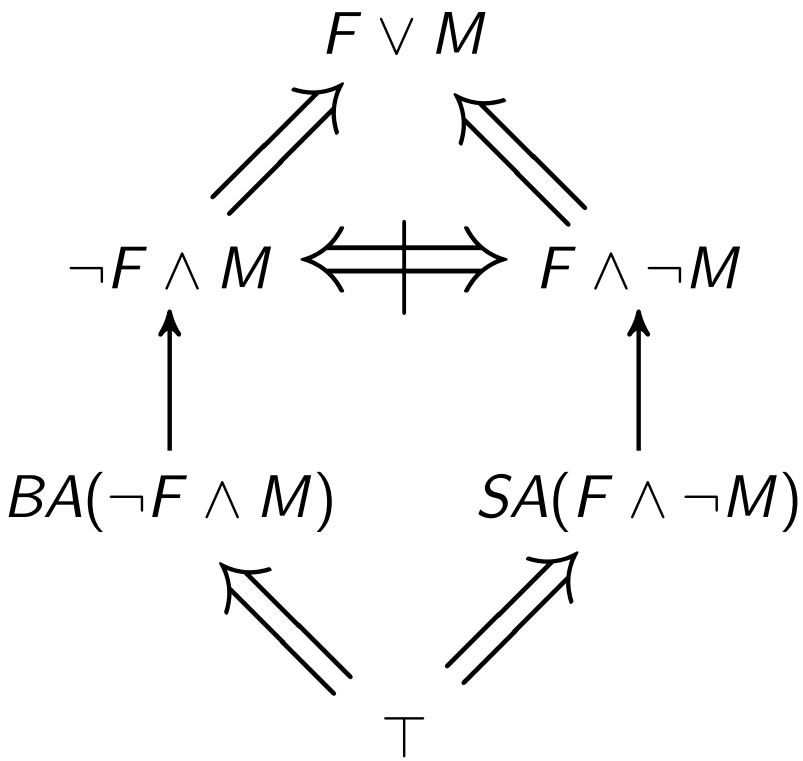


J. Horty. *Skepticism and floating conclusions*. Artificial Intelligence, 135, pp. 55 - 72, 2002.

Floating Conclusions



Floating Conclusions, II



"But if I were told of some other individual that he is both a Quaker and a Republican, I would not be sure what to conclude. It is possible that this individual would adopt an extreme position, as either a dove or a hawk. But it seems equally reasonable to imagine that such an individual, rather than being pulled to one extreme of the other, would combine elements of both views into a more balanced, measured position falling toward the center of the political spectrumperhaps believing that the use of military force is sometimes appropriate, but only as a response to serious provocation."

J. Horty. *Skepticism and floating conclusions*. Artificial Intelligence, 135, pp. 55 - 72, 2002.