Reasoning with Probabilities

Outline of a ESSLLI 2009 Course

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June 29, 2009

This is an outline for the course "Reasoning with Probabilities" taught at the European Summer School for Logic, Language and Information in Bordeaux, France on July 26 - 31, 2009 (ESSLLI 2009). This document contains an extended outline of the course including pointers to relevant literature. The course website:

ai.stanford.edu/∼epacuit/classes/esslli/epprob.html

will contain the slides and a schedule (updated each day). Enjoy the course and please remember to ask questions during the lecture!

This is an advanced but self-contained course. Students will be expected to have had some exposure to (dynamic) epistemic logic and probabilistic logic. In particular, it will be assumed that students have already been introduced to epistemic logic and some of its dynamic extensions (i.e., public announcement logic); and although we will introduce many basic concepts of probabilistic theory (e.g., measure spaces), it will be expected that students have had previous exposure to probabilistic models of uncertainty. Below is some additional reading material related to some of the topics we will discuss in this workshop. Below is a (very) brief outline of the course:

Day 1: Introduction and Background

Day 2: Probabilistic Epistemic Logics

Day 3: Dynamic Probabilistic Epistemic Logics

Day 4: Reasoning with Probabilities

Day 5: Conclusions and General Issues

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Course Description

Both logic and probability provide powerful tools for reasoning about uncertainty in a dynamic environment. Our goal in this course is to examine logical frameworks that incorporate probabilistic modeling of multiagent uncertainty. We will then see how merging these two perspectives on uncertainty can help clarify various conceptual issues and puzzles (such as the Monty Hall puzzle or the sleeping beauty problem). The primary objective is to explore the formal tools used by logicians, computer scientists, philosophers and game theorists for modeling uncertainty. We will focus on both the important conceptual issues (eg., Dutch book arguments, updating with probability zero events and higher-order probabilities) and the main technical results (eg., completeness and decidability of probabilistic modal logics).

Extended Outline

Day 1, July 27: Introduction and Background

Epistemic Logic and Probability Measures: We will provide a brief introduction to epistemic logic (including common knowledge) and some basics mathematical facts about probability measures. There are many textbooks that will provide more extensive discussions of these issues. We can recommend the following:

- Joe Halpern (2003). *Reasoning about Uncertainty*, The MIT Press.
- Patrick Billingsley (1995). *Probability and measure, 3rd edition*, Wiley.
- Hans van Ditmarsch, Wiebe van der Hoek and Barteld Kooi (2007). *Dynamic Epistemic Logic*, Springer.
- Yoav Shoham and Kevin Leyton-Brown (2009). *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations*. Cambridge University Press, Chapters 13 & 14. (www.masfoundations.org)

Probabilities and Beliefs: Probability is a widely used to formally represent an agent's beliefs. We will discuss a number of conceptual and technical issues that have that arise when using probability to represent beliefs in a multiagent situation including (synchronic) Dutch book arguments (Hajek, 2008), higher order probabilities and type spaces (Gaiffman, 1986; Siniscalchi, 2007; Harsanyi, 1967; Aumann, 1999a,b), and common p-beliefs (Monderer and Samet, 1989). See the article below for more information:

- Franz Huber, "Formal Representations of Belief", The Stanford Encyclopedia of Philosophy (Summer 2009 Edition), Edward N. Zalta (ed.), http://plato.stanford.edu/entries/formal-belief/.

Day 2, July 28: Probabilistic (Epistemic) Logics

Probabilistic Logics: We will discuss different logical systems that have been proposed for reasoning about probabilities including an extended discussion of the completeness proofs. There are two main papers that we will discuss:

- Ron Fagin, Joe Halpern and Nimrod Megiddo (1990). A Logic for Reasoning about Probabilities, *Information and Computation*, **87:1**, pp. 78 - 128.
- Aviad Heifetz and Phillipe Mongin (2001). Probability logic for type spaces, *Games and Economic Behavior*, **35**, pp. 31–53.

Probabilistic Epistemic Logics: We then extend the logical systems discussed above to include operators to reason about *knowledge*:

- Ron Fagin and Joe Halpern (1994). Reasoning about Knowledge and Probability, *Journal of the ACM* **41:2**, pp. 340 - 367.

Day 3, July 29: Dynamic Probabilistic Epistemic Logics

This lecture will focus on dynamic version of probabilistic epistemic logics. In particular we will quickly introduce *dynamic epistemic logic*, Bayesian conditioning and Jeffrey updates. We will primarily focus on the following papers:

- Johan van Benthem, Jelle Gerbrandy, and Barteld Kooi (forthcoming). Dynamic update with probabilities (an early version appeared in *Proceedings of LOFT'06*, Liverpool, 2006).
- Joshua Sack (2009). Extending probabilistic dynamic epistemic logic, *Synthese: Knowledge, Rationality and Aciton*, **169**, pp. 241 - 257.

Day 4, July 30: Reasoning with Probabilities

Reasoning with probability zero events: A number of formal models have been proposed that can handle incoming information that is assigned probability zero (note that standard Bayesian conditioning is not defined for such events). We will introduce these different approaches:

- Joe Halpern (2003). Lexicographic probability, conditional probability and nonstandard probability, an early version appeared in *Proceedings of the 8th conference on Theoretical Aspects of Rationality and Knowledge.*

Note that these frameworks are closely related to the logics of belief revision discussed in the ESSLLI course: Dynamic Logics for Interactive Belief Revision taught by Alexandru Baltag and Sonja Smets (cf. Baltag and Smets, 2007)

Reasoning about probability over time: There are a number of subtle issues that arise when reasoning about knowledge, probability and time. We will discuss a number of these issues including adding temporal operators to dynamic probabilistic epistemic logics, the sleeping beauty and dychronic Dutch book, the sleeping beauty problem (see, for example, Elga, 2000; Halpern, 2005; Lewis, 2001) and the related forgetful passenger/absent minded driver problems (see, for example Grove and Halpern, 1997; Piccione and Rubinstein, 1997; Aumann et al., 1997; Board, 2003) and dychronic Dutch book arguments (Skyrms, 1987).

Day 5, July 31: Conclusions and General Remarks

Time permitting, we will conclude by discussing a number of further issues: including defeasible inference (Arló-Costa and Parikh, 2005), logics for reasoning about evidence (Halpern and Pucella, 2006), logics for reasoning about expectations (Halpern and Pucella, 2007) and generalizations of the probabilistic logics we discussed above (Zhou, 2007; Goldblatt, 2008; Moss and Viglizzo, 2004).

Some Puzzles

During the course we will discuss a number of puzzles that have circulated in the literature. This section will list some of the most popular puzzles that have been discussed.

Monty Hall Puzzle The original formulation (from *Ask Marilyn* in *Parade Magazine*) is

> Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind th others, goats. You pick a door, say number 1, and the host, who knows what's behind the doors, opens another door, say number 3, which has a goat. He says to you, "Do you want to pick door number 2?" Is it to your advantage to switch your choice of doors?

The Two-Envelope Puzzle Here is a formulation from (Samet et al., 2004):

> There are two envelopes with money in them. The sum of money in one of the envelopes is twice as large as the other sum. Each of the envelopes is equally likely to hold the larger sum. You are assigned at random one of the envelopes and may take the money inside. However, before you open your envelope you are offered the possibility of switching the envelopes and taking the money inside the other one. Should you switch?

The Sleeping Beauty Puzzle Here is a formulation from (Elga, 2000):

> Some researchers are going to put you to sleep. During the two days that your sleep will last, they will briey wake you up either once or twice, depending on the toss of a fair coin (heads: once; tails: twice). After each waking, they will put you back to sleep with a drug that makes you forget that waking. When you are rst awakened, to what degree ought you believe that the outcome of the coin toss is heads?

There is quite an extensive literature that this puzzle has generated, which we will not survey here (see Halpern, 2005; Lewis, 2001, and references therein)

The Absent-Minded Driver Related to the sleeping beauty problem is the absent-minded driver puzzle. The original formulation is from (Piccione and Rubinstein, 1997):

> An individual is sitting late at night in a bar planning his midnight trip home. In order to get home he has to take the highway and get off at the second exit. Turning at the rst exit leads into a disastrous area (payoff 0). Turning at the second exit yields the highest reward (payoff 4). If he continues beyond the second exit, he cannot go back and at the end of the highway he will nd a motel where he can spend the night (payoff 1). The driver is absentminded and is aware of this fact. At an intersection, he cannot tell whether it is the rst or the second intersection and he cannot remember how many he has passed (one can make the situation more realistic by referring to the 17th intersection). While sitting at the bar, all he can do is to decide whether or not to exit at an intersection. We exclude at this stage the possibility that the decision maker can include random elements in his strategy.

This has also generated a large literature (see Grove and Halpern, 1997; Aumann et al., 1997; Board, 2003, and references therein).

References

Arló-Costa, H. and R. Parikh (2005). Conditional probability and defeasible inference. *Journal of Philosophical Logic 34*, 97 – 119.

Aumann, R. (1999a). Interactive epistemology I: Knowledge. *International Journal of Game Theory 28*, 263–300.

Aumann, R. (1999b). Interactive epistemology I: Probability. *International Journal of Game Theory 28*, 301 – 314.

Aumann, R., S. Hart, and M. Perry (1997). The absent-minded driver. *Games and Economic Behaviour 20*, 102 – 116.

Baltag, A. and S. Smets (2007). From conditional probability to the logic of doxastic actions. In *Proceedings of TARK*.

Board, O. (2003). The not-so-absent-minded driver. *Research in Economics 57*, 189 – 200.

Elga, A. (2000). Self-locating belief and the speeping beauty problem. *Analysis 62*, 292 – 296.

Gaiffman, H. (1986). A thoery of higher order probabilities. In J. Halpern (Ed.), *Proceedings of Theoretical Aspects of Reasoning about Knolwedge (TARK 1986)*, pp. 275 – 292.

Goldblatt, R. (2008). Deduction systems for coalgebras over measurable spaces. *Journal of Logic and Computation*.

Grove, A. and J. Halpern (1997). On the expected value of games with absetnindedness. *Games and Economic Behaviour 20*, 51 – 65.

Hajek, A. (2008). Dutch book arguments. In P. Anand, P. Pattanaik, and C. Puppe (Eds.), *The Oxford Handbook of Rational and Social CHoice*. Oxford University Press.

Halpern, J. (2005). Sleeping beauty reconsidered: Conditiona and reflection in asynchronous systems. In T. Gendler and J. Hawthorne (Eds.), *Oxford Studies in Epistemology*, Volume 1, pp. 111 – 142.

Halpern, J. and R. Pucella (2006). A logic for reasoning about evidence. *Journal of Artificial Intelligence Research 25*.

Halpern, J. and R. Pucella (2007). Characterizing and reasoninga bout probabilistic and non-probabilistic expectation. *Journal of the ACM V.*

Harsanyi, J. (1967). Games with incomplete informations played by 'bayesian' players. *Management Science 14*, 159–182, 320–334, 486–502.

Lewis, D. (2001). Sleeping beuaty: reply to elga. *Analysis 61*, 171 – 176.

Monderer, D. and D. Samet (1989). Approximating common knowledge with common beliefs. *Games and Economic Behaviour 1*, 170 – 190.

Moss, L. and I. Viglizzo (2004). Harsanyi type spaces for final coalgebras constructed from satisfied theories. In *Electronic notes in theoretical computer science*, Volume 106, pp. 279 – 295.

Piccione, M. and A. Rubinstein (1997). On the interpretation of decision problems with imperfect recall. *Games and Economic Behaviour 20*, 3 – 24.

Samet, D., I. Samet, and D. Schmeidler (2004). One observation behind two envelope puzzles. *American Mathematical Monthly 111.*

Siniscalchi, M. (2007). Epistemic game theory: Beliefs and types. In *The New Palgrave Dictionary of Economics.*

Skyrms, B. (1987). Dynamic coherence and probability kinematics. *Philosophy of Science 54*(1), 1 – 20.

Zhou, C. (2007). *Complete deductive systems for probability logic with application to Harsanyi type spaces.* Ph. D. thesis, Indiana University.