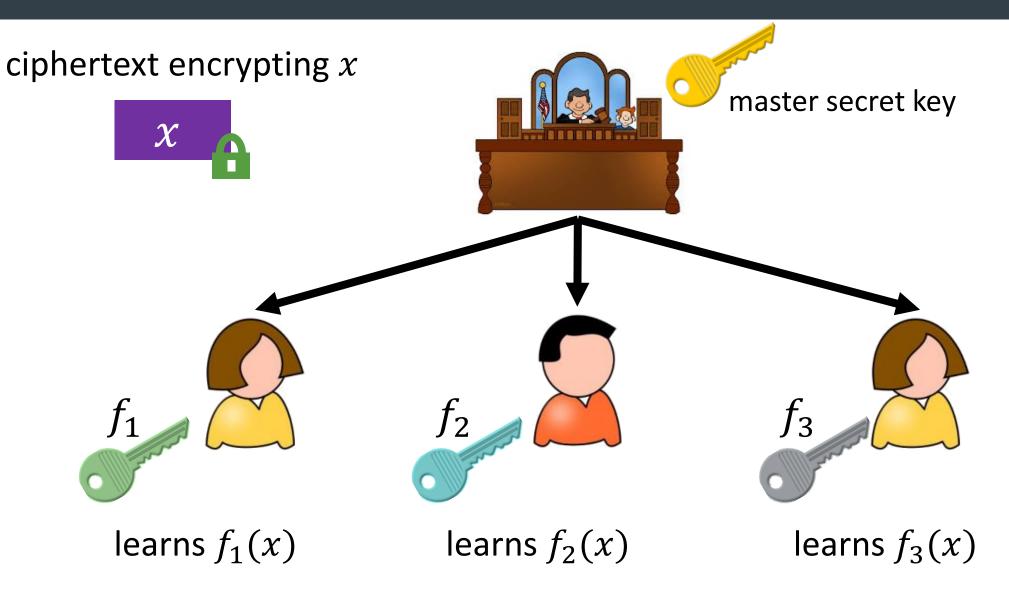
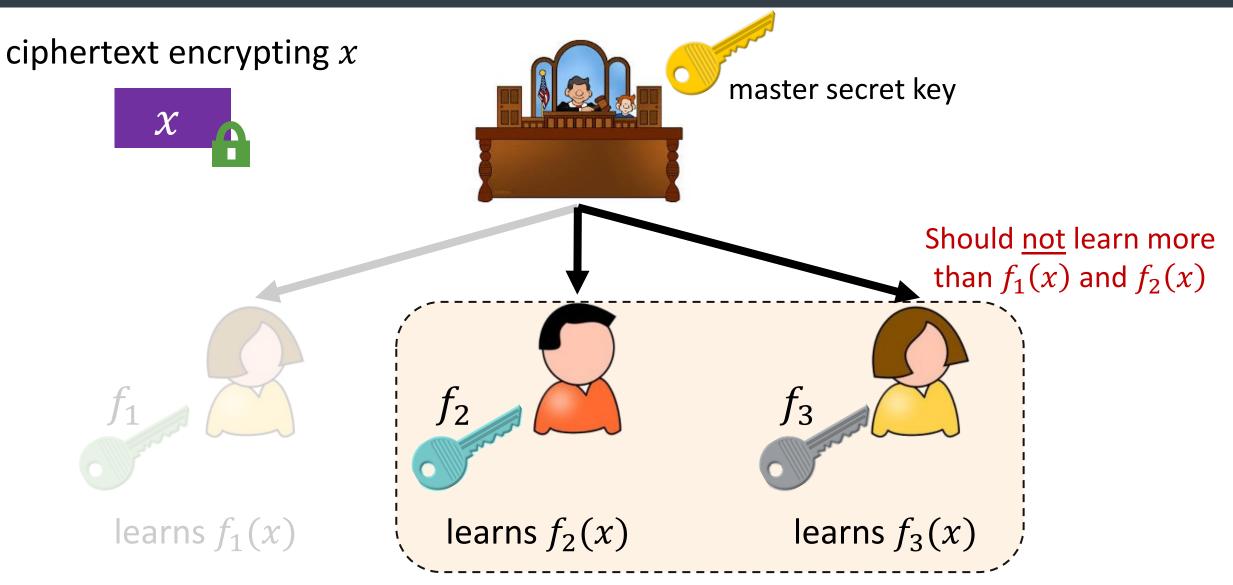
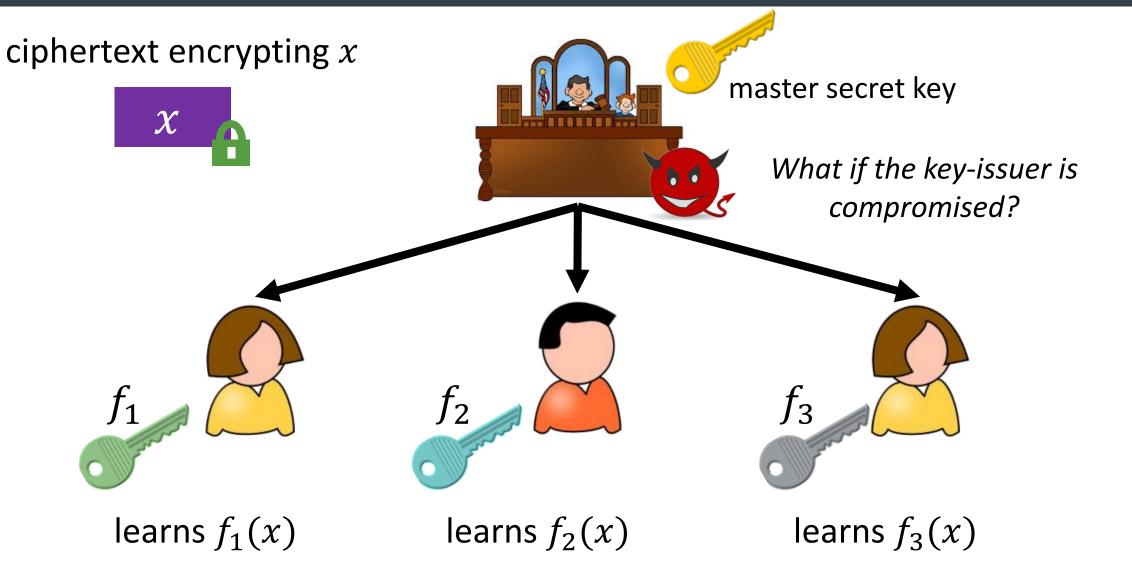
Removing Trust Assumptions from Functional Encryption

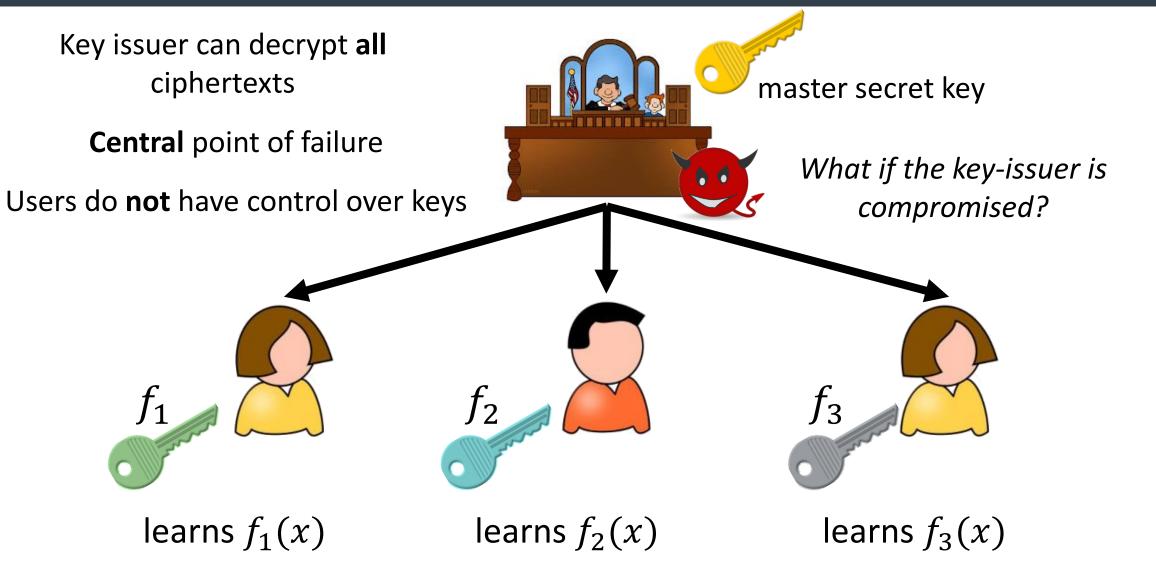
David Wu December 2023

based on joint works with Cody Freitag, Rachit Garg, Susan Hohenberger, George Lu, and Brent Waters









Functional Encryption vs. Public-Key Encryption

Public-key encryption is **decentralized**



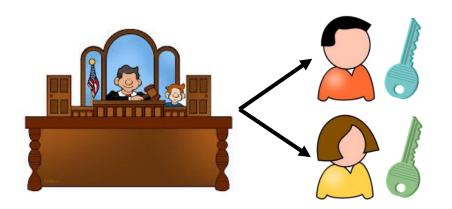




Can we get the best of both worlds?

Every user generates their own key (no coordination or trust needed) Does **not** support fine-grained decryption

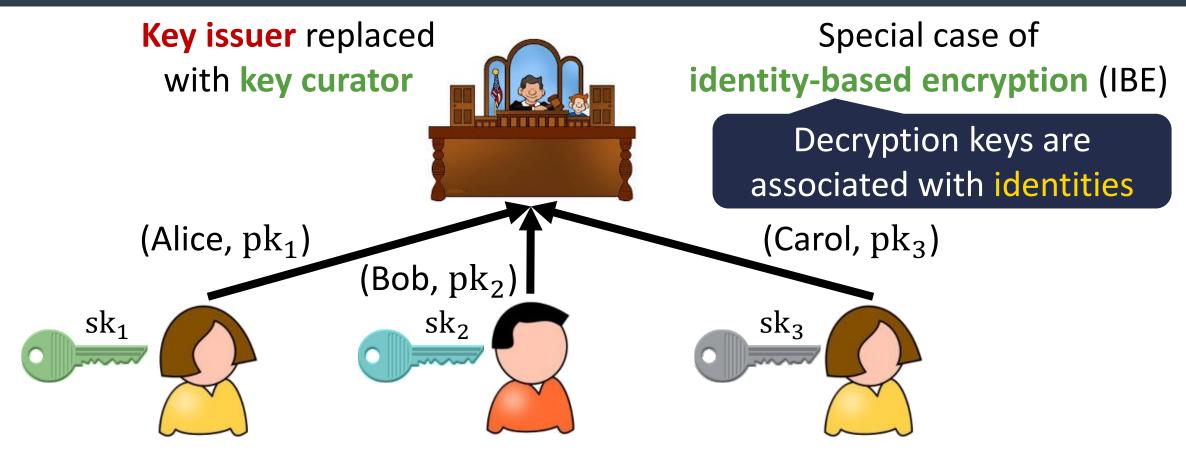
Functional encryption is centralized



Central (trusted) authority generates individual keys

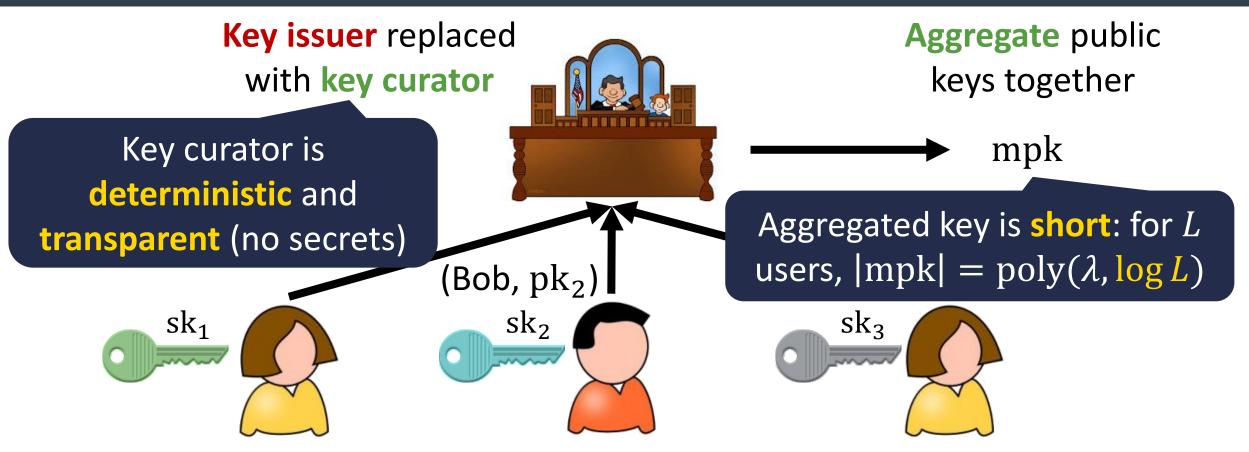
Supports fine-grained decryption capabilities

[GHMR18]

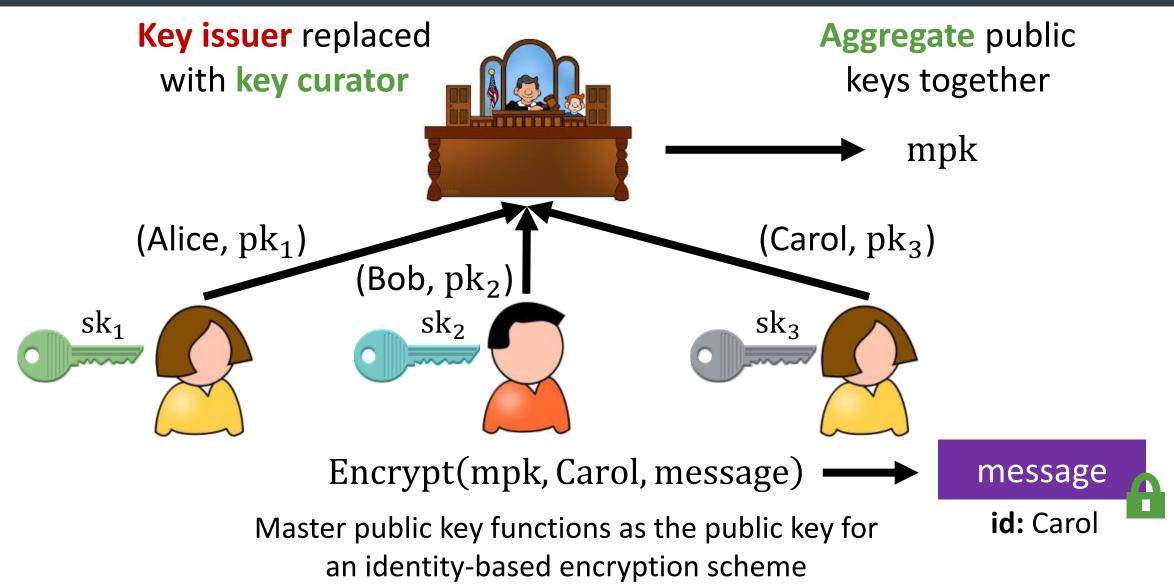


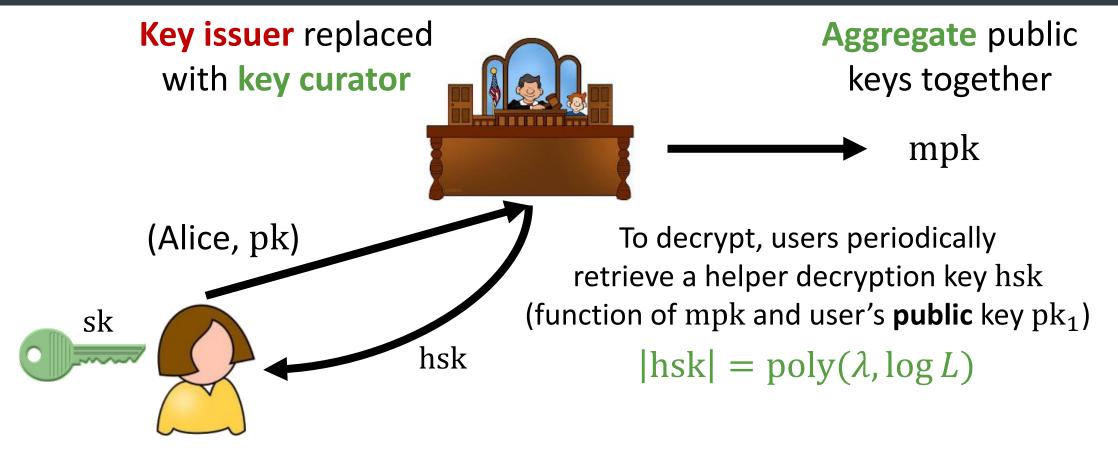
Users chooses their <u>own</u> public/secret key and **register** their public key with the curator

[GHMR18]



Users chooses their <u>own</u> public/secret key and **register** their public key with the curator

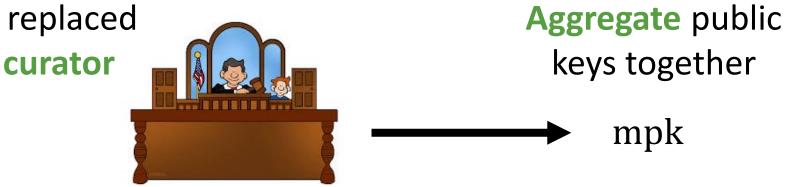




Note: As users join, the master public key is updated, so users **occasionally** need to retrieve a new helper decryption key $\# \text{ key updates per user} = \text{poly}(\lambda, \log L)$

[GHMR18]

Key issuer replaced with key curator

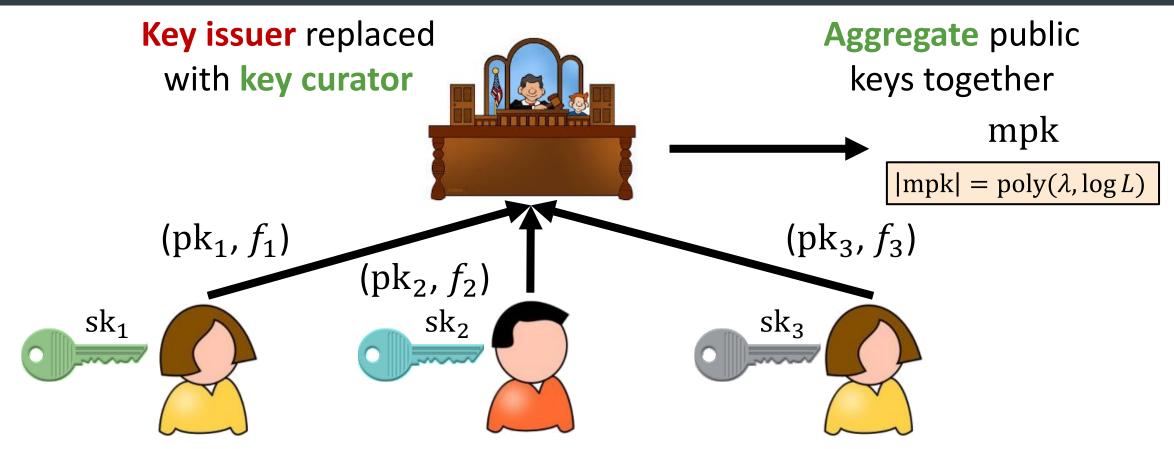


- Initial constructions based on indistinguishability obfuscation or hash garbling (based on CDH, QR, LWE) – all require non-black-box use of cryptography
- High concrete efficiency costs: ciphertext is 4.5 TB for supporting 2 billion users [CES21]

Can we construct RBE schemes that only need black-box use of cryptography?

Can we construct support more general policies (beyond identity-based encryption)?

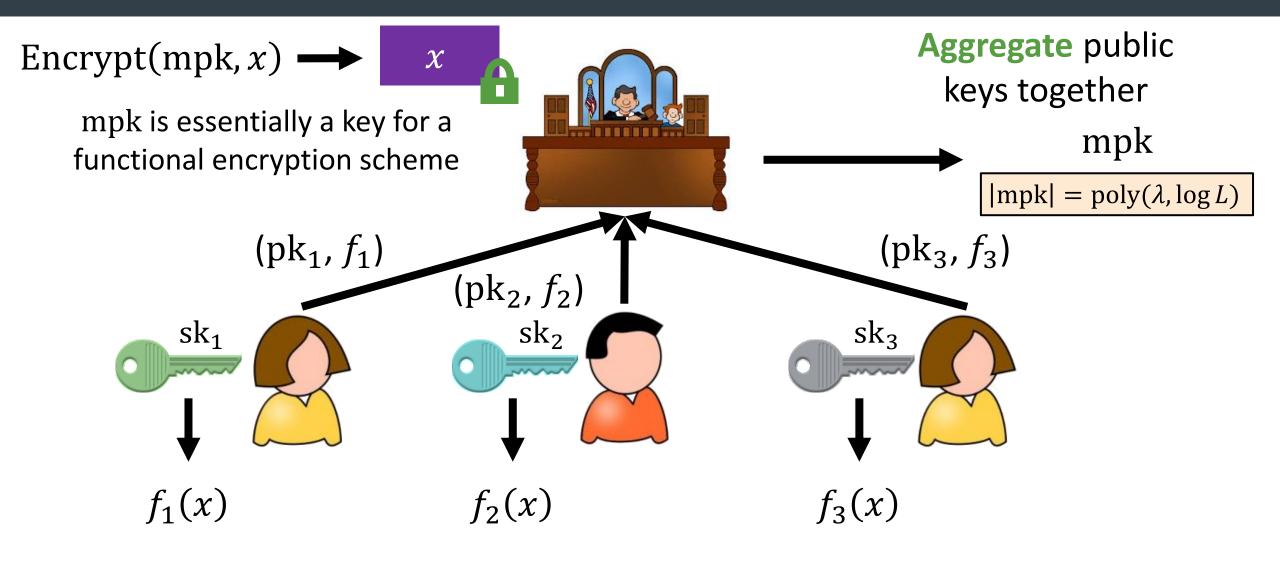
Removing Trust from Functional Encryption



Users chooses their own key and **register** the public key (together with **function** *f*) with the curator

Note: *f* could also be chosen by the key curator

Removing Trust from Functional Encryption



Registered Functional Encryption

Can we construct RBE schemes that only need black-box use of cryptography? YES!

Can we construct support more general policies (beyond identity-based encryption)? YES!

Registration-based encryption [GHMR18, GHMMRS19, GV20, CES21, DKLLMR23, GKMR23, ZZGQ23, FKP23]

Registered attribute-based encryption (ABE)

- Monotone Boolean formulas [HLWW23, ZZGQ23]
- Inner products [FFMMRV23, ZZGQ23]
- Arithmetic branching program [ZZGQ23]
- Boolean circuits [HLWW23, FWW23]

Distributed/flexible broadcast [BZ14, KMW23, FWW23, GLWW23]

Registered functional encryption

- Linear functions [DPY23]
- Boolean circuits [FFMMRV23, DPY23]

Lots of progress in this past year!

<u>Underlined schemes</u> only need **black-box** use of cryptography

Registered Functional Encryption

Can we construct RBE schemes that only need black-box use of cryptography? YES!

Can we construct support more general policies (beyond identity-based encryption)? YES!

Registration-based encryption [GHMR18, GHMMRS19, GV20, CES21, DKLLMR23, GKMR23, ZZGQ23, FKP23]

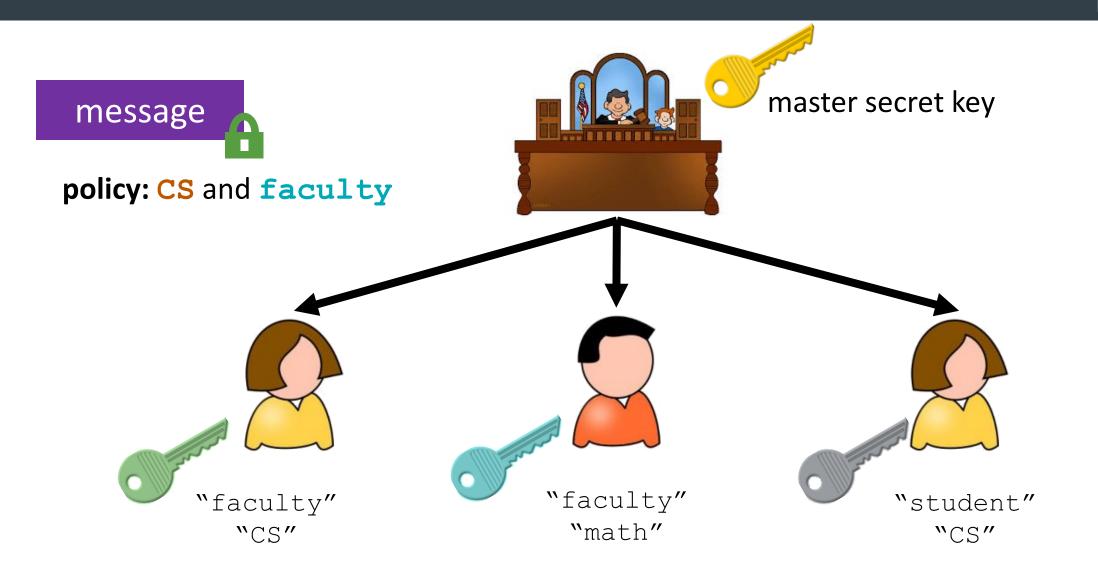
Registered attribute-based encryption (ABE)		
 Monotone Boolean formulas [<u>HLWW23</u>, <u>ZZGQ23</u>] 		Lots of progress in
 Inner products [FFMMRV23, ZZGQ23] 		
 Arithmetic branching program [ZZGQ23] 		this past year!
 Boolean circuits [HLWW23, FWW23] 	This talk	
Distributed/flexible broadcast [BZ14, KMW23, FWW23, GLWW23]	This talk	

Registered functional encryption

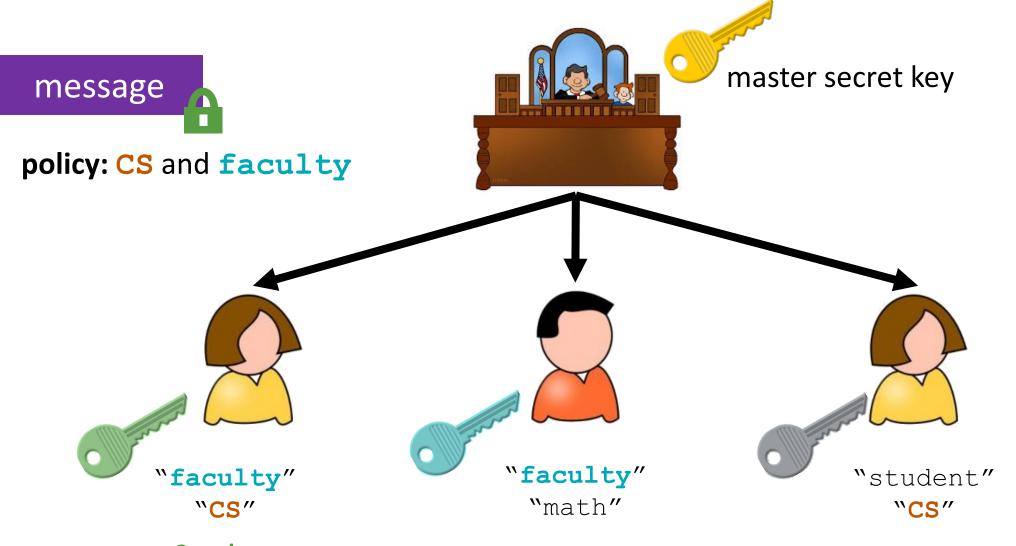
- Linear functions [DPY23]
- Boolean circuits [FFMMRV23, DPY23]

<u>Underlined schemes</u> only need **black-box** use of cryptography

[SW05, GPSW06]

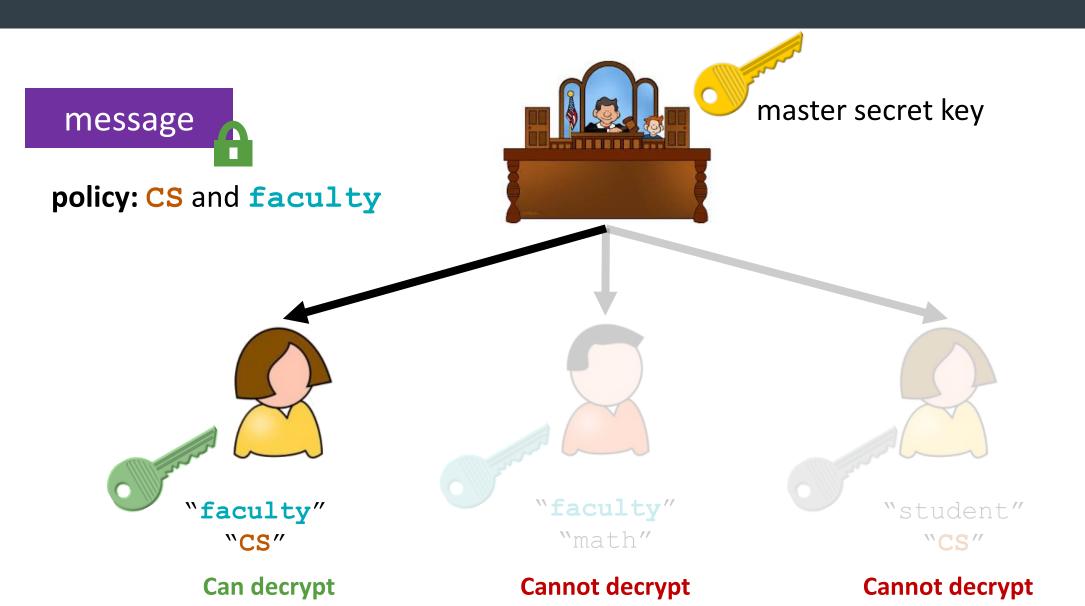


[SW05, GPSW06]

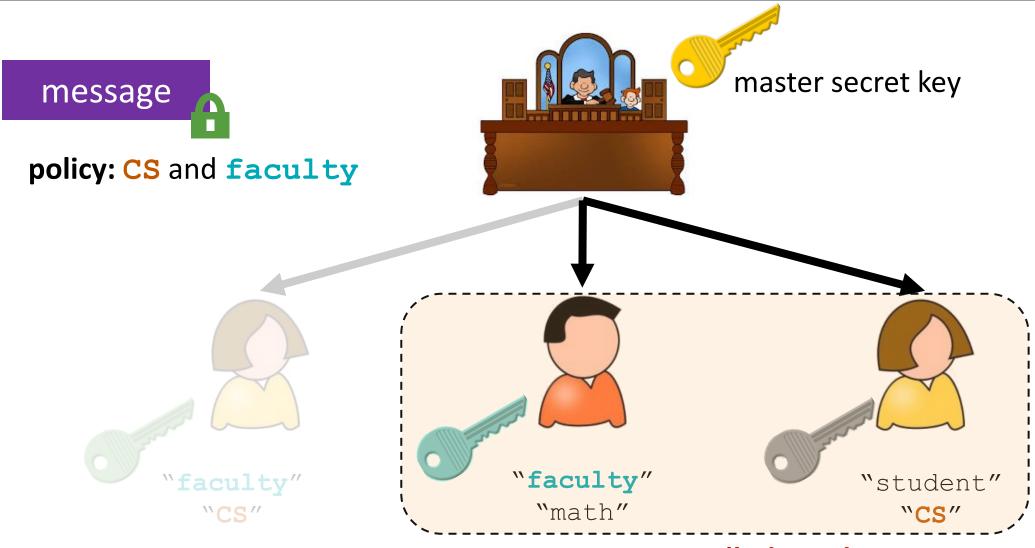


Can decrypt

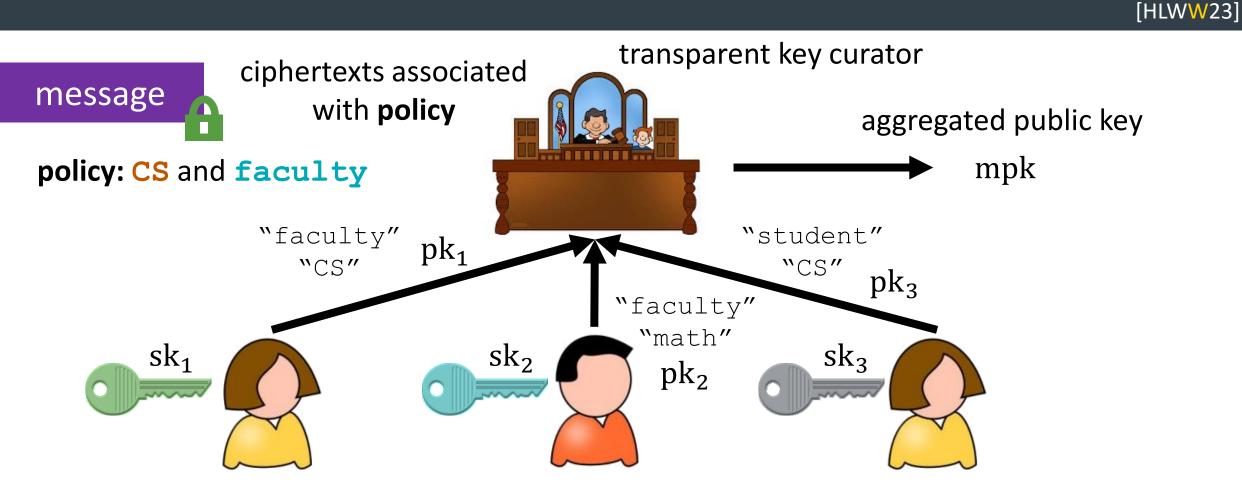
[SW05, GPSW06]



[SW05, GPSW06]



Users <u>cannot</u> collude to decrypt



Users chooses their <u>own</u> public/secret key Users join the system by registering their public key along with a set of attributes

Simplification: assume that all of the users register at the same time (rather than in an

[HLWW23]

online fashion)

Slotted registered ABE:

Let *L* be the number of users

Each slot associated with a public key pk and a set of attributes S

- $|mpk| = poly(\lambda, |\mathcal{U}|, \log L)$
- $|\text{hsk}_i| = \text{poly}(\lambda, |\mathcal{U}|, \log L)$
- λ : security parameter
- \mathcal{U} : universe of attributes

Simplification: assume that all of the users register at the **same** time (rather than in an online fashion)

Slotted registered ABE:

Let *L* be the number of users

$$pk_1, S_1$$
 pk_2, S_2 pk_3, S_3 pk_4, S_4 \cdots pk_L, S_L
Aggregate
 mpk
 hsk_1, \dots, hsk_L

Each slot associated with a <u>public key</u> pk and a set of attributes S

Encrypt(mpk, P, m) \rightarrow ct

 $\text{Decrypt}(\text{sk}_i, \text{hsk}_i, \text{ct}) \rightarrow m$

Encryption takes master public key and policy *P* (no slot)

[HLWW23]

Decryption takes secret key sk_i for some slot and the helper key hsk_i for that slot

Simplification: assume that all of the users register at the **same** time (rather than in an online fashion)

Slotted registered ABE:

Let *L* be the number of users

$$pk_1, S_1$$
 pk_2, S_2 pk_3, S_3 pk_4, S_4 ... pk_L, S_L Aggregatempk $bsk_1, ..., bsk_L$

Each slot associated with a <u>public key</u> pk and a set of attributes S

 $Encrypt(mpk, P, m) \rightarrow ct$

 $\text{Decrypt}(\text{sk}_i, \text{hsk}_i, \text{ct}) \rightarrow m$

Main difference with registered ABE: Aggregate takes all *L* keys <u>simultaneously</u>

[HLWW23]

Let *L* be the number of users

Aggregate

Slotted scheme does *not* support online registration

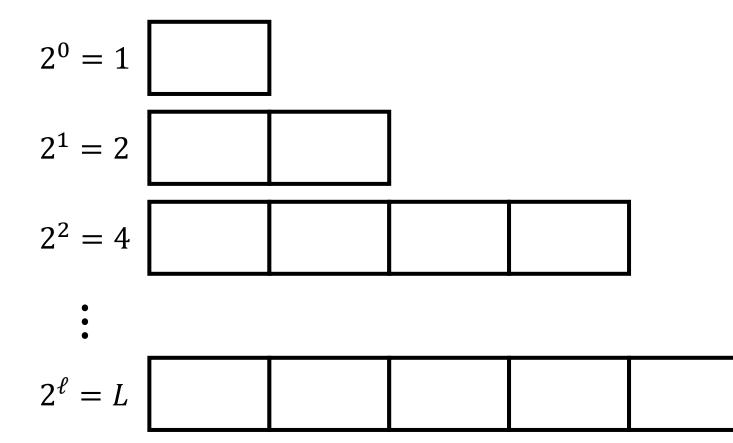
Solution: use "powers-of-two" approach (like [GHMR18])

[HLWW23]

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To support $L = 2^{\ell}$ users: maintain ℓ slotted schemes

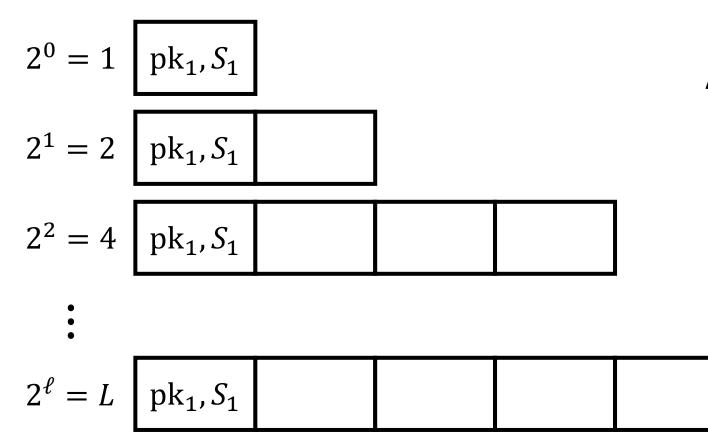
Initially: all slots are empty $mpk = \bot$



[HLWW23]

Solution: use "powers-of-two" approach (like [GHMR18])

To support $L = 2^{\ell}$ users: maintain ℓ slotted schemes



Initially: all slots are empty $mpk = \bot$

Add key to each scheme with available slot



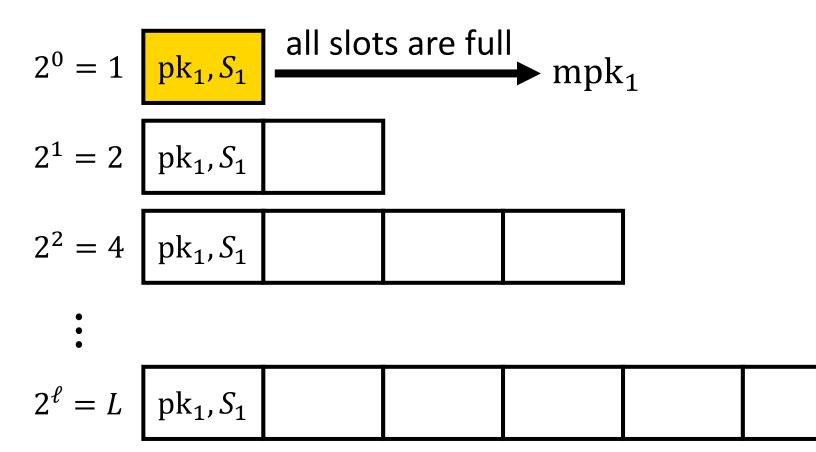




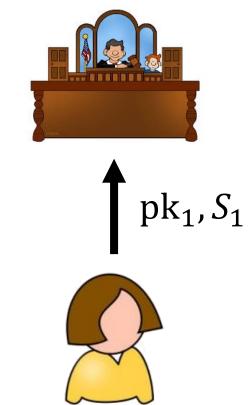
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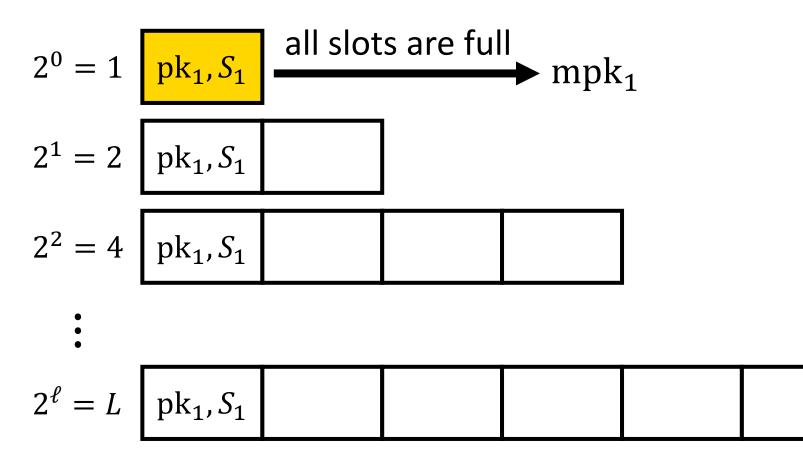
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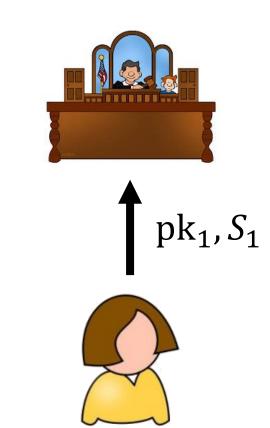
[HLWW23]

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To support $L = 2^{\ell}$ users: maintain ℓ slotted schemes



Initially: all slots are empty mpk = (mpk₁)



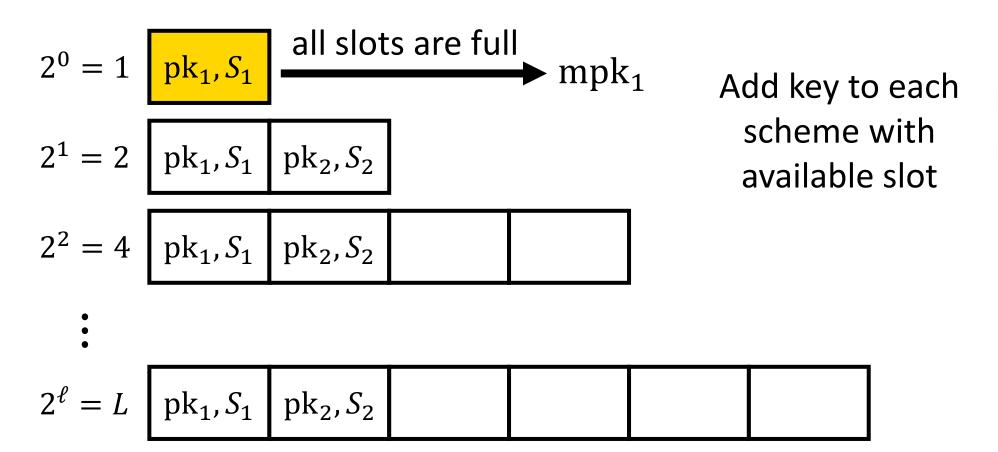
[HLWW23]

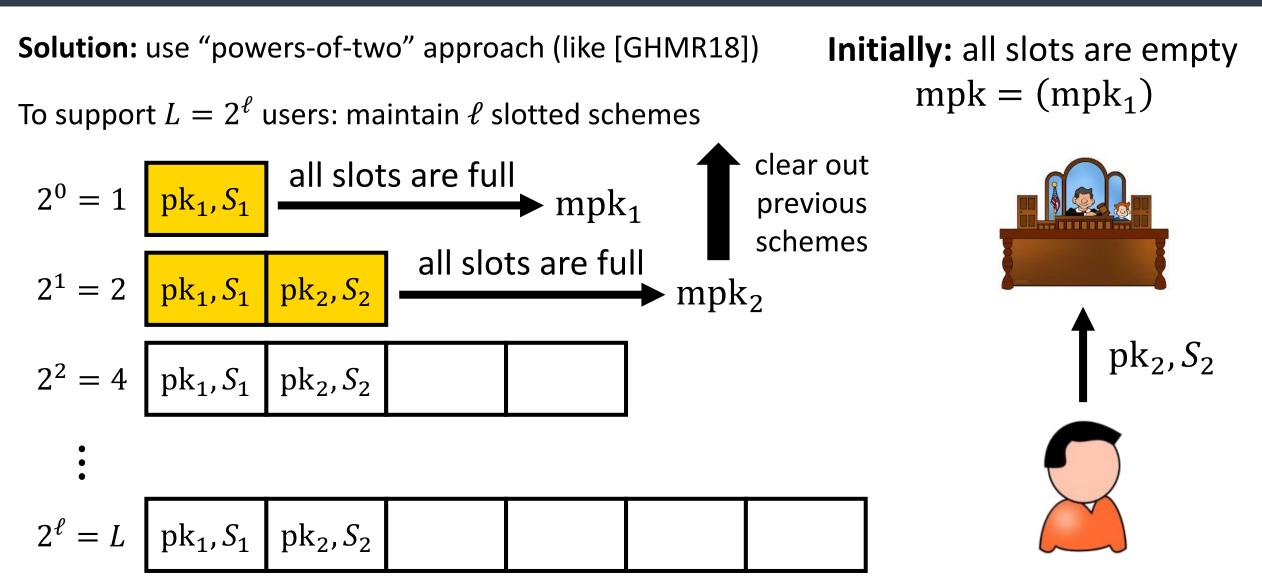
 pk_2, S_2

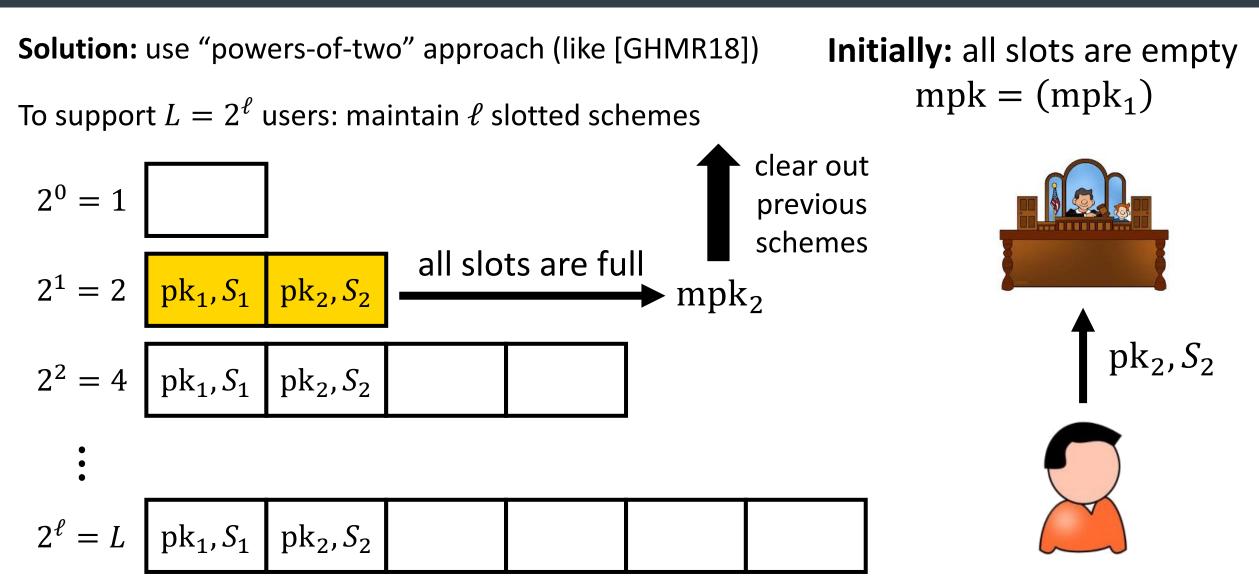
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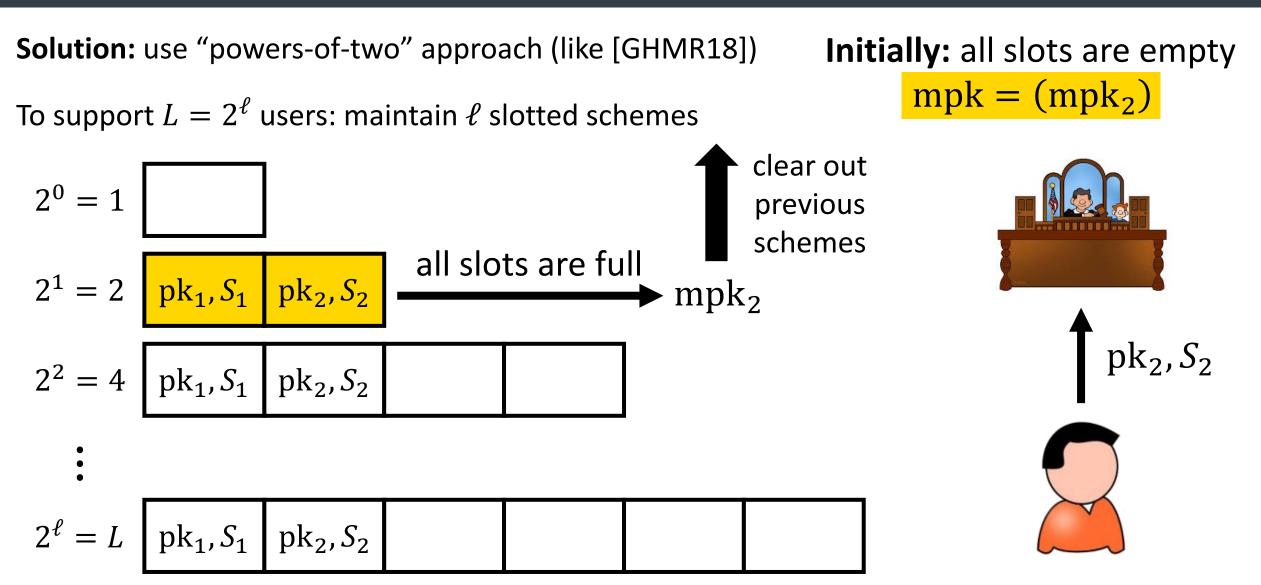
To support $L = 2^{\ell}$ users: maintain ℓ slotted schemes

Initially: all slots are empty $mpk = (mpk_1)$







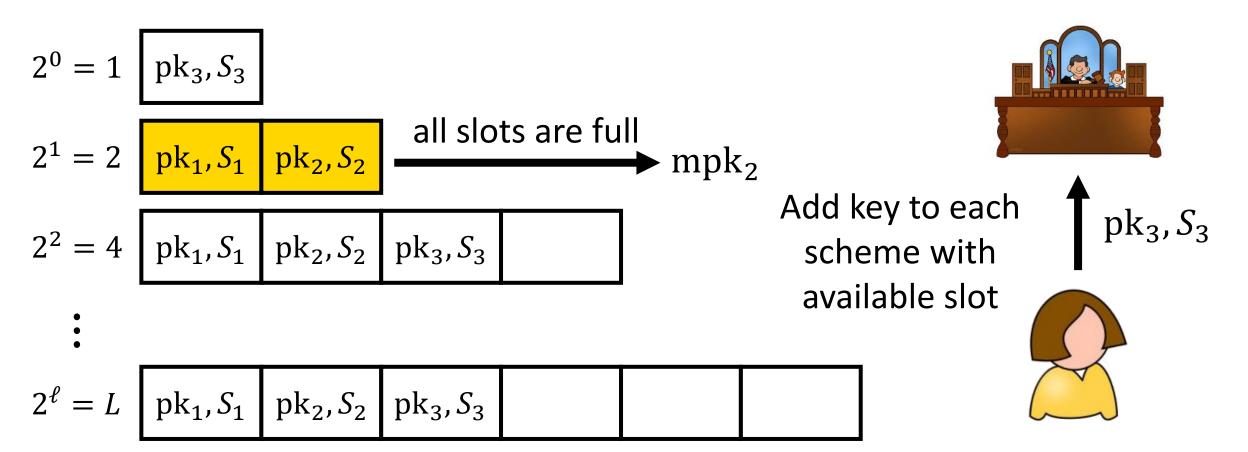


[HLWW23]

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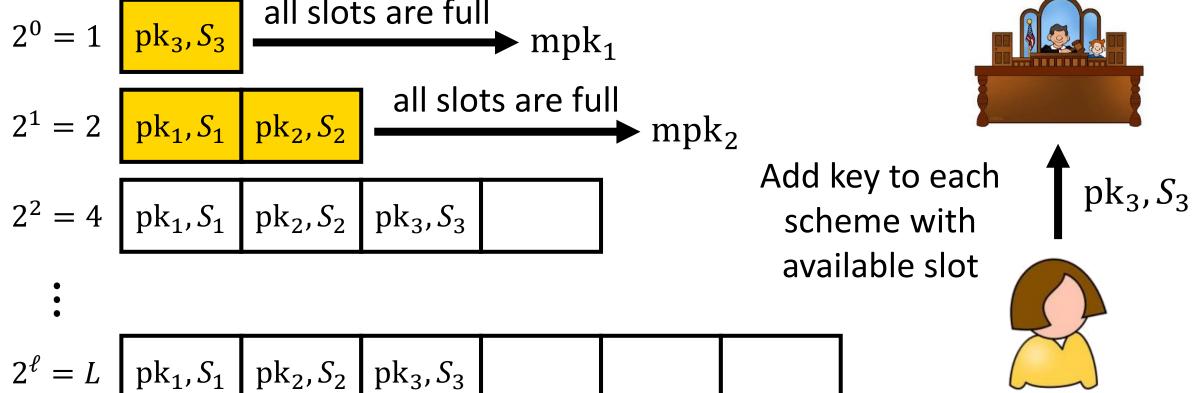
To support $L = 2^{\ell}$ users: maintain ℓ slotted schemes

Initially: all slots are empty $mpk = (mpk_2)$



[HLWW23]

Solution: use "powers-of-two" approach (like [GHMR18]) To support $L = 2^{\ell}$ users: maintain ℓ slotted schemes $2^{0} = 1$ pk_{3}, S_{3} all slots are full pk_{1}, S_{3} pk_{1} pk_{2}



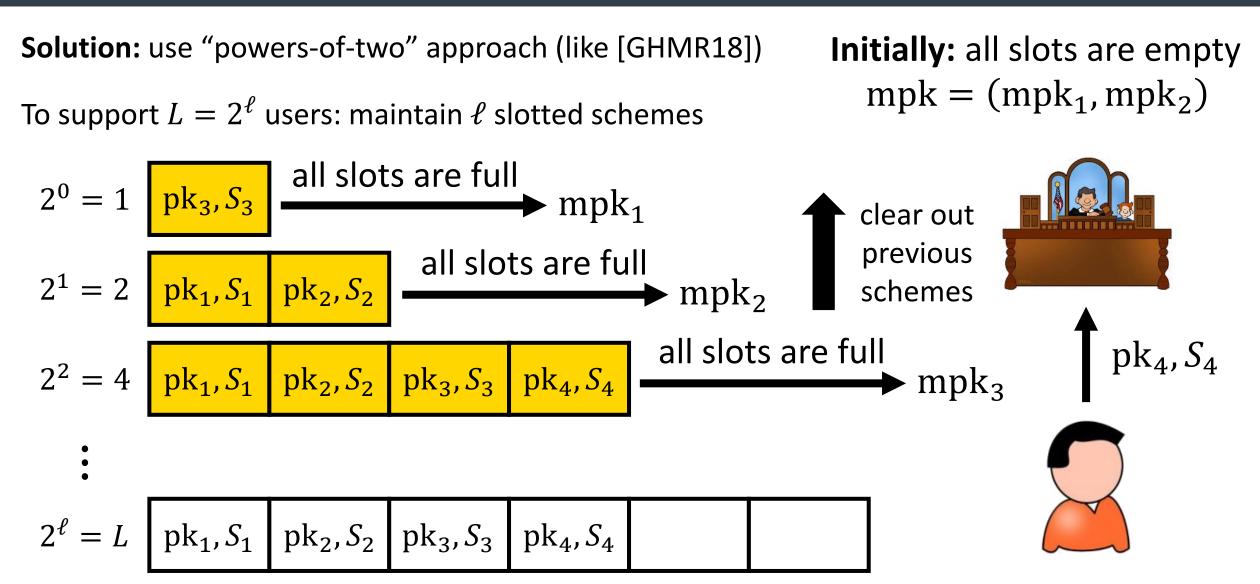
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Solution: use "powers-of-two" approach (like [GHMR18]) **Initially:** all slots are empty $mpk = (mpk_1, mpk_2)$ To support $L = 2^{\ell}$ users: maintain ℓ slotted schemes $2^0 = 1$ pk₃, S₃ all slots are full pk_1 $2^1 = 2$ pk_1, S_1 pk_2, S_2 all slots are full \rightarrow mpk₂ Add key to each pk₃, S₃ $2^2 = 4$ pk₁, S₁ pk₂, S₂ pk₃, S₃ scheme with available slot $2^{\ell} = L \quad pk_1, S_1 \quad pk_2, S_2 \quad pk_3, S_3$

[HLWW23]

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[HLWW23]

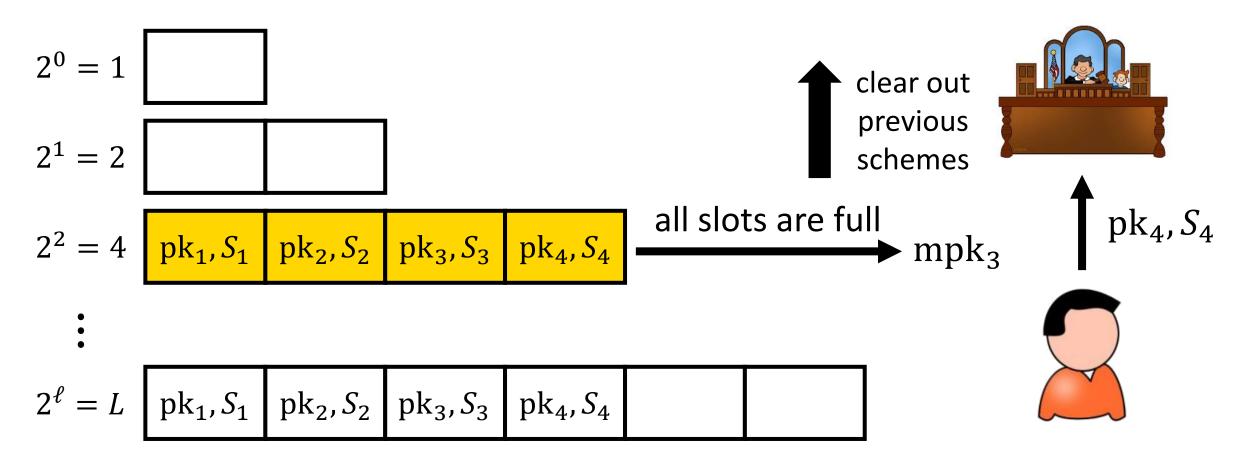


[HLWW23]

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To support $L = 2^{\ell}$ users: maintain ℓ slotted schemes

Initially: all slots are empty $mpk = (mpk_1, mpk_2)$



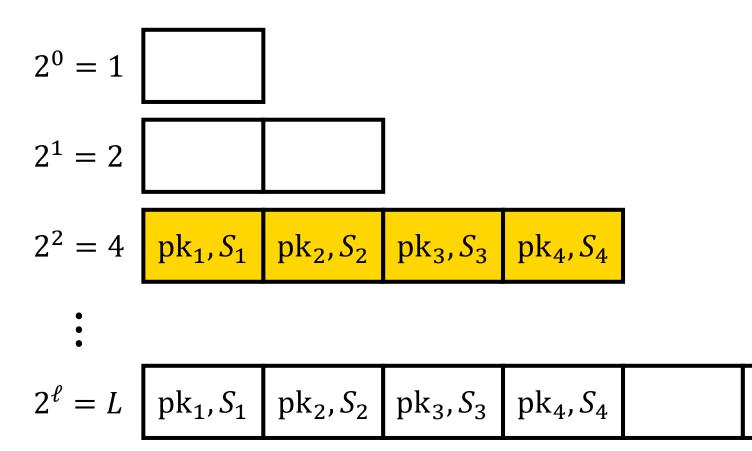
[HLWW23]

Solution: use "powers-of-two" approach (like [GHMR18]) **Initially:** all slots are empty $mpk = (mpk_3)$ To support $L = 2^{\ell}$ users: maintain ℓ slotted schemes $2^0 = 1$ clear out previous schemes $2^1 = 2$ all slots are full pk₄, *S*₄ $2^2 = 4$ pk₁, S₁ pk₂, S₂ pk₃, S₃ pk₄, S₄ ► mpk₃ $2^{\ell} = L$ $pk_1, S_1 \quad pk_2, S_2 \quad pk_3, S_3 \quad pk_4, S_4$

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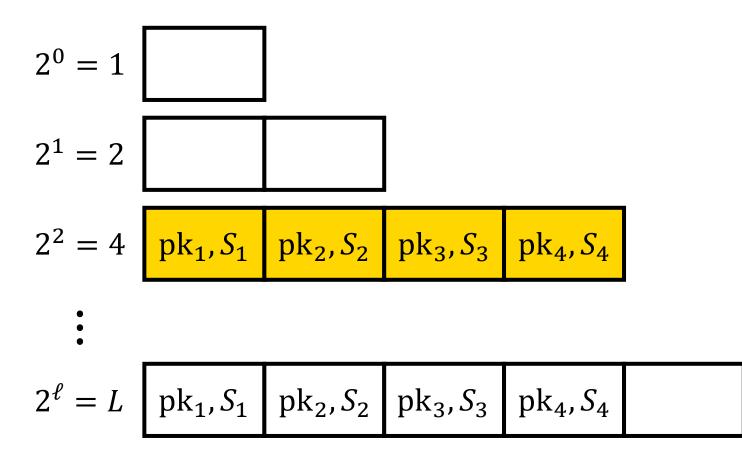
Ciphertext is an encryption to <u>each</u> public key

log *L* overhead

[HLWW23]

Solution: use "powers-of-two" approach (like [GHMR18])

To support $L = 2^{\ell}$ users: maintain ℓ slotted schemes



Initially: all slots are empty $mpk = (mpk_3)$

Update needed whenever user's key moves from scheme *i* to scheme *j* > *i*

At most $\ell = \log L$ updates

Constructing Slotted Registered ABE

Construction will rely on (composite-order) pairing groups (\mathbb{G}, \mathbb{G}_T)

Pairing is an **efficiently-computable** bilinear map $e: \mathbb{G} \to \mathbb{G}_T$ from \mathbb{G} to \mathbb{G}_T : $e(g^x, g^y) = e(g, g)^{xy}$

Multiplies exponents in the target group

Outline of Slotted Registered ABE

[HLWW23]

Scheme will rely on a **structured** common reference string (CRS)

Slot components: each slot $i \in [L]$ will have a set of associated group elements (denoted A_i)

$$A_1$$
 A_2 A_3 A_4 \cdots A_L

Attribute components: each attribute $w \in \mathcal{U}$ will have a group element U_w

User's individual public/secret key is an ElGamal key-pair

$$sk = r$$
, $pk = g^r$

Aggregated public key is just the product of every user's public key:

$$mpk = \prod_{i \in [L]} g^{r_i}$$

Similar aggregation for attribute components

Outline of Slotted Registered ABE

[HLWW23]

Scheme will rely on a **structured** common reference string (CRS)

Slot components: each slot $i \in [L]$ will have a set of associated group elements (denoted A_i)

Attribute components: each attribute $w \in \mathcal{U}$ will have a group element U_w

Decryption enforces the following two requirements:

Slot requirement: Decrypter know a secret key associated with the public key for some slot i^*

Attribute requirement: Attributes associated with slot i^* satisfy the decryption policy

In the construction, message is "blinded" by v_1v_2 , where v_1 can be computed with knowledge of a secret key associated with a slot i^* and v_2 can be computed if the attributes for slot i^* satisfy the policy

Outline of Slotted Registered ABE

[HLWW23]

Scheme will rely on a **structured** common reference string (CRS)

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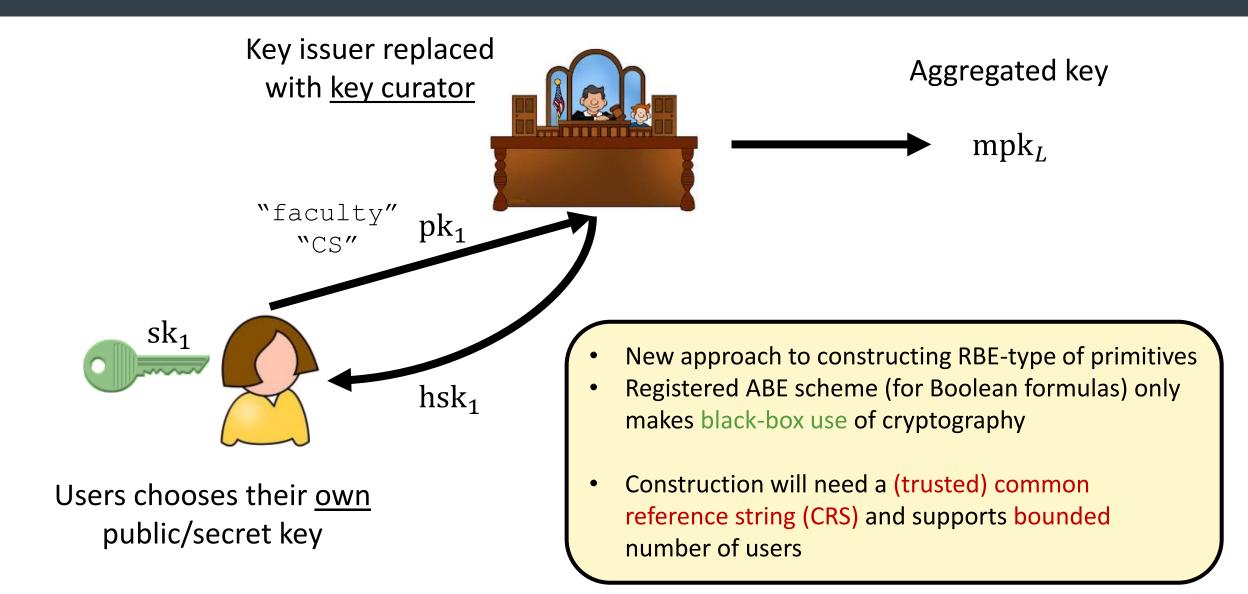
Need to be careful to defend against collusions [see paper for details] ving two requirements:

v a secret key associated with the public key for some slot i^st

associated with slot i^* satisfy the decryption policy

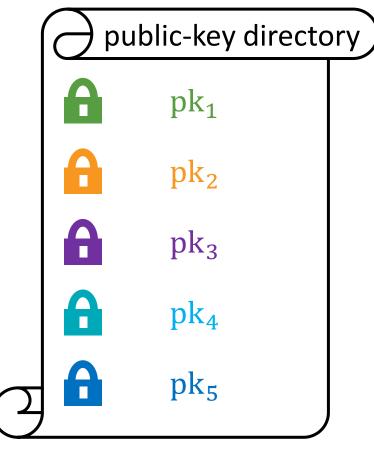
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Registered ABE Summary



An Application to Broadcast Encryption

Registered ABE is a useful building block for other trustless cryptographic systems



Suppose we want to encrypt a message to $\{pk_1, pk_3, pk_4\}$

[FWW23]

Public-key encryption: ciphertext size grows with the size of the set

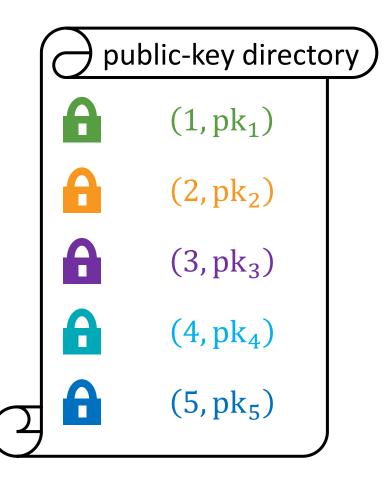


Broadcast encryption: achieve *sublinear* ciphertext size, but requires central authority

Independent, user-generated keys

An Application to Broadcast Encryption

Distributed broadcast encryption [BZ14]



Each user chooses its own public key, and each key has a **unique** index Encrypt(pp, $\{pk_i\}_{i \in S}, m$) \rightarrow ct Can encrypt a message *m* to any set of public keys **Efficiency:** $|ct| = |m| + poly(\lambda, log|S|)$ Decrypt(pp, $\{pk_i\}_{i \in S}$, sk, ct) $\rightarrow m$ Any secret key associated with broadcast set can decrypt Decryption does requires knowledge of public keys in

[FWW23]

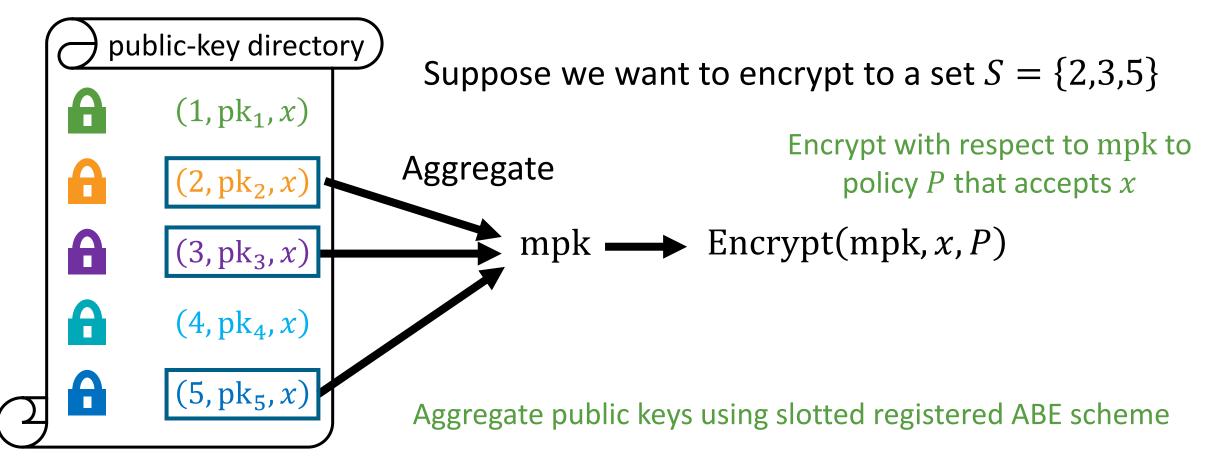
broadcast set

Distributed Broadcast from Slotted Registered ABE

[FWW23]

Consider a registered ABE scheme with a single dummy attribute x

Public key for an index *i* is a key for **slot** *i* with **attribute** *x*

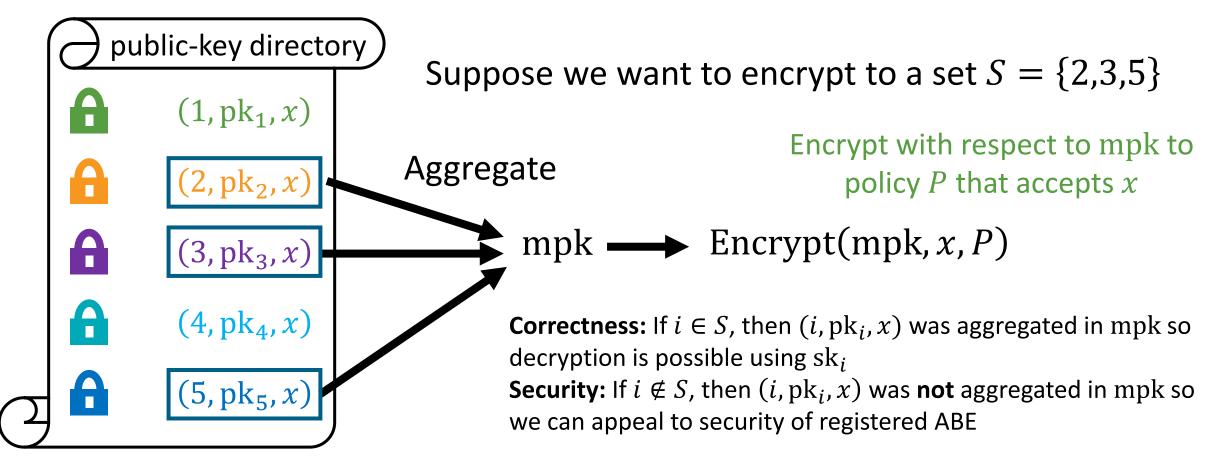


Distributed Broadcast from Slotted Registered ABE

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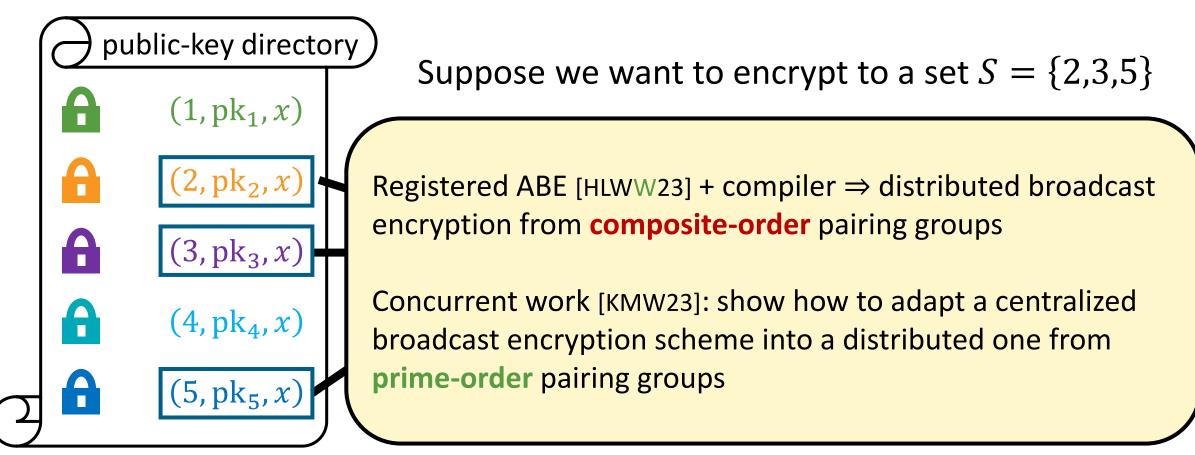


Distributed Broadcast from Slotted Registered ABE

[FWW23]

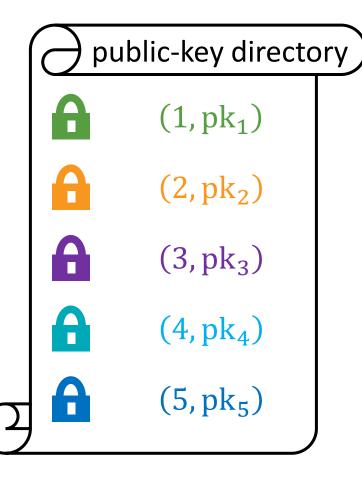
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[FWW23, GLWW23]

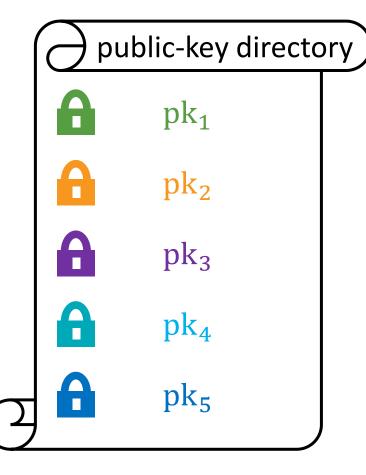
Distributed broadcast encryption still requires **some** coordination



Users have to generate public keys for **distinct** slots (for correctness), so public-key directory needs to be **centralized**

[FWW23, GLWW23]

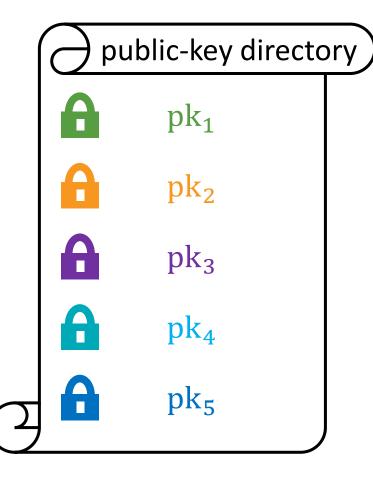
Distributed broadcast encryption still requires **some** coordination



Users have to generate public keys for **distinct** slots (for correctness), so public-key directory needs to be **centralized**

Flexible broadcast encryption: no notion of slots, can encrypt to an *arbitrary* set of public keys

Distributed broadcast encryption still requires **some** coordination



 $Encrypt(pp, \{pk_i\}_{i \in S}, m) \to ct$

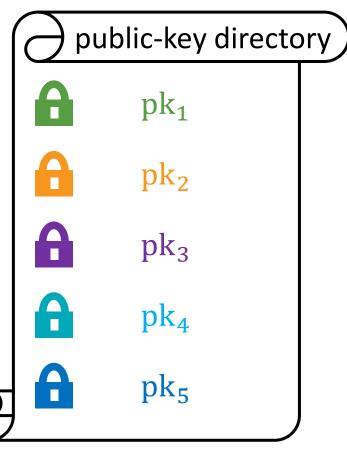
Can encrypt a message m to any set of public keys

```
Efficiency: |ct| = |m| + poly(\lambda, log|S|)
```

```
Decrypt(pp, \{pk_i\}_{i \in S}, sk, ct) \rightarrow m
```

Any secret key associated with broadcast set can decrypt Decryption does requires knowledge of public keys in broadcast set

Distributed broadcast encryptic

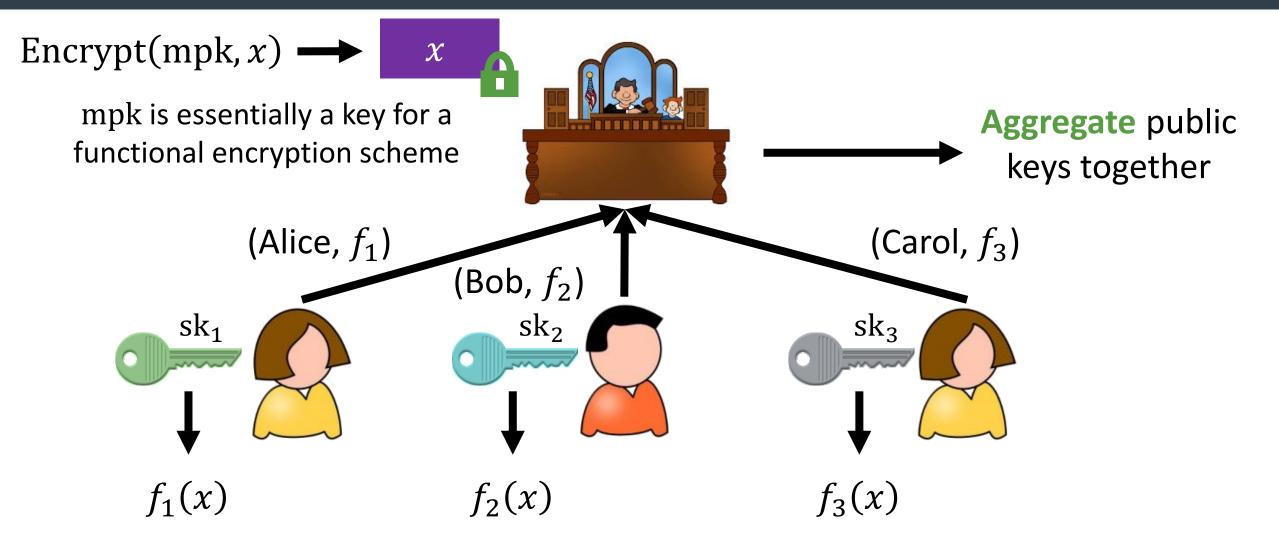


some coordination public parameters Encrypt(pp, $\{pk_i\}_{i \in S}, m$) \rightarrow ct Can encrypt a message *m* to any set of public keys **Efficiency:** $|ct| = |m| + poly(\lambda, log|S|)$ Decrypt(pp, $\{pk_i\}_{i \in S}$, sk, ct) $\rightarrow m$ Any secret key associated with broadcast set can decrypt

Decryption does requires knowledge of public keys in broadcast set

[GLWW23]: **distributed** broadcast encryption \Rightarrow **flexible** broadcast encryption

Removing Trust from Functional Encryption



Goal: Support capabilities of functional encryption **without** a trusted authority

Open Problems

Schemes with short CRS or unstructured CRS without non-black-box use of cryptography Existing constructions have long structured CRS (typically quadratic in the number of users)

Lattice-based constructions of registered FE (and special cases of FE) Registration-based encryption known from LWE [DKLLMR23] Registered ABE for circuits known from evasive LWE (via witness encryption) [FWW23]

Key revocation and verifiability

Defending against possibly malicious adversaries

Improve concrete efficiency for registered FE schemes

Current bottlenecks include large CRS and large public keys

Thank you!

References

- **[BSW11]** Dan Boneh, Amit Sahai, and Brent Waters. Functional Encryption: Definitions and Challenges. TCC 2011.
- [BZ14] Dan Boneh and Mark Zhandry. Multiparty Key Exchange, Efficient Traitor Tracing, and More from Indistinguishability Obfuscation. CRYPTO 2014.
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