Scaling Verifiable Computation Using Efficient Set Accumulators

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Problem: Verifiable Storage

- Represent a large storage (e.g. array) with a small digest
- Verifiably read and update the digest

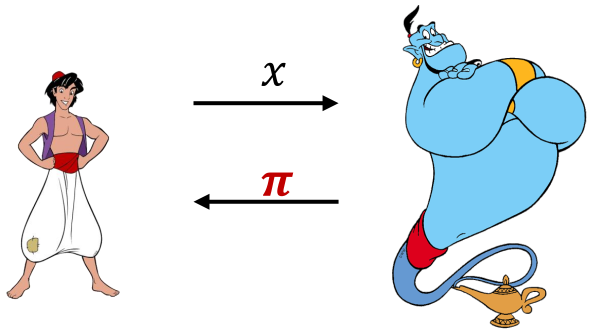
$$d \leftarrow Digest(A)$$

<u>Prover(A, d)</u>		<u>Verifier(d)</u>
$v \leftarrow A[i]$	$\xrightarrow{i, v, \pi_r}$	$Verify_{read}(d, i, v, \pi_r)$
$A[i_w] \leftarrow v_w$	$\xrightarrow{d', i_w, v_w, \pi_w}$	$Verify_{update}(d, i_w, v_w, d', \pi_w)$

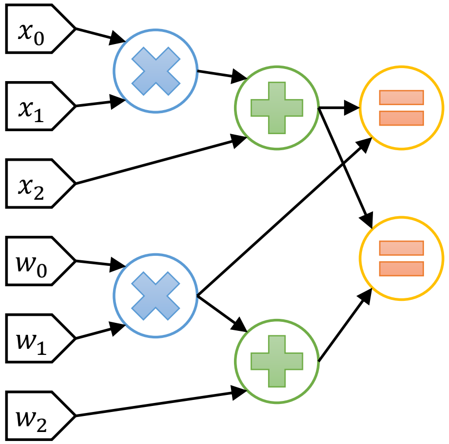
Context: Verifiable outsourcing/cryptographic proof systems

Our Work: Concretely cheaper verifiable storage using RSA accumulators

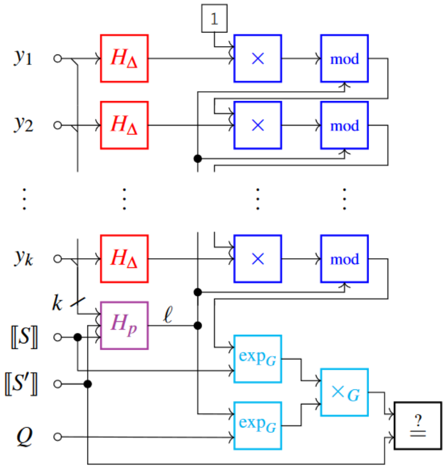
Cryptographic Proof Systems



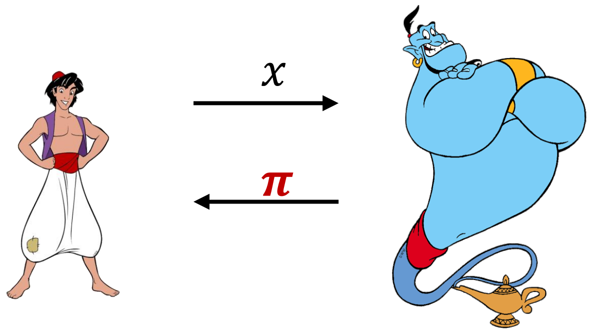
Programming Them



RSA Accumulators



Cryptographic Proof Systems



Programming Them

RSA Accumulators

NP Proof Systems

$L \in NP$
$(x \in L)$?

$x \longrightarrow$

$\longleftarrow w$

$w \leftarrow ?$

$\exists\, w. \quad \boldsymbol{V_L}(x, w)$?

Properties

- $|w| \in poly(|x|)$
- $T_{V_L} \in poly(|x|)$
- Aladdin learns w

Cryptographic Proof Systems: Abstract

$L \in NP$
$(x \in L)?$

x $\longrightarrow$

$\boldsymbol{\pi}$ $\longleftarrow$

$w \leftarrow ?$
$s.t.\ V_L(x, w) = \top$
$\boldsymbol{\pi} \leftarrow \boldsymbol{Prove}_{V_L}(x, w)$

$\boldsymbol{Verify}_{V_L}(\boldsymbol{\pi}, x \in L)$

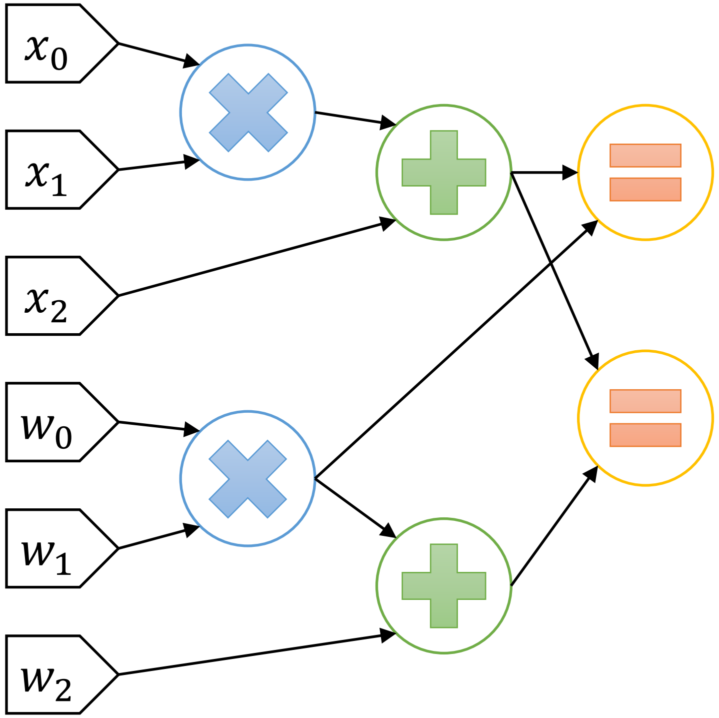
Extra Properties
- $|\pi| \in O(1)$
- $T_{Verify} \in O(|x|)$
- (Aladdin doesn't learn w)
- $T_{Prove} \in poly(T_{V_L})$

Using PCPs
+
Cryptography

Cryptographic Proof Systems: Concrete

L must be verifiable by an arithmetic constraint system (arithmetic circuit)

$$V_L(x, w)$$



Rank-1 Constraint Systems (R1CS)

- Constraints have the form
  $$A \times B = C$$
  where A, B, C are linear combinations of variables
- Prover time proportional to constraint count.

$$x_0(1 - x_0) = 0$$

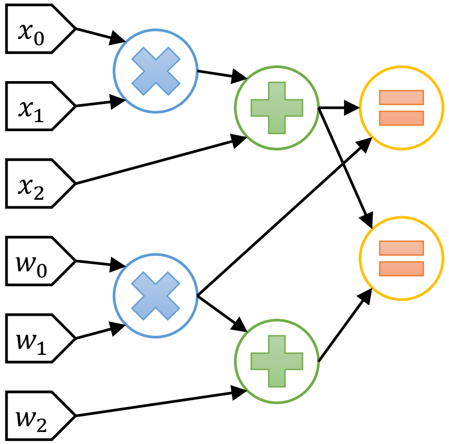
$$0 = w_0 + 2w_1 + 4w_2 - x$$

$$x_0 x_1 = w$$

$$x_0 x_1 x_2 = w$$

Cryptographic Proof Systems

Programming Them



RSA Accumulators

What Does Programming in R1CS Mean?

<u>Abstract Constraint</u>

$$z < 16$$

Variables encoded as field variables

Predicates encoded as constraints

"Programming"

<u>Rank-1 Constraints</u>

$$\begin{gathered} A_1 \times B_1 = C_1 \\ A_2 \times B_2 = C_2 \\ A_3 \times B_3 = C_3 \\ \vdots \\ A_n \times B_n = C_n \end{gathered}$$

Constraints may use witness variables

Inequality in R1CS

Abstract Constraint

$$z < 16$$

Encoded as the field variable $\boldsymbol{z}$

Rank-1 Constraints

$$w_0 \times (1 - w_0) = 0$$
$$w_1 \times (1 - w_1) = 0$$
$$w_2 \times (1 - w_2) = 0$$
$$w_3 \times (1 - w_3) = 0$$
$$0 = w_0 + 2w_1 + 4w_2 + 8w_3 - z$$

Polynomial Multiplication

Abstract Constraint

$$f(x) \cdot g(x) = h(x)$$

Each coefficient is a field variable:

- $f(x) = f_0 + f_1 x + f_2 x^2$
- $g(x) = g_0 + g_1 x + g_2 x^2$
- $h(x) = h_0 + h_1 x + h_2 x^2 + h_3 x^3 + h_4 x^4$

Rank-1 Constraints

$$(f_0 + f_1 + f_2)(g_0 + g_1 + g_2) = h_0 + h_1 + h_2 + h_3 + h_4$$
$$(f_0 + 2f_1 + 4f_2)(g_0 + 2g_1 + 4g_2) = h_0 + 2h_1 + 4h_2 + 8h_3 + 16h_4$$
$$(f_0 + 3f_1 + 9f_2)(g_0 + 3g_1 + 9g_2) = h_0 + 3h_1 + 9h_2 + 27h_3 + 81h_4$$
$$(f_0 + 4f_1 + 16f_2)(g_0 + 4g_1 + 16g_2) = h_0 + 4h_1 + 16h_2 + 64h_3 + 256h_4$$
$$(f_0 + 5f_1 + 25f_2)(g_0 + 5g_1 + 25g_2) = h_0 + 5h_1 + 25h_2 + 125h_3 + 625h_4$$

Check $f(a) \cdot g(a) = h(a)$
for different a

Big Natural Multiplication

Abstract Constraint

$$x \cdot y = z$$

Represent naturals with limbs, base b. Each limb is a field element.

- $x = x_0 + x_1 b + x_2 b^2$
- $y = y_0 + y_1 b + y_2 b^2$
- $\mathrm{z} = z_0 + z_1 b + z_2 b^2 + z_3 b^3 + z_4 b^4 + z_5 b^5$

Rank-1 Constraints Sketch

$$carry\Big(nat\big(poly(x) \times poly(y)\big)\Big) = z$$

~ a ripple-carry adder from digital architecture (range checks!)

Big Natural Division

Abstract Constraint

$$\lfloor y/x \rfloor = q$$

Represent naturals with limbs, base b. Each limb is a field element.

- $x = x_0 + x_1 b + x_2 b^2$
- $y = y_0 + y_1 b + y_2 b^2$
- $q = q_0 + q_1 b + q_2 b^2$

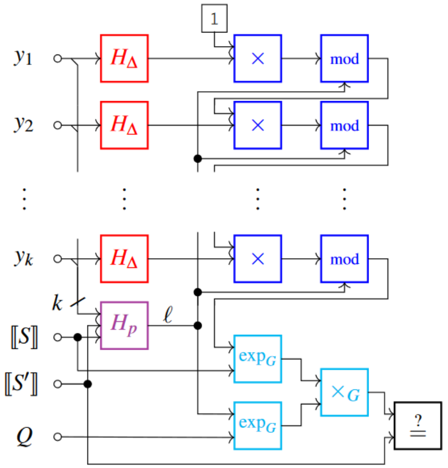
Rank-1 Constraints Sketch

$$\exists r. \quad y = xq + r$$

Cryptographic Proof Systems

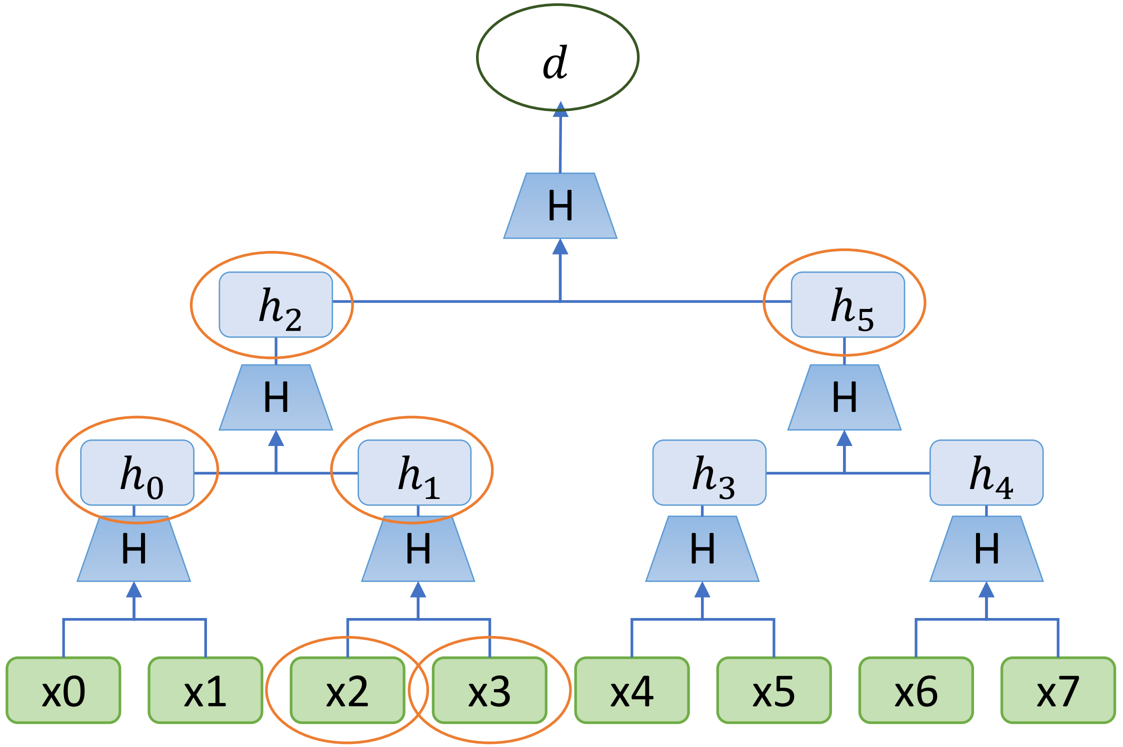
Programming Them

RSA Accumulators



The Competition: Merkle Trees

- Based on a hash function $H: F \times F \to F$
  - Collision-Resistant
- Reduce the array to a single value with a hash-tree
- Proofs based on paths in the tree



Verification cost: (roughly) $\boldsymbol{k \log m}$ **hashes**
for k updates and a storage of capacity m.

RSA Accumulators

- Based on RSA groups
  - The integers modulo pq: the produce of two unknown primes.
  - Hard to compute roots.
    - x^n is easy, $\sqrt[n]{x}$ is hard.
- The digest of an RSA Accumulator is

The stored elements

$$d = g^{\prod_i H_\Delta(y_i)}$$

Fixed generator

A (special) hash function

RSA Accumulator Proofs

- Insertion proof:
  - Verifier checks an exponentiation

$$d' = d^{H_\Delta(y)}$$

- Removal proof:
  - Insertion in reverse
- Membership proof:
  - A removal proof, but the new digest is forgotten
  - Sound because computing roots is hard!

Batched RSA Accumulator Proofs

- Batches require two small exponentiations [BBF 18]/[Wes 18]
  - Requires a hash function to prime numbers (for non-interactivity)

Prover | | Verifier

$$d' = d^{\prod_i H_\Delta(y_i)}$$

$$\ell \leftarrow\!\!\!\sim \text{Primes}$$

$$\xleftarrow{\ \ell\ }$$

$$Q \leftarrow d^{\left\lfloor \prod_i H_\Delta(y_i) / \ell \right\rfloor}$$

$$\xrightarrow{\ Q\ }$$

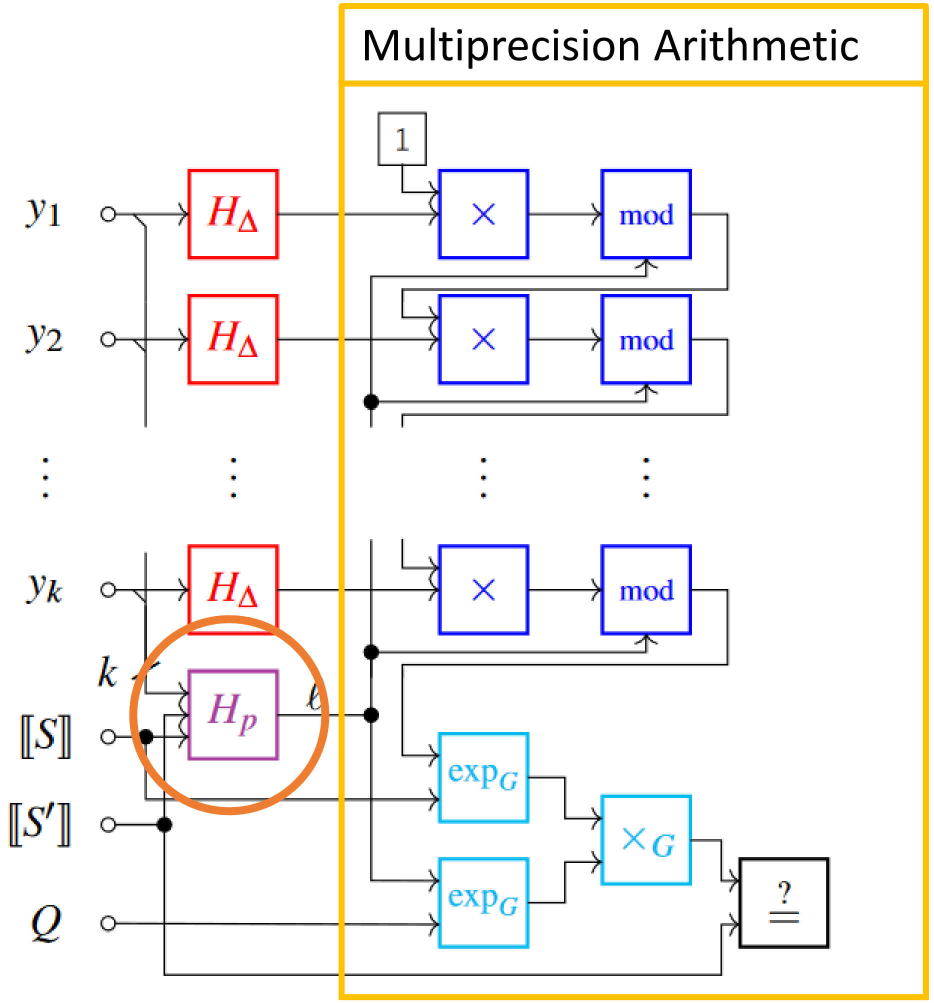
$$d' = Q^\ell \cdot d^{\prod_i H_\Delta(y_i) \% \ell}$$

Verification cost: $\boldsymbol{k}$ (**hashes** & **modular** $\times$) $+$ **2 exponentiations** for k updates and a storage of capacity m.

RSA Accumulator Circuit Overview

$\ell \leftarrow H_p(\ldots)$

$d' = Q^{\ell} \cdot d^{\prod_i H_\Delta(y_i) \% \ell}$



Traditional Hash-to-Prime

- Rejection sampling of primes
- Miller Rabin primality test
  - Probabilistic!
  - $2^{-\lambda}$ soundness uses $O(\lambda)$, $\tilde{O}(\lambda)$-bit exponentiations
  - Many constraints

procedure **HashToPrime**(x):

  $g \leftarrow PRG(seed = x)$

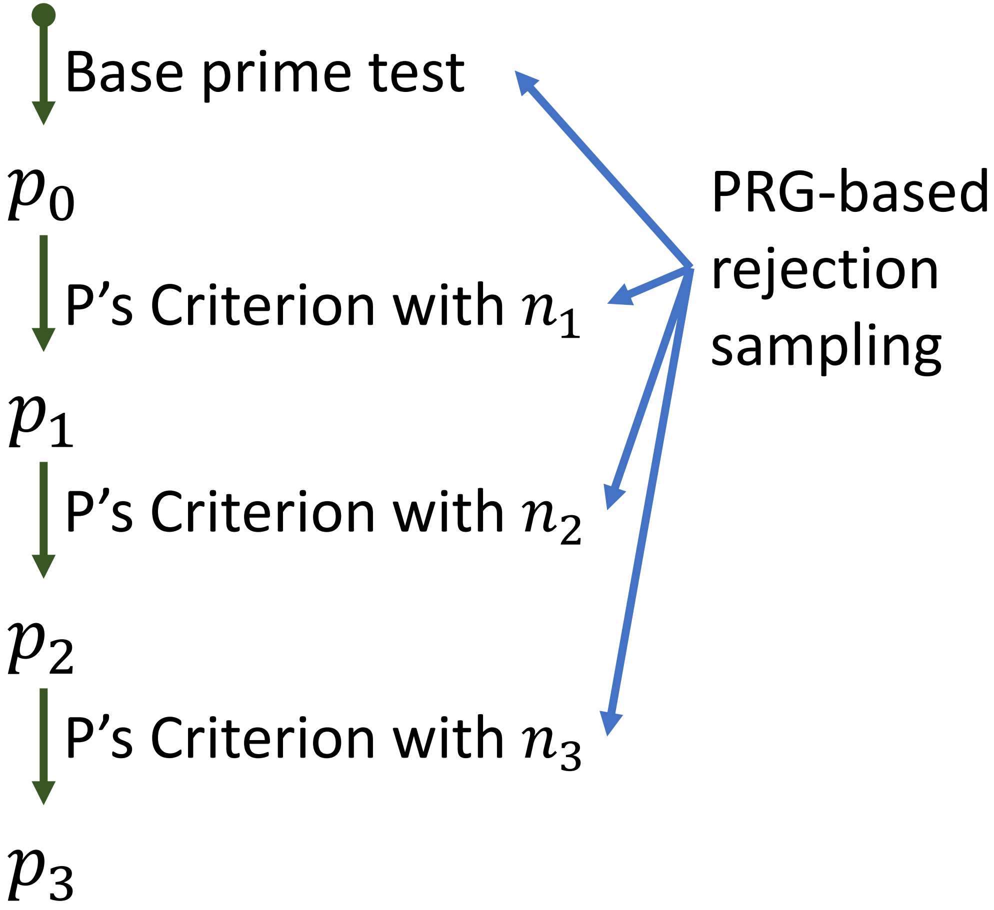
  while g.output() is composite:

    g.advance()

  Return g.output()

Pocklington Prime Generation

- Pocklington's criterion:
  - If
    - p is prime
    - $n < p$
    - $\exists a.\, a^{np} \equiv_{np+1} 1 \wedge \gcd(a^n - 1, np + 1) = 1$
  - Then $np + 1$ is prime
- Basis for a recursive primality certificate
  - Idea: Rejection sampling of prime certificates



Many fewer constraints than Miller-Rabin, and provably prime

Other Techniques and Tricks

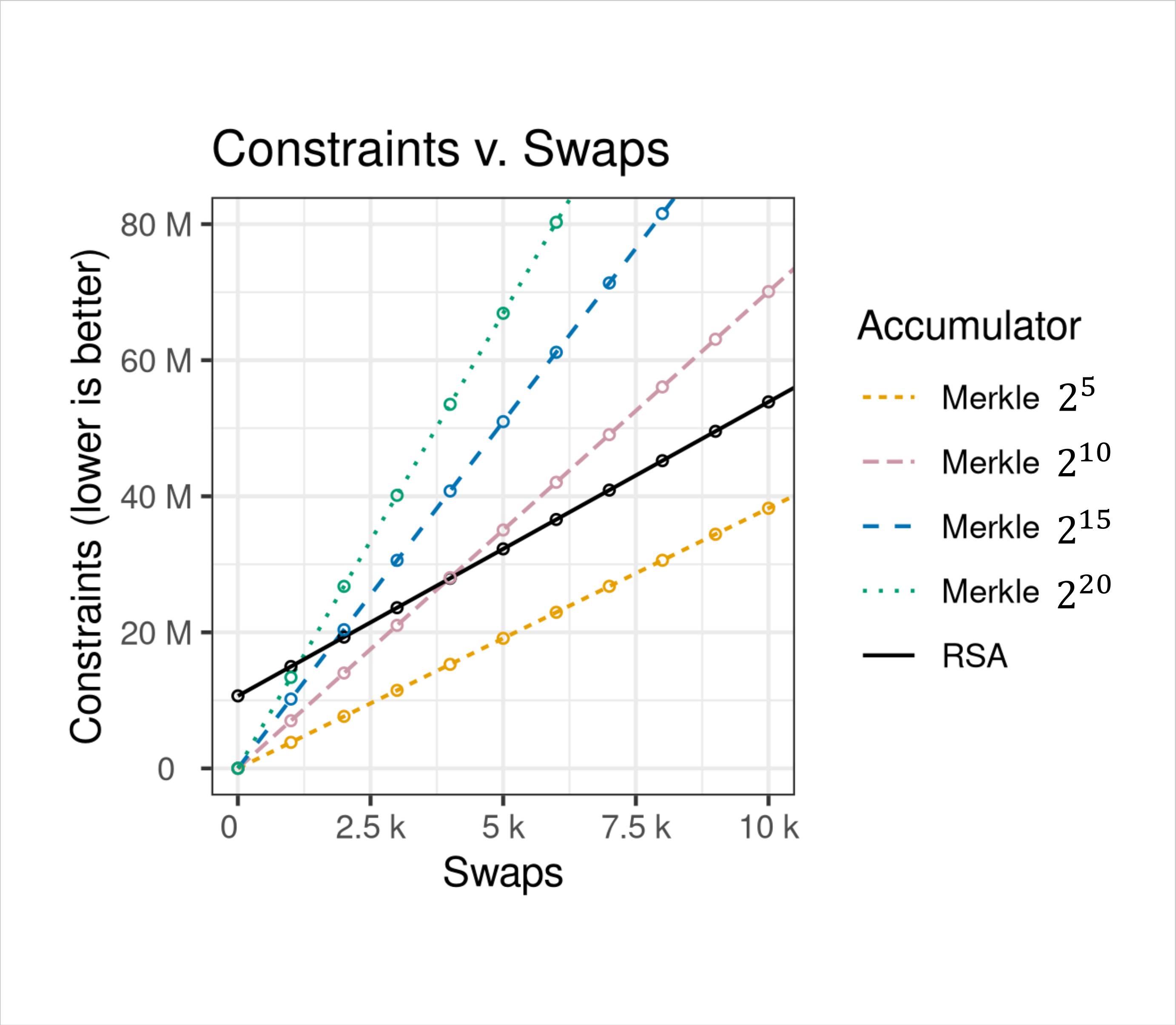
- Optimizations for multiprecision arithmetic in constraints
  - Based on xjSnark [KPS 18]

- A new hash function, conjectured to be division-intractable

- Precise semantics for batching dependent accesses.

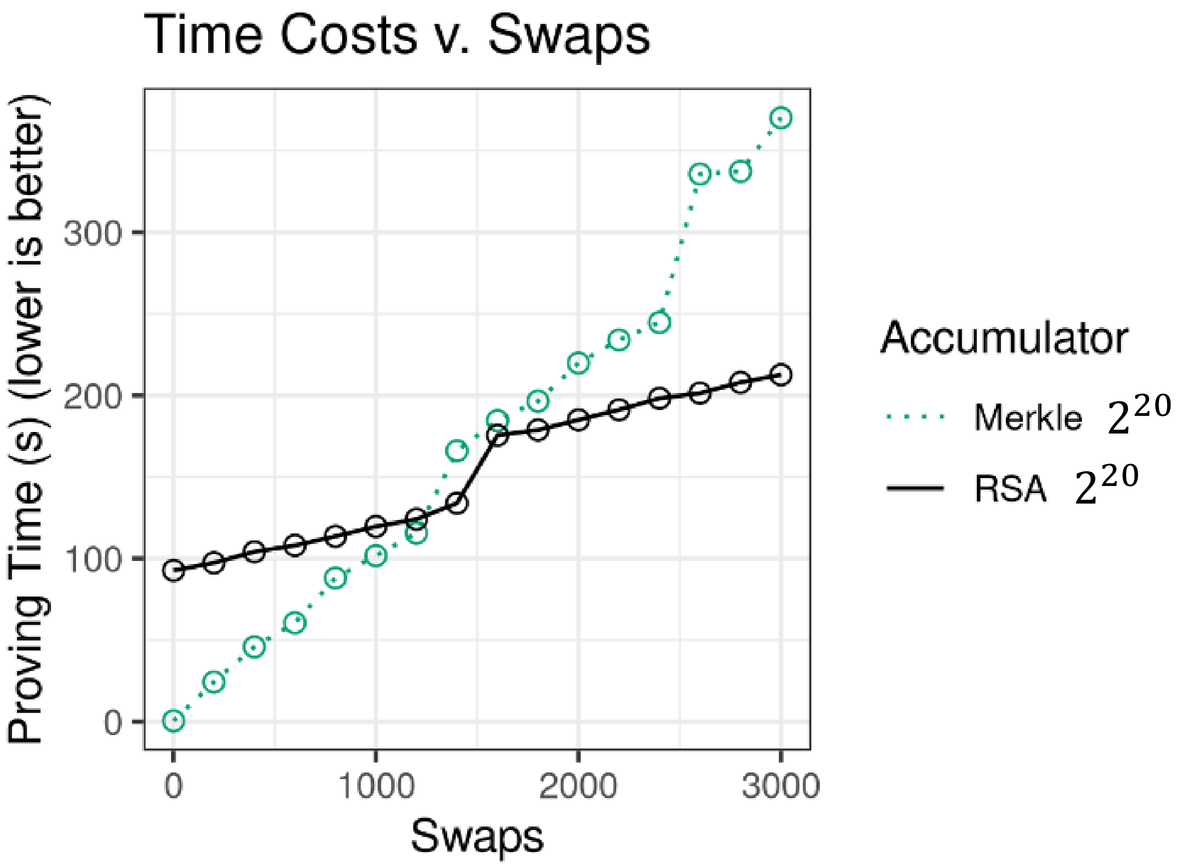
Evaluation: Constraints

- Implementation in Bellman, using Groth16.
- Consider storage of varying size
- Perform varying numbers of *swaps* (remove x, add y)
- Measure constraints
- Crossover occurs at a few thousand operations



Evaluation: Prover Time

- Includes RSA accumulator removal time (≈43s)
  - Computing d' such that $d = d'^{\prod_i H_\Delta(y_i)}$
  - Independent of batch size, linear in storage size.
- Machine info:
  - 48 logical cores
  - 132GB memory



Future Directions

- Better investigation of concrete prover costs
- Integration with the proof system
  - Direct support for range-proofs ($z < 2^{32}$)
  - Arithmetic circuits over $\mathbb{Z}/pq\mathbb{Z}$ (crazy?)
- Managing non-proof prover costs
  - Multi-tiered accumulators?
  - Hybrid RSA-Merkle accumulators?

Summary

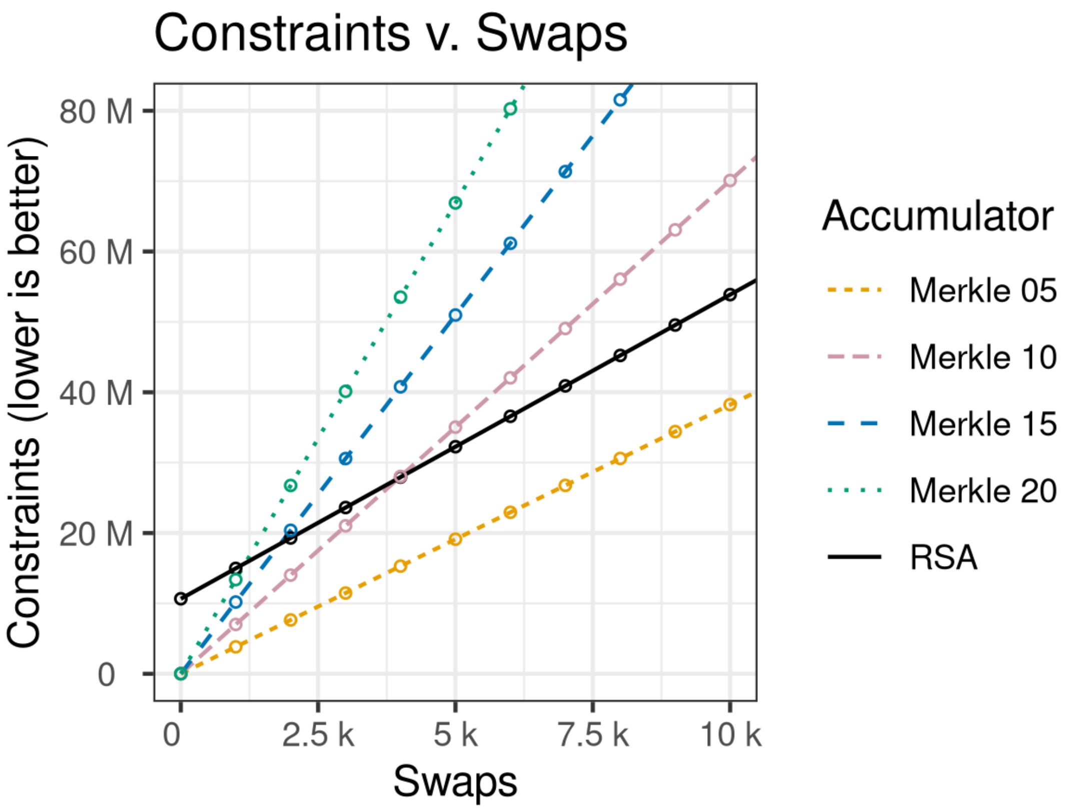
Research Question

Do RSA accumulators use fewer constraints than Merkle Trees?

Techniques

- Multiprecision arithmetic
- Division-intractable hashing
- Hashing to prime numbers
- Semantics of dependent accesses

Conclusions



Paper: [ia.cr/2019/1494](ia.cr/2019/1494)
Implementation: github.com/alex-ozdemir/bellman-bignat