Programmable Cryptography: The **Full Stack**

(via cryptography, compilers, and verification)

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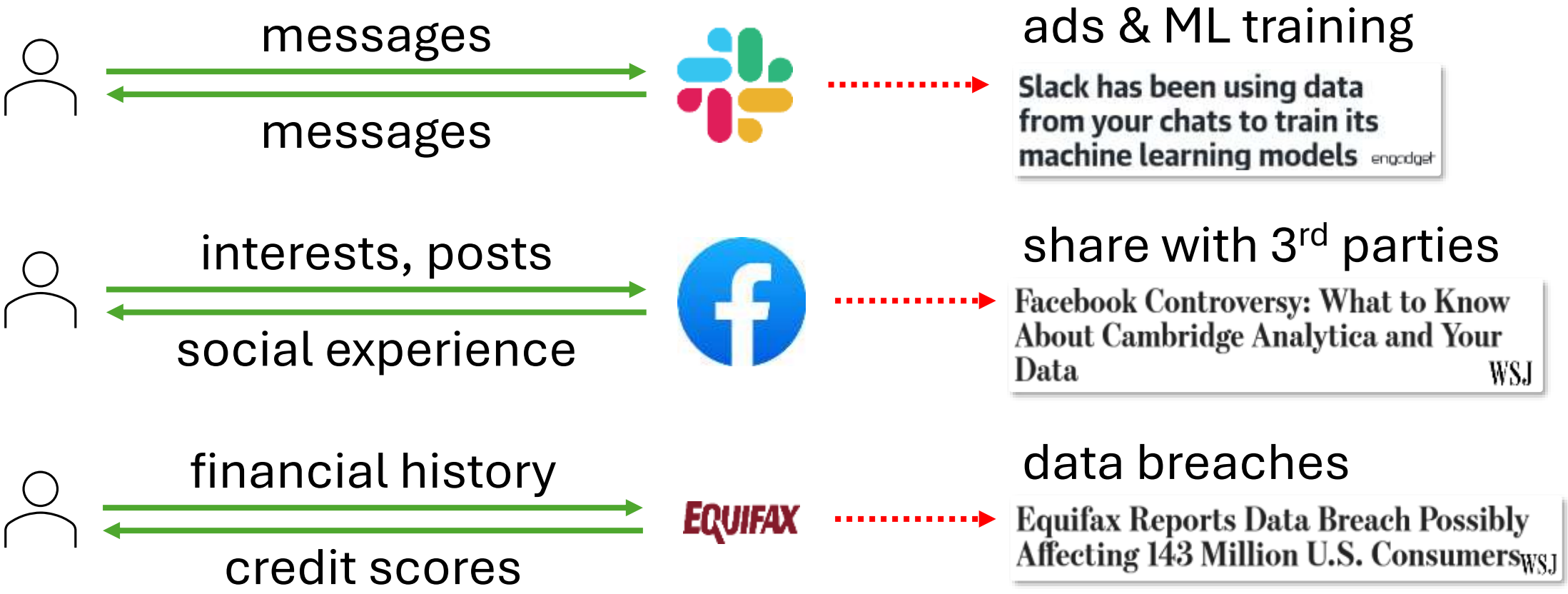
May 30th, 1pm PT

Based on joint work with: Clark Barrett, Dan Boneh, Fraser Brown, Gereon Kremer, Cesare Tinelli, Riad S. Wahby

To use the internet is to be vulnerable



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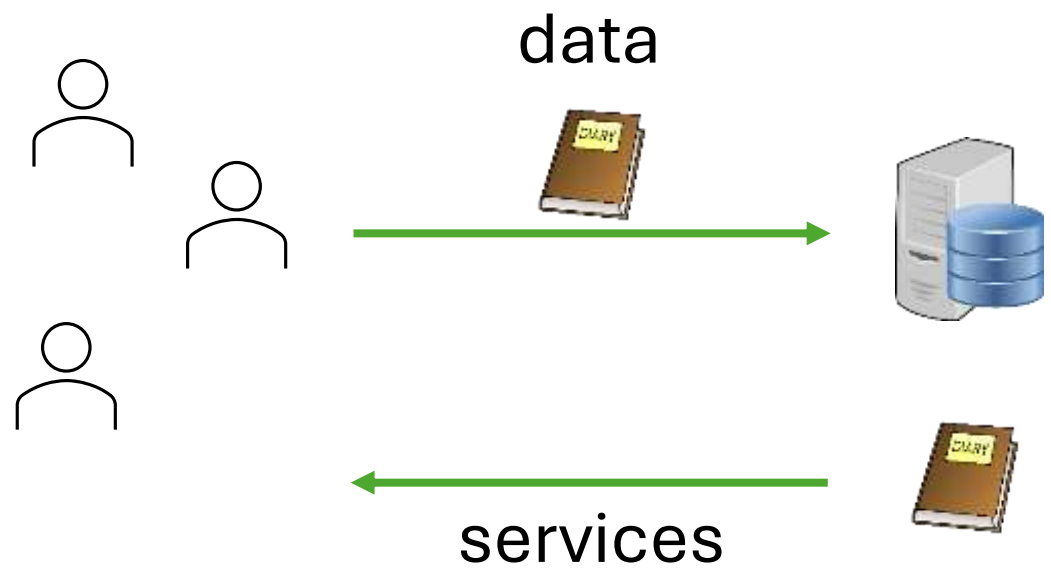
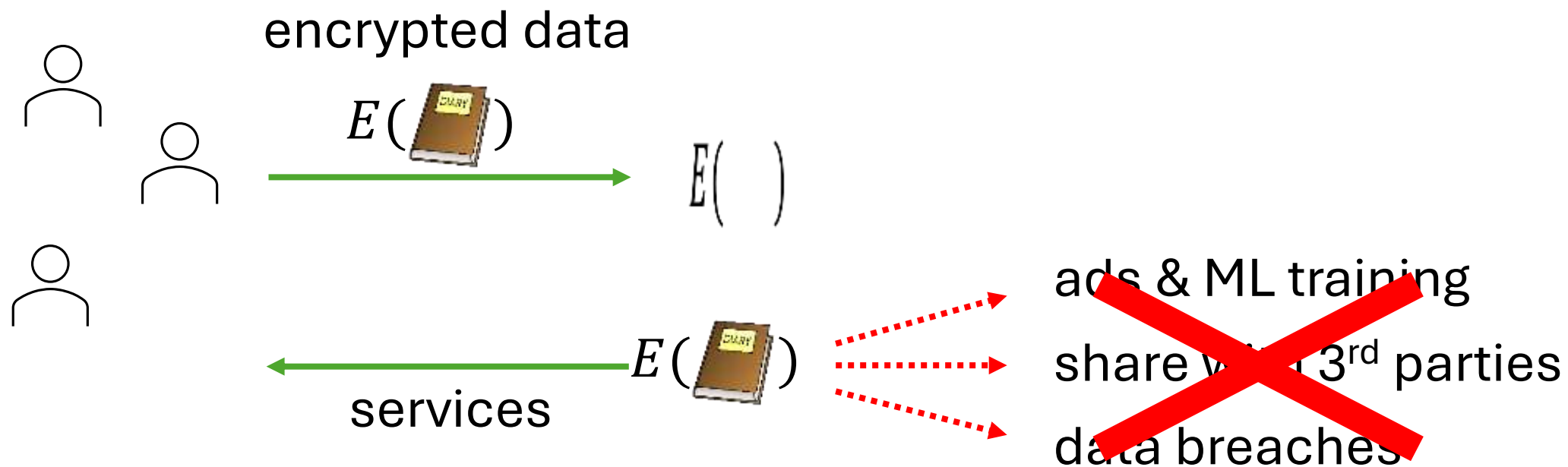


ads & ML training

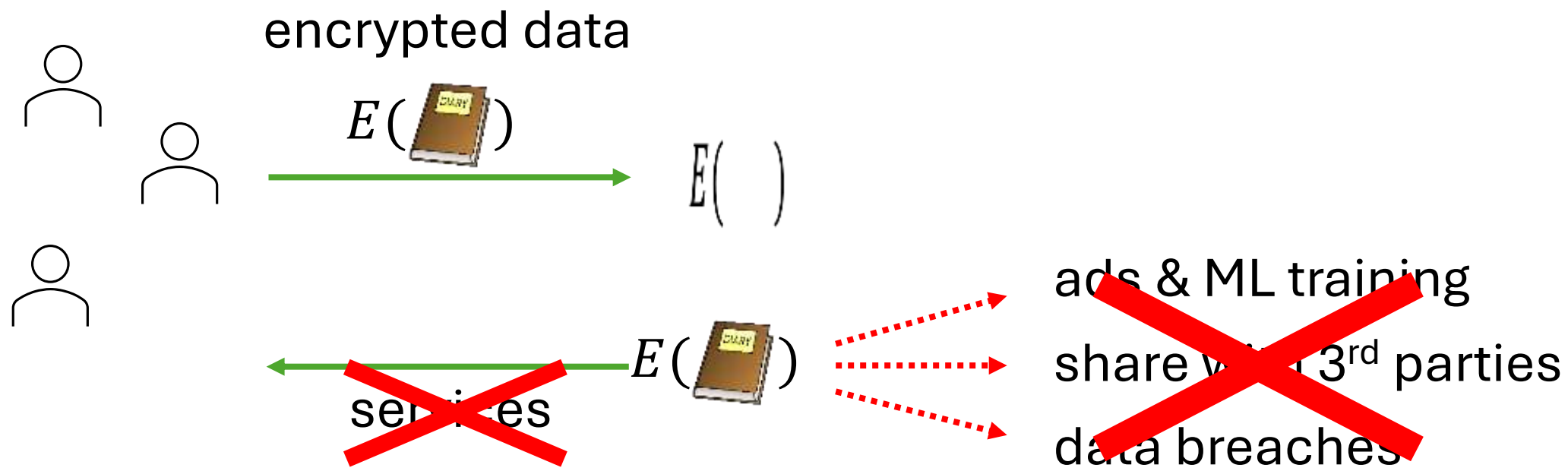
share with 3rd parties

data breaches

Encryption secures data



Encryption secures data, but prevents use



Programmable cryptography enables use

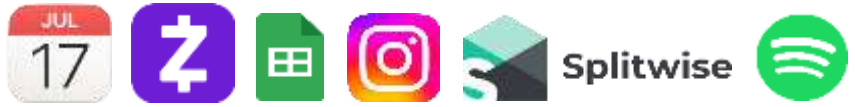
A **programmable cryptosystem** securely executes a developer-defined program on secret data.

Examples:

FHE	ZKP	MPC
Fully Homomorphic Encryption	Zero Knowledge Proof	Multi Party Computation
compute on encrypted data	**prove** properties of secret data	**compute** on distributed data
$E(x) \rightarrow E(f(x))$	$E(x) \rightarrow \phi(x)$	$x \parallel y \rightarrow f(x, y)$

Encryption secures ***data***.

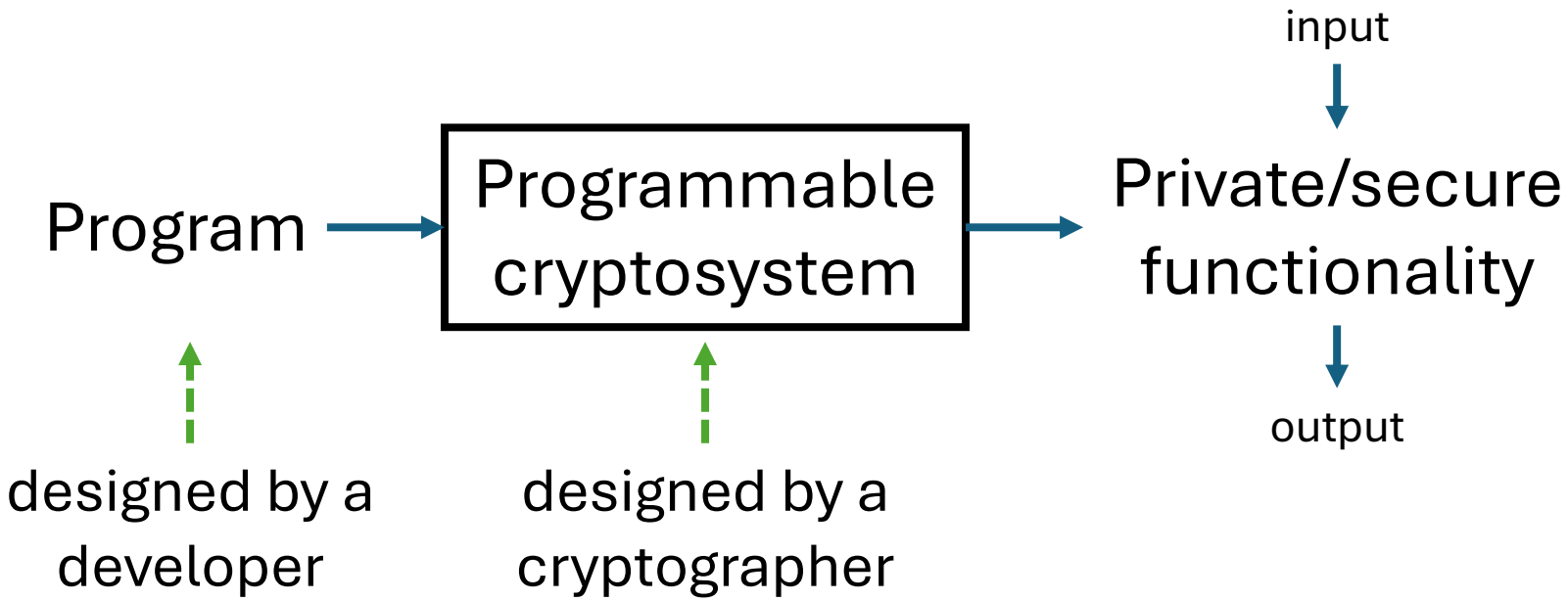
Programmable cryptography secures ***data in use***.



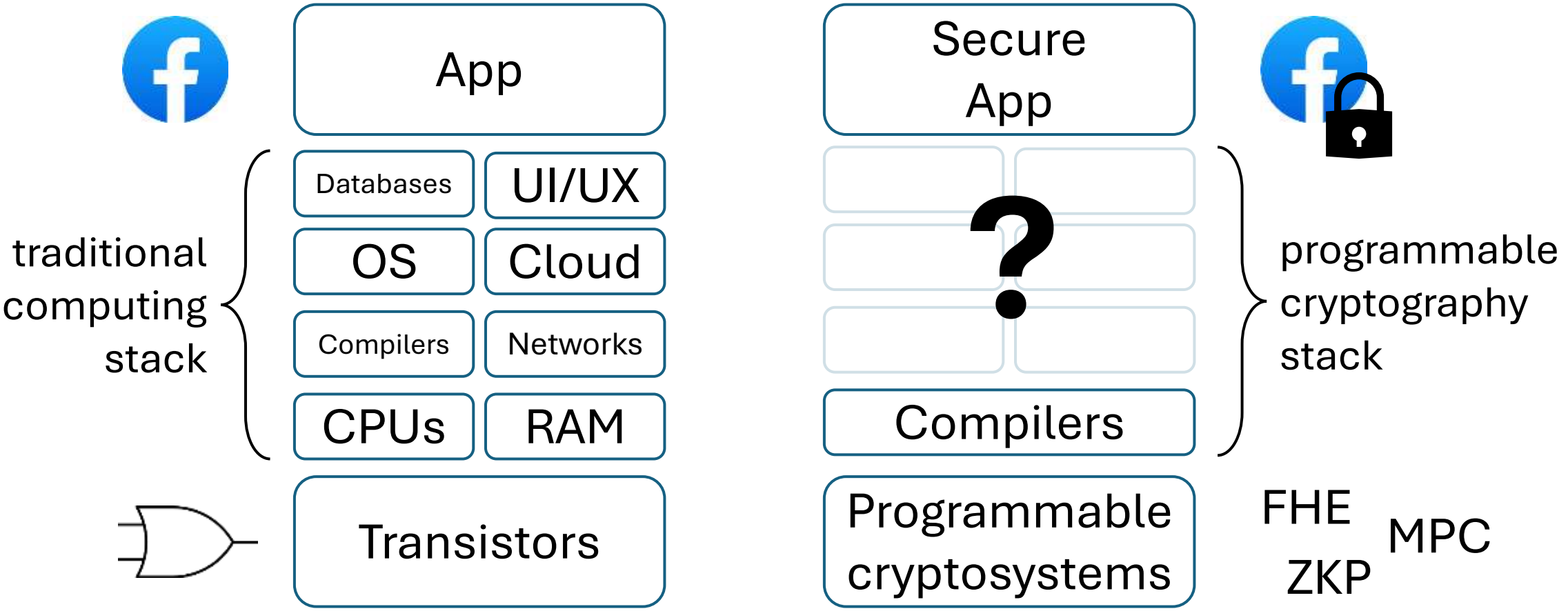
[Gentry'09] [RivAdlDer'78] [GolMicRac'85] [Yao'82]

Programmable cryptography is general

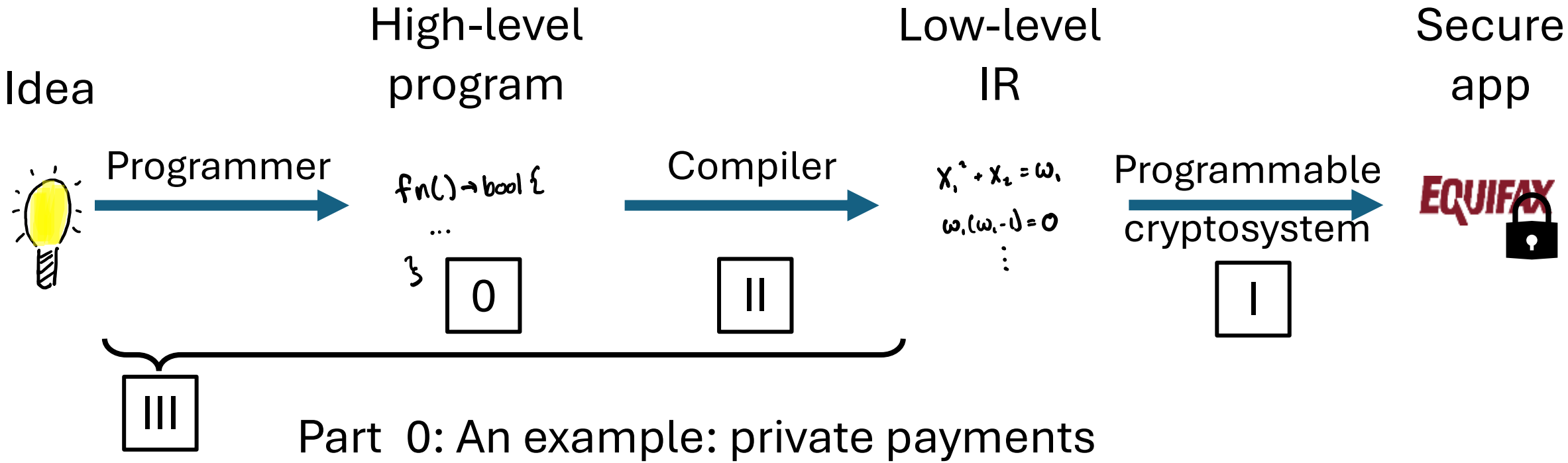
A **programmable cryptosystem** securely executes a **developer-defined program** on secret data.



Programmable cryptography needs a **stack**



My PhD work:



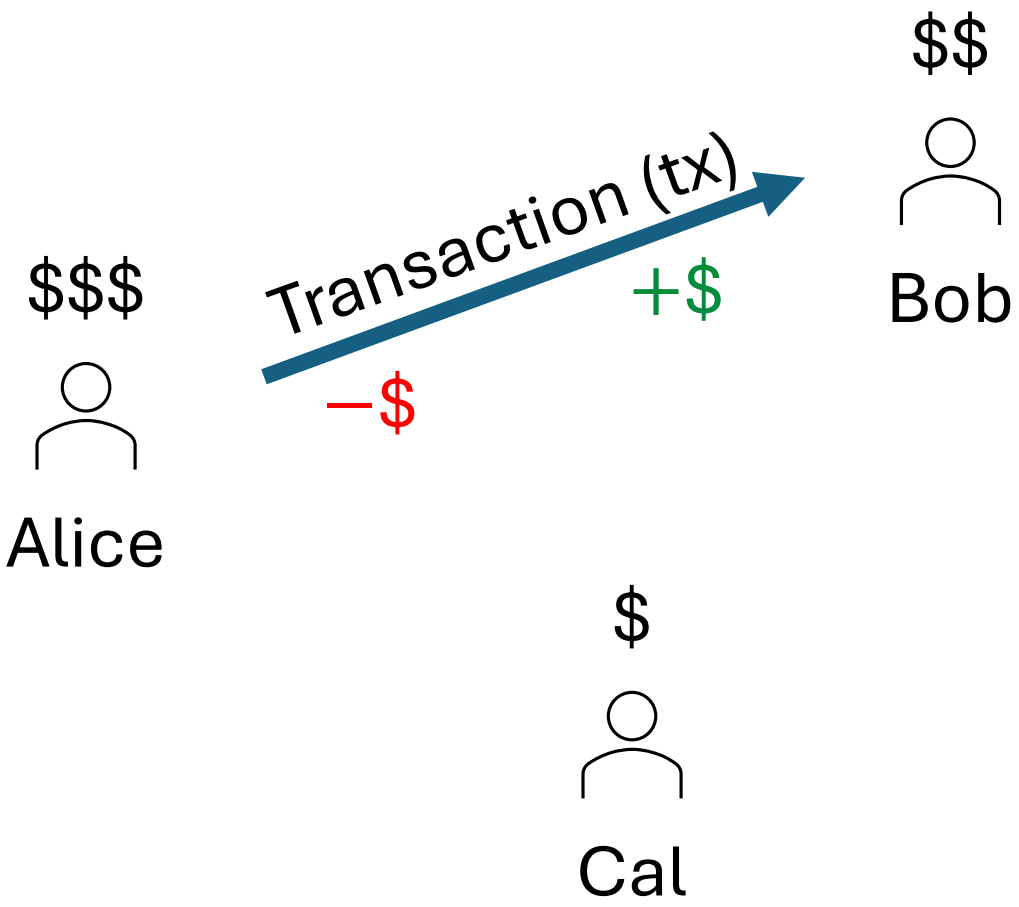
Part 0: An example: private payments

Part I: Collaborative zero knowledge (cryptography)

Part II: Common compiler infrastructure (compilers)

Part III: Verification via finite field solvers (verification)

Running Example: Private Payments



Security properties:

Privacy:
- only owners know balances
- only participants know tx amounts

Integrity:
- money is conserved
- all balances are positive
- participants authorize every tx

First attempt: payments from a public log

Alice has d_A \$.

Alice sends $\delta \leq [0, d_A]$ \$.

Public log

$A: d_A$

$B: d_B$

$A: d_A - \delta$

$B: d_B + \delta$

σ_A, σ_B

Bob has d_B \$.

all data is public
⇒ no ***privacy***

users check log
⇒ easy ***integrity***

Second attempt: just encrypt the log?

Alice has d_A \$.

$e_A = E(k_A, d_A)$

Alice sends $\delta \leq [0, d_A]$ \$.

$e'_A = E(k_A, d_A - \delta)$

Public log

e_A

e_B

e'_A

e'_B

σ_A, σ_B

Bob has d_B \$.

$e_B = E(k_B, d_B)$

$e'_B = E(k_B, d_B + \delta)$

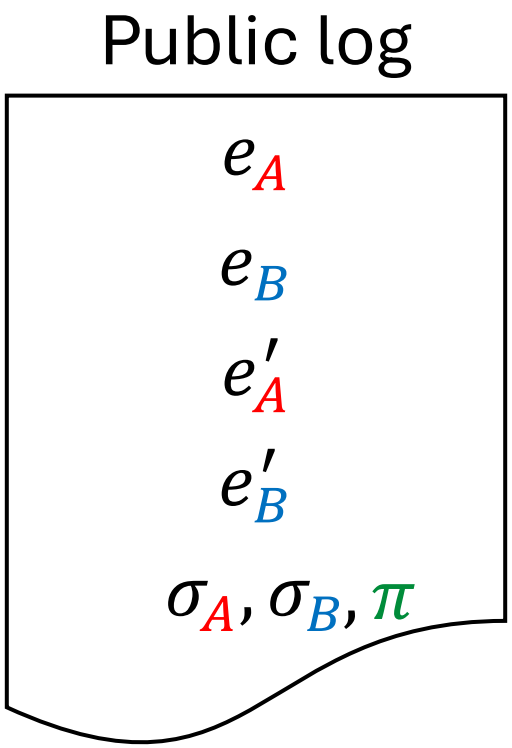
What if $e'_B = E(k_B, d_B + 2\delta)$?

Balances is encrypted
⇒ easy ***privacy***

Can't validate txs
⇒ no ***integrity***

*Let E be a suitable cipher (e.g., AEAD + CMT-4). "Confidential Transactions" [Maxwell '15]

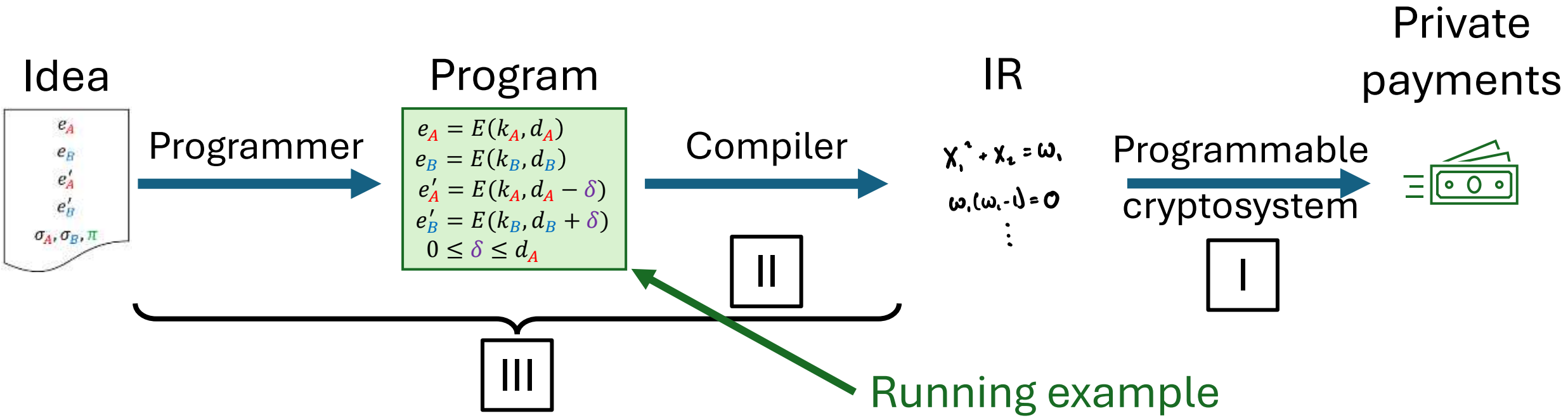
Fixing it, with programmable cryptography



Idea: add a zero knowledge proof (ZKP) π that the updates are correct.

$$e_A = E(k_A, d_A)$$
$$e_B = E(k_B, d_B)$$
$$e'_A = E(k_A, d_A - \delta)$$
$$e'_B = E(k_B, d_B + \delta)$$
$$0 \leq \delta \leq d_A$$

The full stack of programmable cryptography



Part I: Collaborative zero knowledge

Part II: Common compiler infrastructure

Part III: Verification via finite field solving

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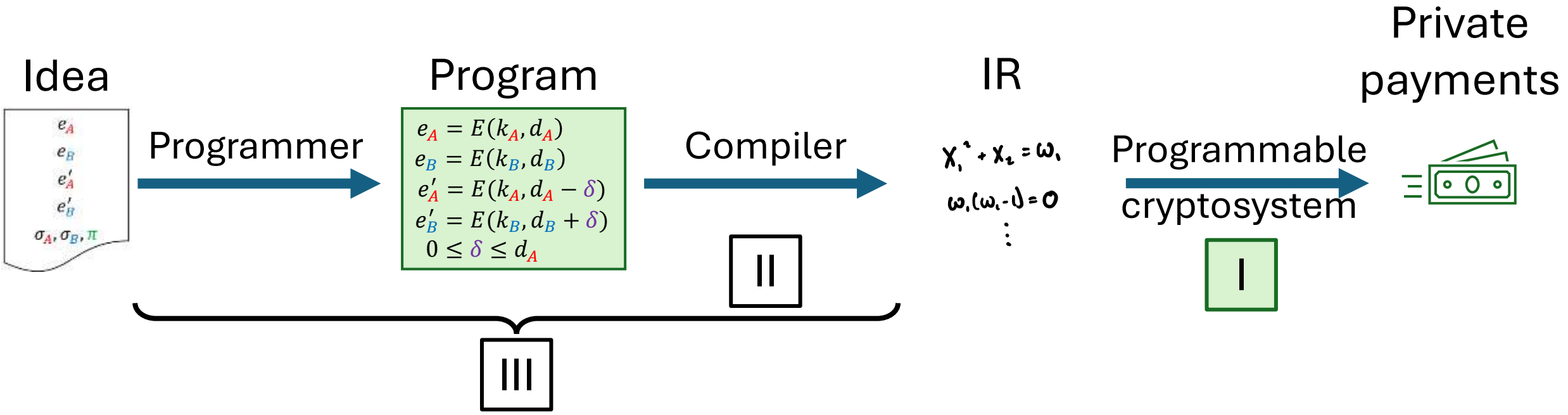


Part I: Collaborative zero knowledge [**Ozdemir**, Boneh; USENIX Security '22]

Part II: Common compiler infrastructure

Part III: Verification via finite field solving

What is a traditional ZKP?

Consider a property $P(x, w)$

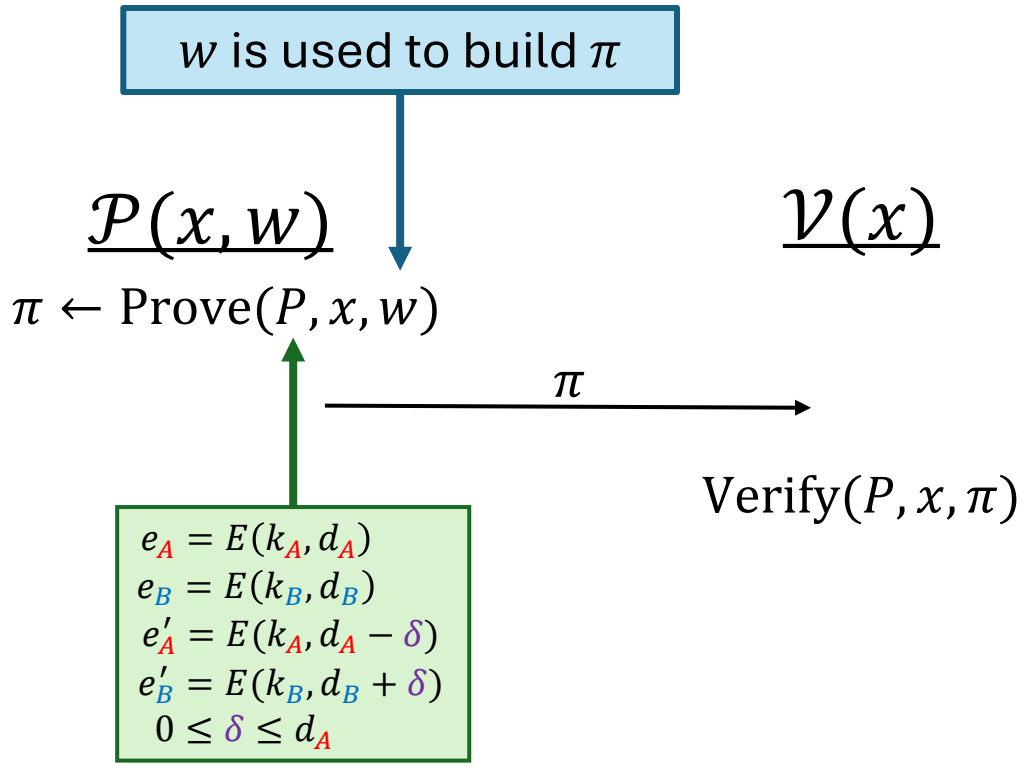
public data x (e_A, e_B, e'_A, e'_B)

secret data w $(d_A, d_B, \delta, k_A, k_B)$

A ZKP is two algorithms:

$\text{Prove}(P, x, w) \rightarrow \pi$

$\text{Verify}(P, x, \pi) \rightarrow \{\text{accept}, \text{reject}\}$



Security properties (informal):

complete: a valid w yields a valid π

sound: constructing a valid π is infeasible without a valid w

zero-knowledge: π reveals no information about w

Problem: who creates the ZKP?

$w = (d_A, d_B, \delta, k_A, k_B)$ is used to build π

Alice knows some of w.

Bob knows some of w.

Problem: no **single party** can create a ZKP, because the secrets are **distributed**.

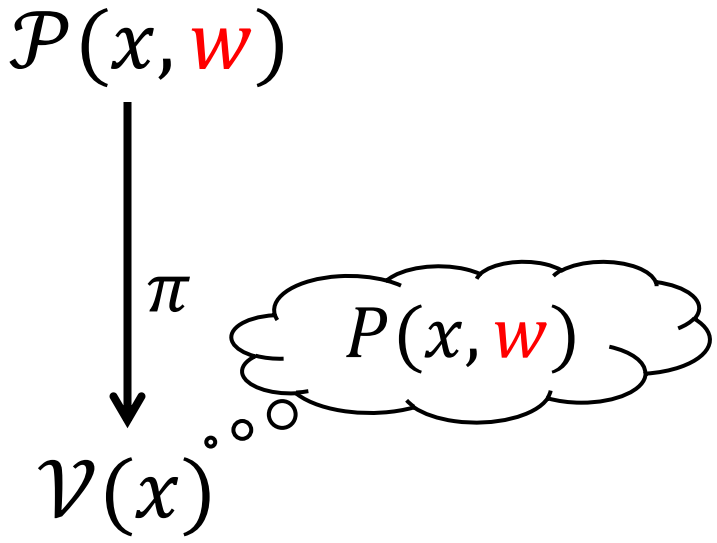
Sending w to one party is **not private**.

Contributions:

1. A new definition: the **collaborative ZKP**
2. A very efficient construction

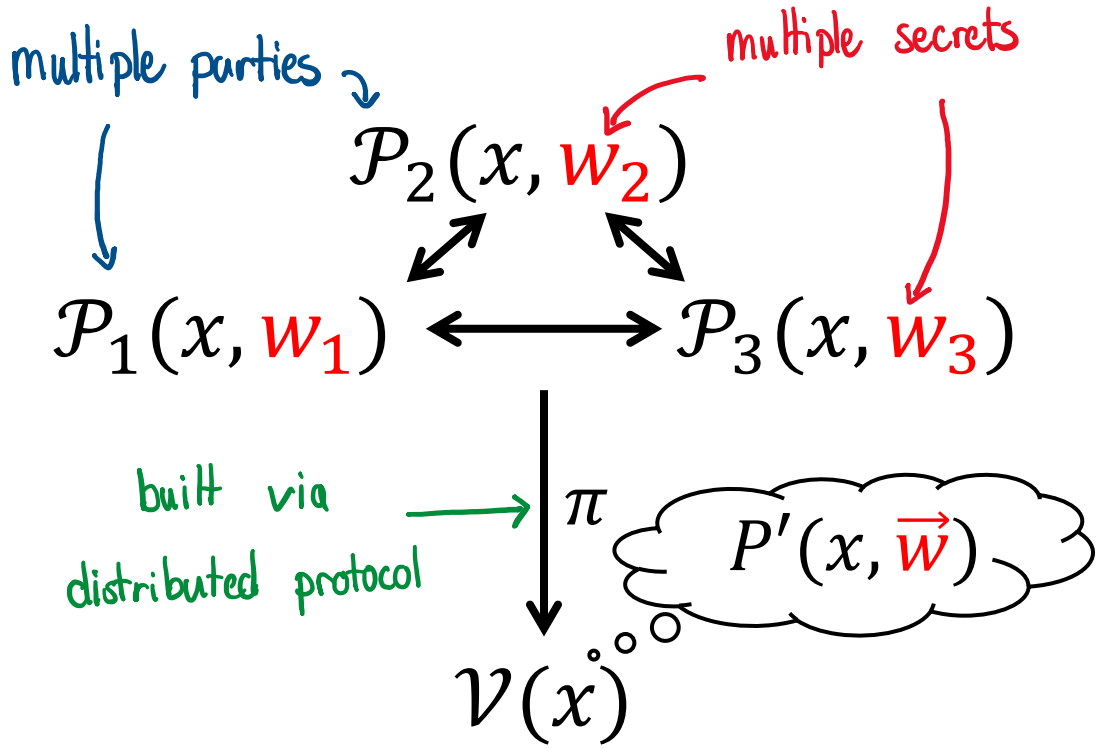
Our definition: Collaborative ZKPs

Prior: ZKPs



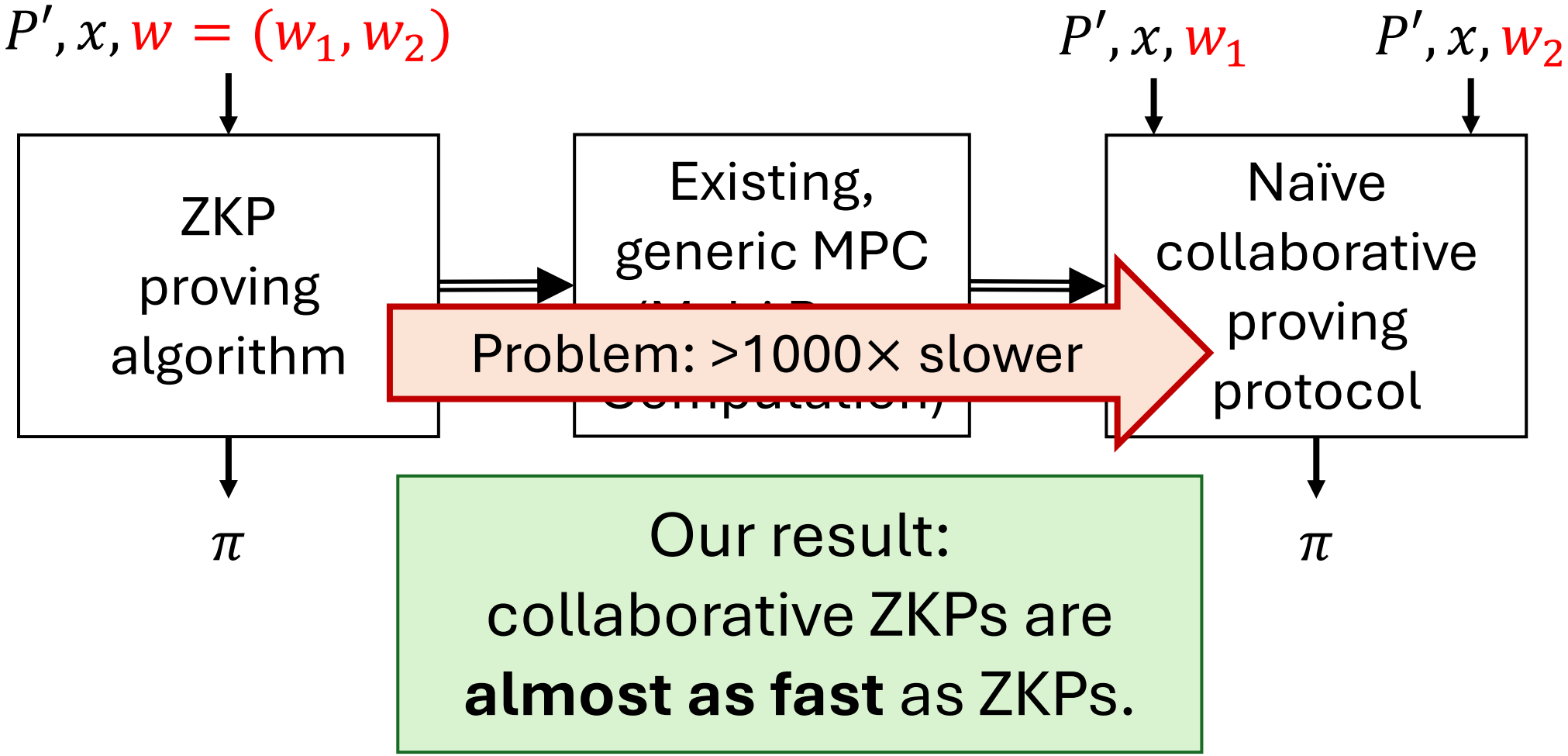
sound, complete, hides w from $\mathcal{V}$

Collaborative ZKPs



also hides w_i from $\mathcal{P}_j$

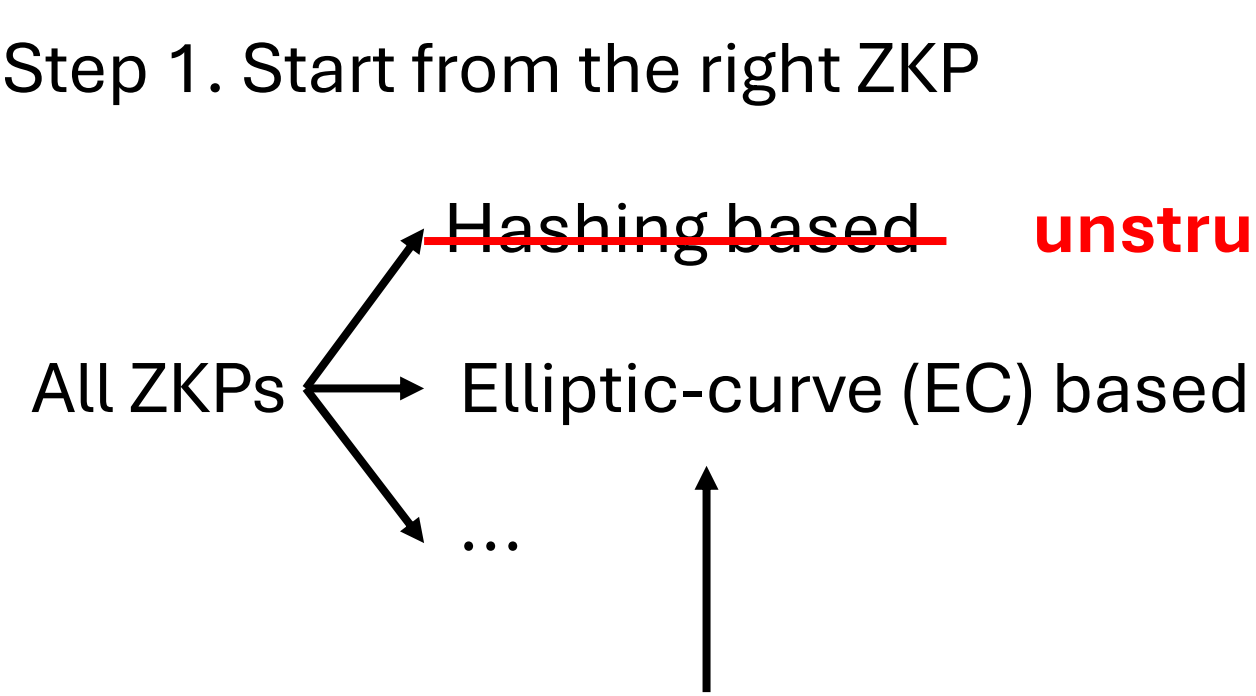
A naïve protocol



Our result:
collaborative ZKPs are
almost as fast as ZKPs.

Choosing the right starting point

Step 1. Start from the right ZKP



the slowest part of proving

unstructured bottleneck

algebraic bottleneck

$$H \leftarrow \boxplus_{i=1}^{N} (z_i \boxdot G_i)$$

scalar
secret
(encodes $\vec{w}$)

EC point
public
(encodes P)

An EC: a set of p points $G, H, \ldots$ with

- addition ($G \boxplus H$)
- multiplication by scalars ($z \boxdot G$)
  - $z \in \mathbb{Z}_p, \ \mathbb{Z}_p = \{0, \ldots, p-1\}$

Intuition: how to handle the bottleneck

Step 2. Solve the bottleneck with $\mathbb{Z}_p$ **and EC** secret sharing

$$H \leftarrow \boxplus_{i=1}^{N} (z_i \boxdot G_i)$$

		Local computation	Result:
Prover 1:	$\forall i$, gets random $z_{i,1}$	$H_1 \leftarrow \boxplus_{i=1}^{N} (z_{i,1} \boxdot G_i)$	
Prover 2:	$\forall i$, gets $z_{i,2} \leftarrow z_i - z_{i,1}$	$H_2 \leftarrow \boxplus_{i=1}^{N} (z_{i,2} \boxdot G_i)$	$H = H_1 \boxplus H_2$

(note: $z_i = z_{i,1} + z_{i,2}$)

A $\mathbb{Z}_p$-linear sharing of z_i

An EC-linear sharing of H

Evaluated the bottleneck **without communication**

Intuition: how to handle the bottleneck

Step 2. Solve the bottleneck with $\mathbb{Z}_p$ **and EC** secret sharing

$H \leftarrow \boxplus_{i=1}^{N} (z_i \boxdot G_i)$

Zooming out:

- This is one (key) step in a big protocol
- Malicious security is necessary
- Sketch: lift SPDZ & GSZ'20 to ECs

Result:

Prover 1: $\forall i$, gets

Prover 2: $\forall i$, gets $z_{i,2} \leftarrow z_i - z_{i,1}$ $H_2 \leftarrow \boxplus_{i=1}^{N} (z_{i,2} \boxdot G_i)$

$H = H_1 \boxplus H_2$

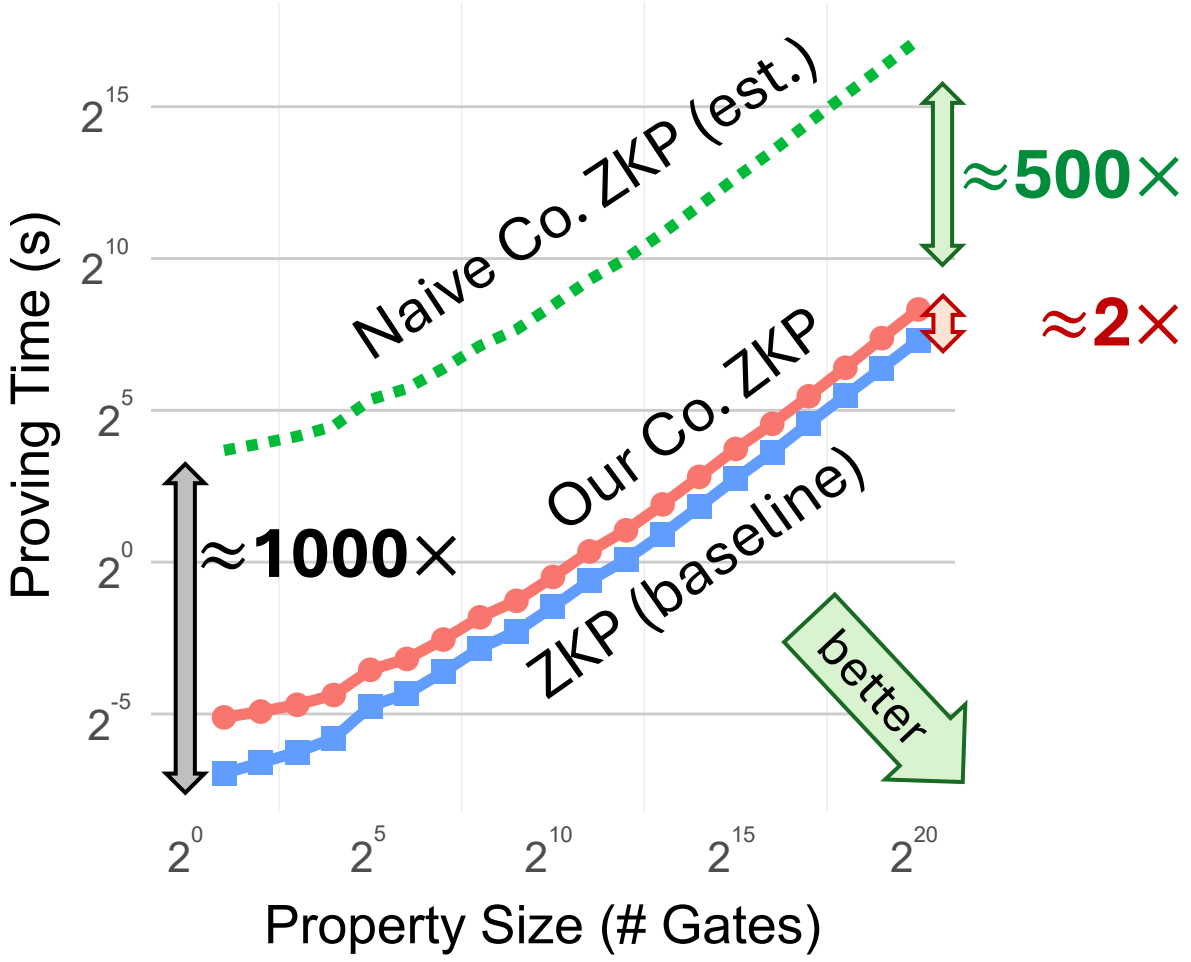
An EC-linear sharing of H

(note: $z_i = z_{i,1} + z_{i,2}$)

A $\mathbb{Z}_p$-linear sharing of z_i

Evaluated the bottleneck ***without communication***

Efficient collaborative ZKPs & their impact



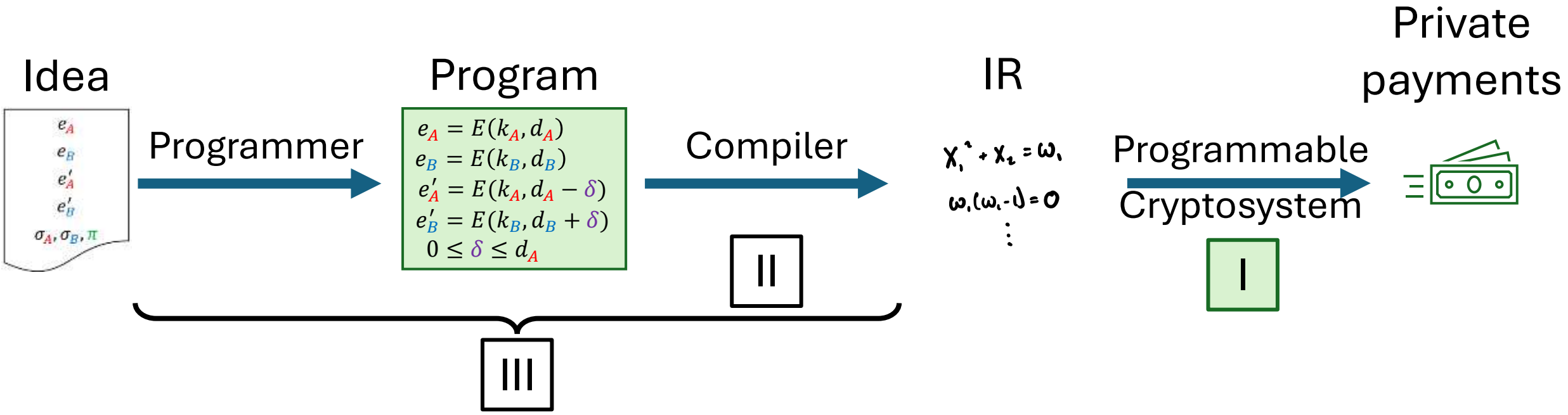
Launched an active research area

[HKKLMRRV'22], [CLM'23], [GGJPS'23], [GJMW'24], [DKR'23], [ZFS'24], [GGSSZ'25], [RGGJS'25],...

Multiple industrial implementations and compilers

Ongoing applications to private markets, ZKP outsourcing, ...

The full stack of programmable cryptography



Part I: Collaborative zero knowledge [**Ozdemir**, Boneh; USENIX Security '22]

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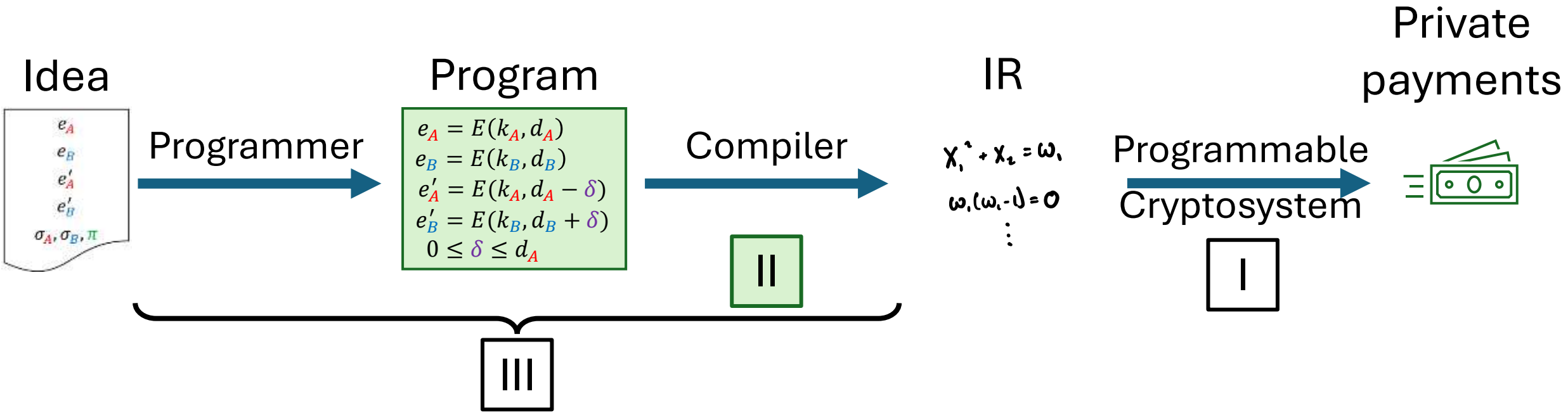


Part I: Collaborative zero knowledge

Part II: Common compiler infrastructure [**Ozdemir**, Brown, Wahby; IEEE S&P '22]

Part III: Verification via finite field solving

Problem: ZKPs for high-level programs

Complex applications require high-level languages

Even **simple** applications do too:

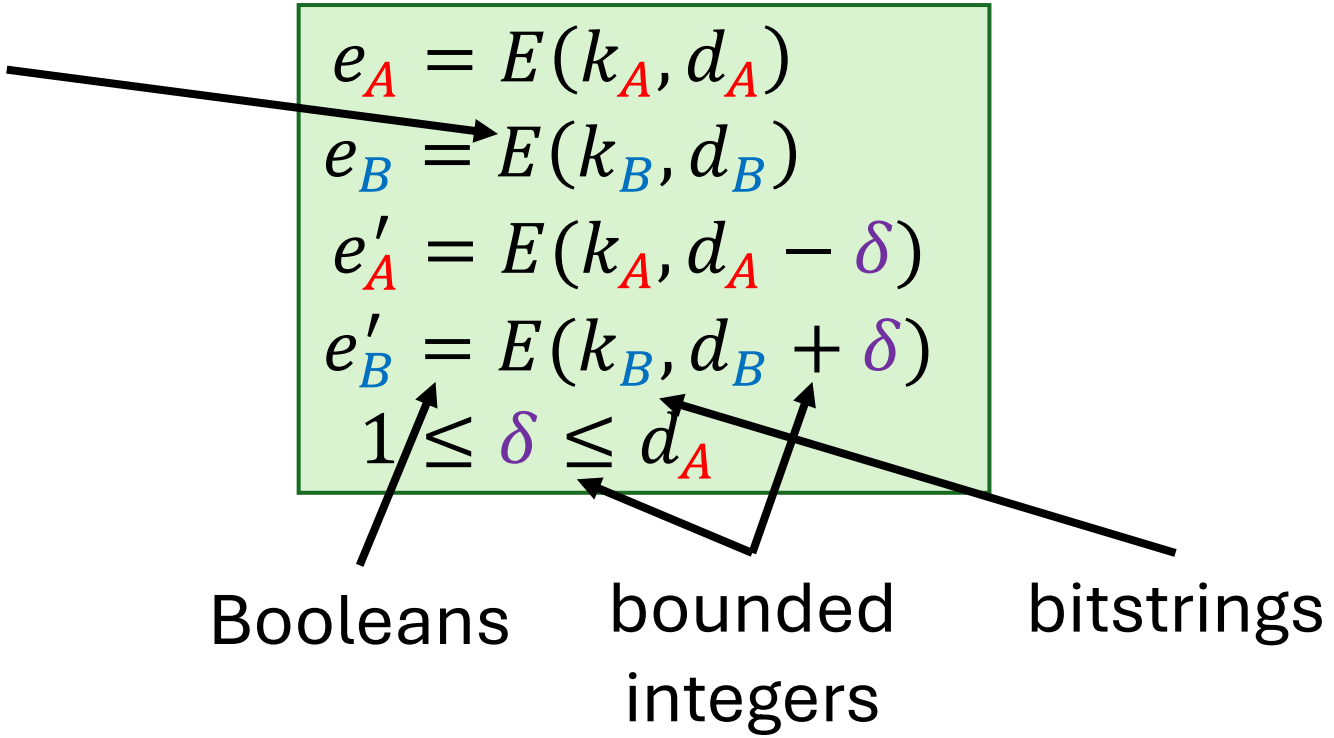
library function

if-statements

loops

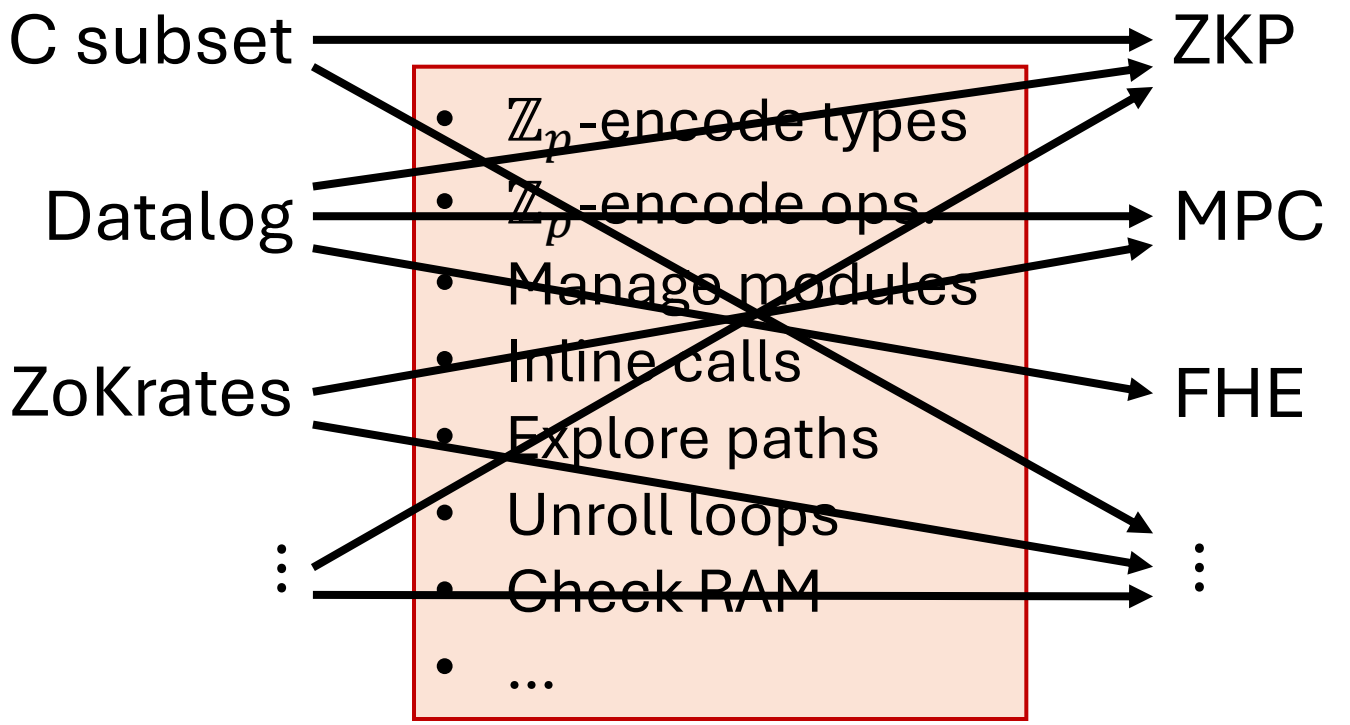
arrays/RAM

custom types



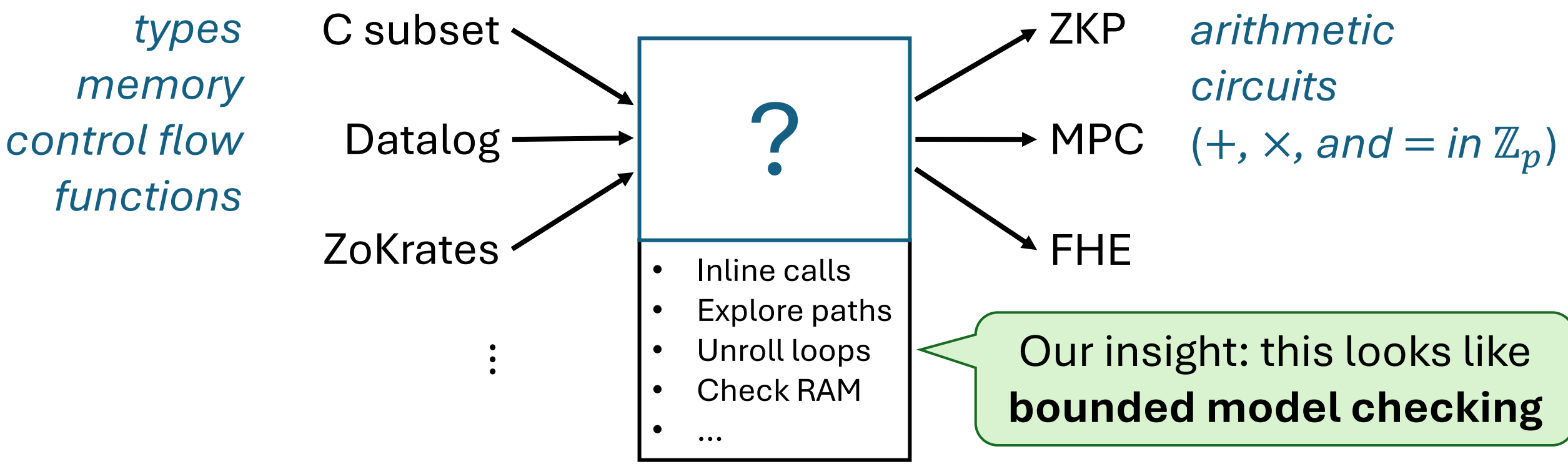
Compilation is hard

types
memory
control flow
functions



arithmetic circuits
(+, ×, and = in $\mathbb{Z}_p$*)*

Can we make common infrastructure?



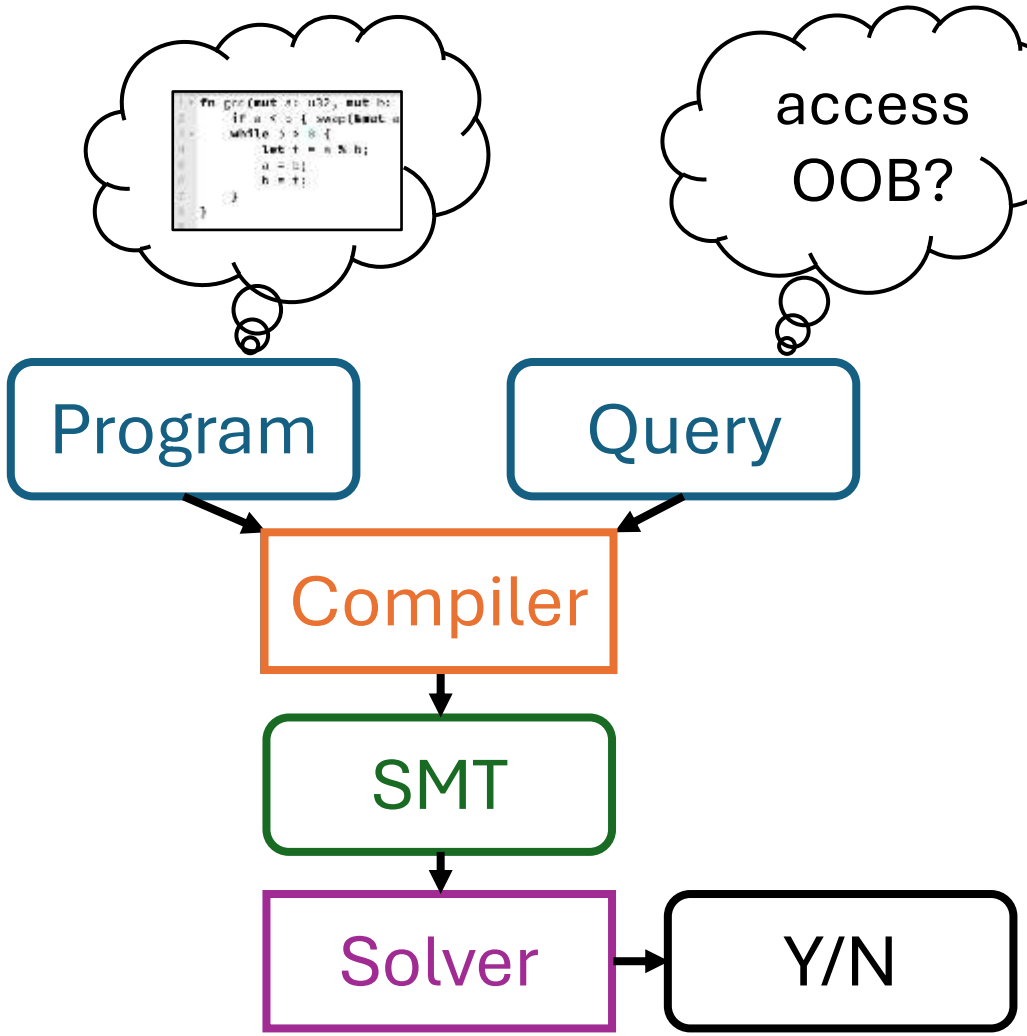
Key challenge: what is the common abstraction?

BMC: bounded model checking is compilation

1. Model the **program** and **query** as an **SMT logical formula**

   SMT: Satisfiability Modulo Theories
   (a many-typed formula)

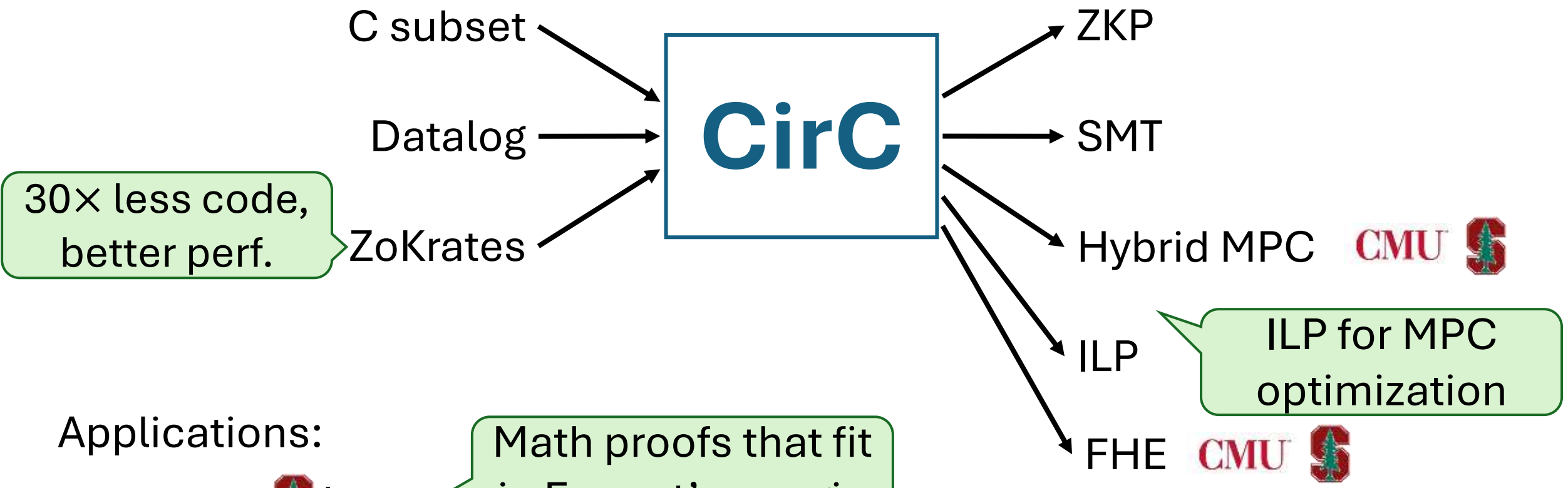
2. Check the formula with a **solver**



[Clarke, Emerson '81]

*the BMC connection is latent in projects like CBMC-GC

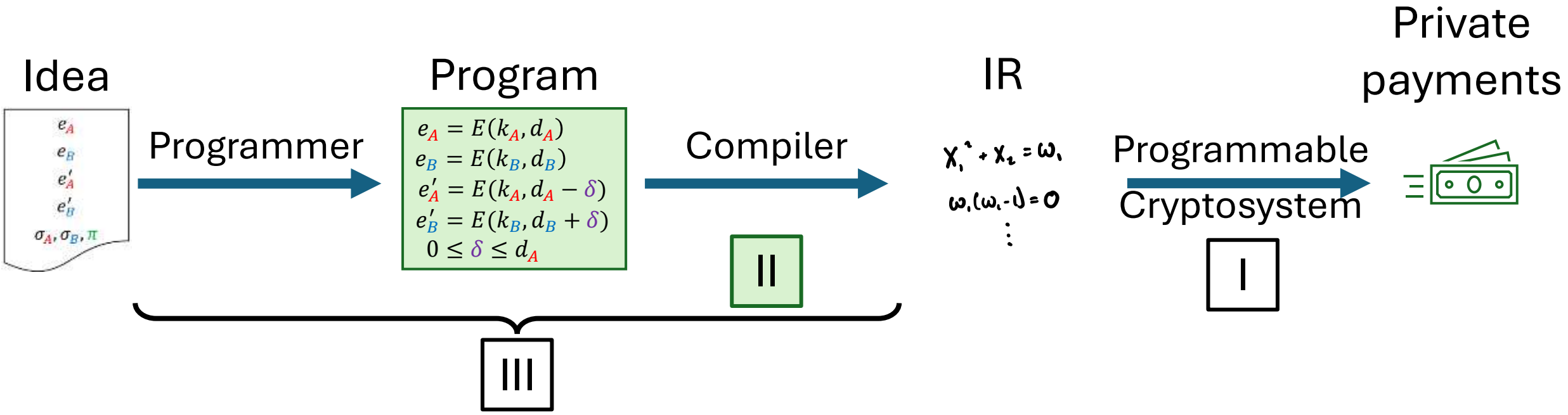
Building CirC (**Cir**cuit **C**ompiler infrastructure)



Applications:

- Lean — Math proofs that fit in Fermat's margin
- Penn — Numeric optimization
- Stanford, Michigan — Signature possession
- NYU — Private middleboxes
- Algorand — Cryptocurrency bridges

The full stack of programmable cryptography



Part I: Collaborative zero knowledge

Part II: Common compiler infrastructure [**Ozdemir**, Brown, Wahby; IEEE S&P '22]

Part III: SMT for finite fields

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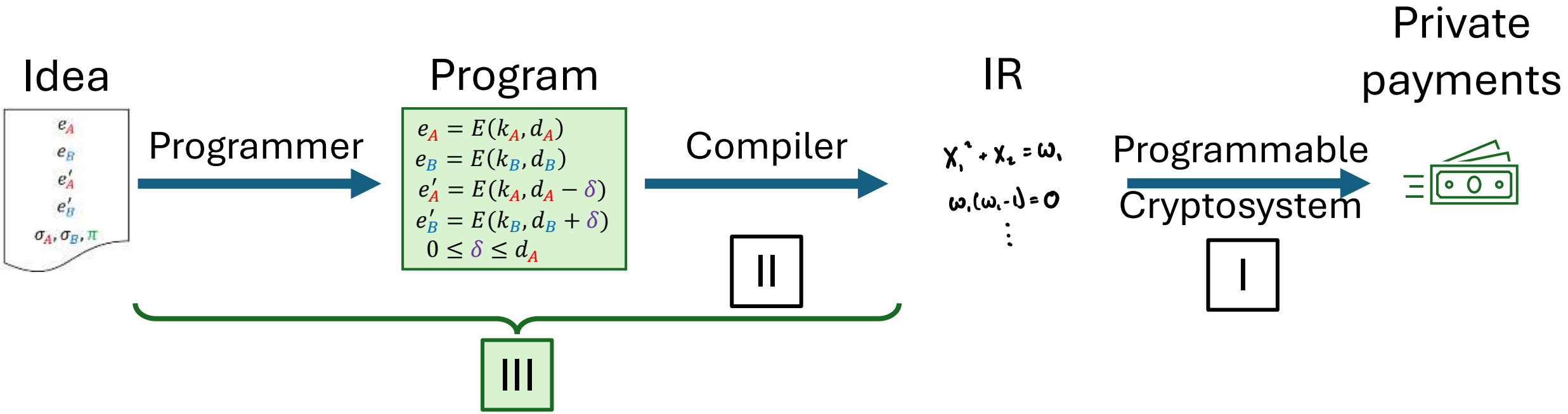

Part I: Collaborative zero knowledge

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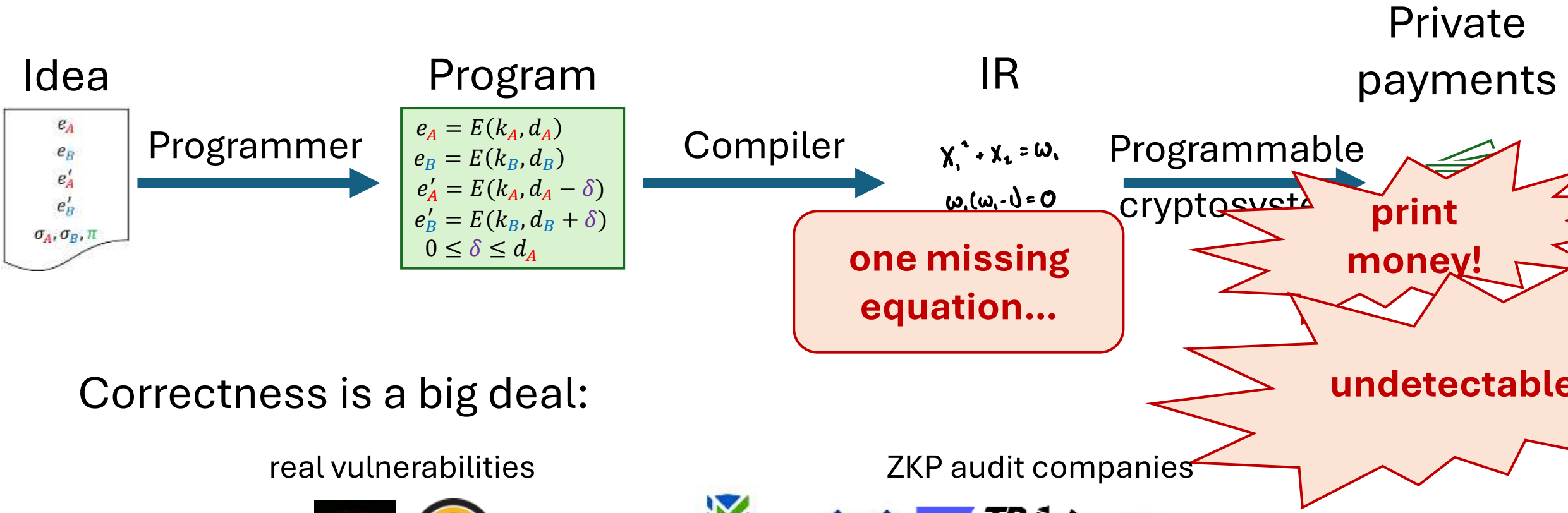
[**Ozdemir**, Kremer, Tinelli, Barrett; CAV '23]

Bugs have serious consequences



Correctness is a big deal:

real vulnerabilities

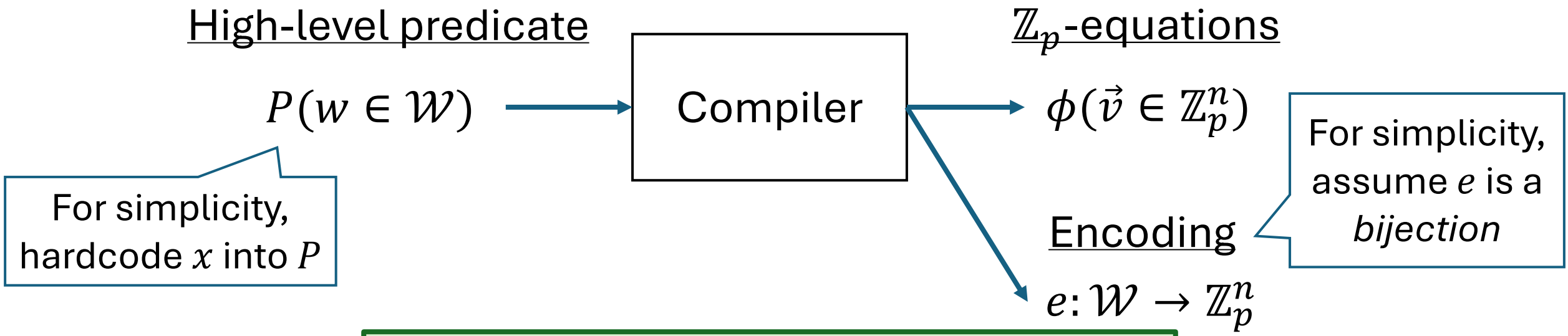
ZKP audit companies

Veridise

TRAIL OF BITS

ZKSECURITY

Definition: ZKP compiler soundness



Definition of **soundness**:
every ϕ-solution encodes a P-solution.

$$\forall \vec{v} \in \mathbb{Z}_p^n, \qquad \phi(\vec{v}) \Rightarrow P\left(e^{-1}(\vec{v})\right)$$

Standard approach: reduction to satisfiability

ϕ is ***sound*** if:

$$\forall \vec{v} \in \mathbb{Z}_p^n, \qquad \phi(\vec{v}) \Rightarrow P\left(e^{-1}(\vec{v})\right)$$

ϕ is ***unsound*** if

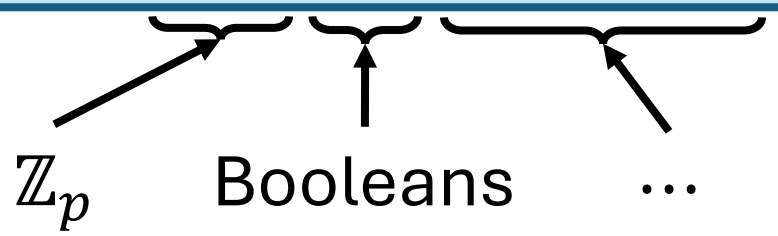
$$\neg\left(\phi(\vec{v}) \Rightarrow P\left(e^{-1}(\vec{v})\right)\right)$$

is **satisfiable**.

We need the right kind of solver

ϕ is ***unsound*** if U is satisfiable:

$$\neg\left(\phi(\vec{v}) \Rightarrow P\left(e^{-1}(\vec{v})\right)\right)$$



$\Rightarrow$ SMT solver

Problem:

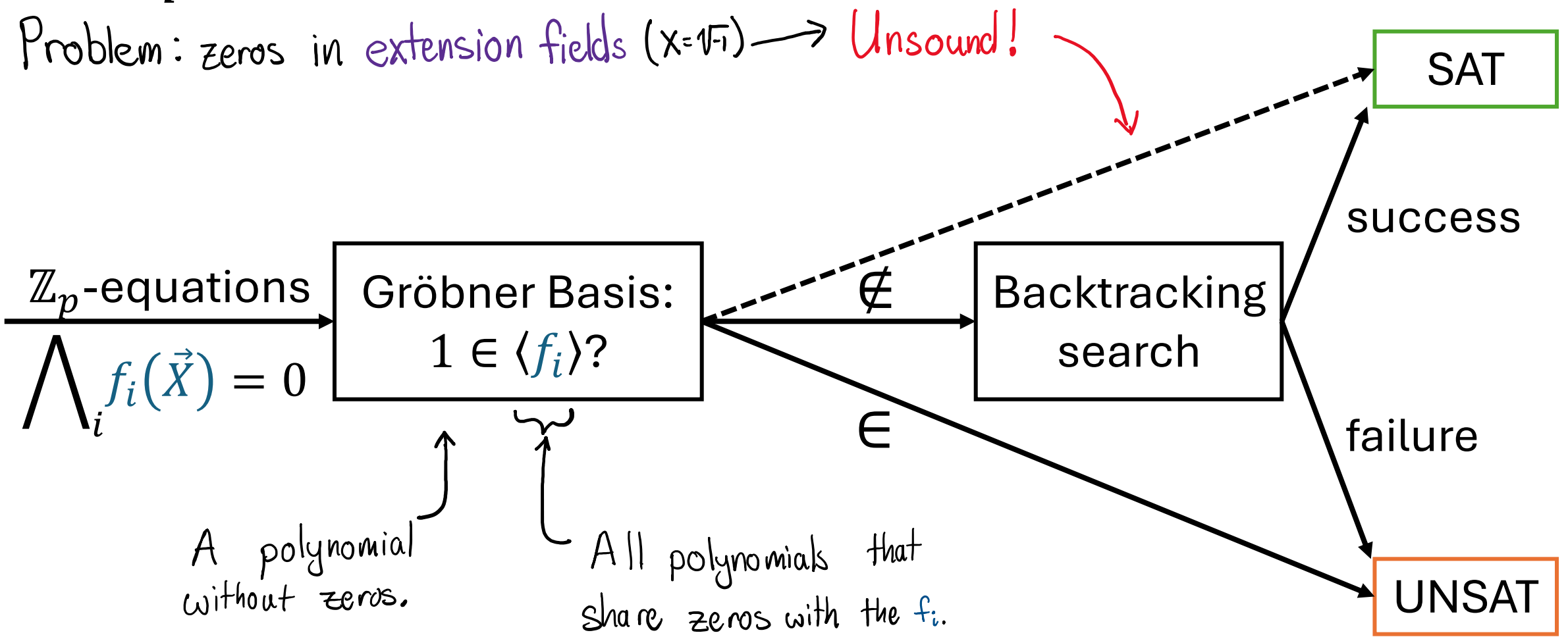
No SMT solvers natively support $\mathbb{Z}_p$

Our contributions:

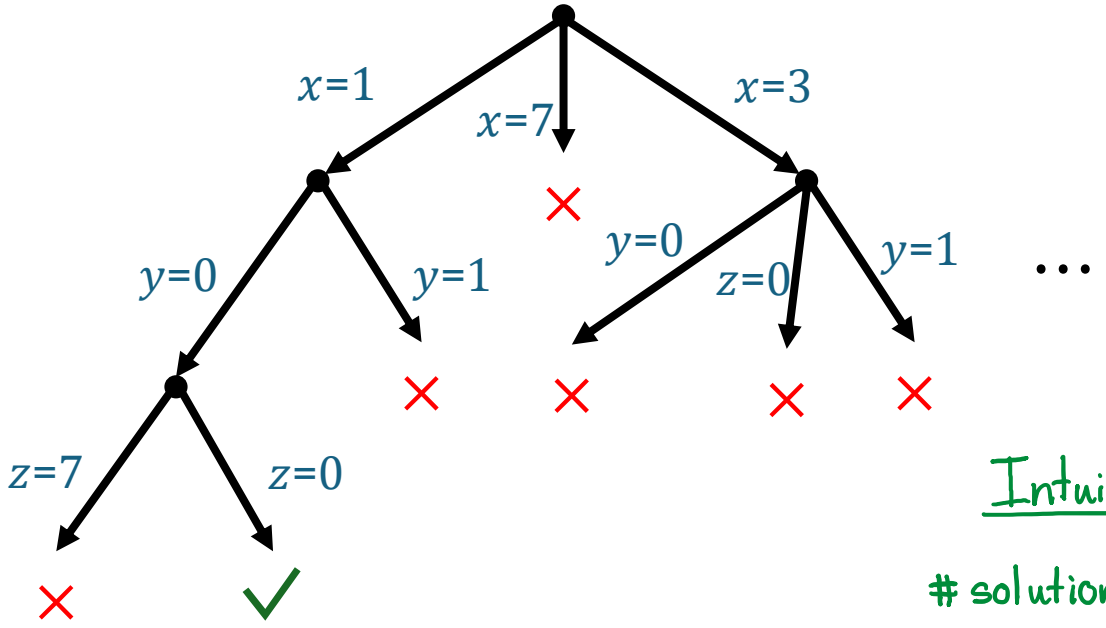
- An SMT theory solver for $\mathbb{Z}_p$
- An implementation in **cvc5**

[Ozdemir et al., CAV '23]

$\mathbb{Z}_p$-solving, Part I



$\mathbb{Z}_p$-solving, Part II: backtracking search



…

Intuition: → # solutions ~ $p^{\dim}$

(false over $\overline{\mathbb{Z}_p}$:)

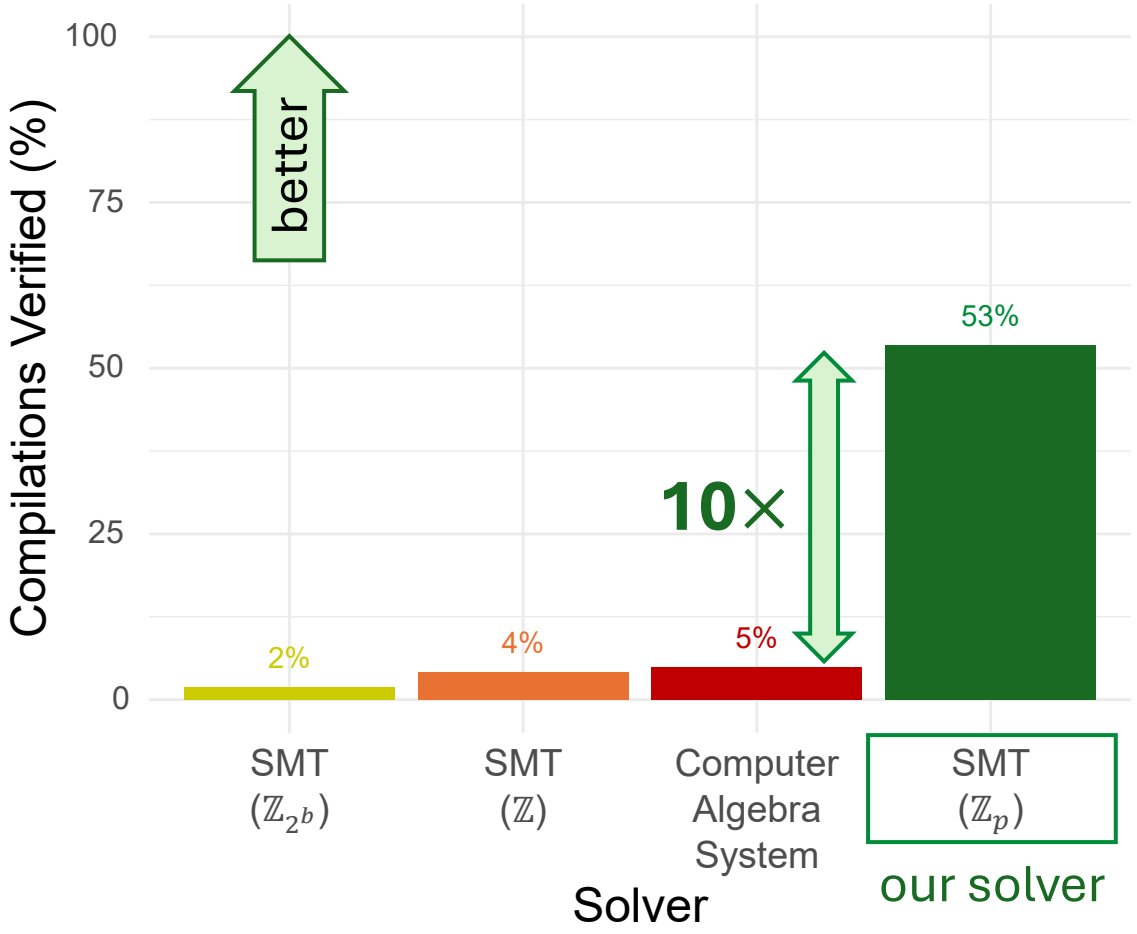
… but B is highly structured?

At each node:

- Compute a GB B
- Branch in one of three ways:
  - If ∃ univariate $p(X_i) \in B$:
    - factor p and branch on X_i roots
  - If $\dim > 0$: random search:
    - $X_1 = 1$,
    - $X_2 = 1$,
    - $X_1 = 2$,
    - $X_2 = 2$, …
  - Else, if $\dim = 0$:
    - Use B to *compute* a univariate $p(X_i) \in \langle B \rangle$

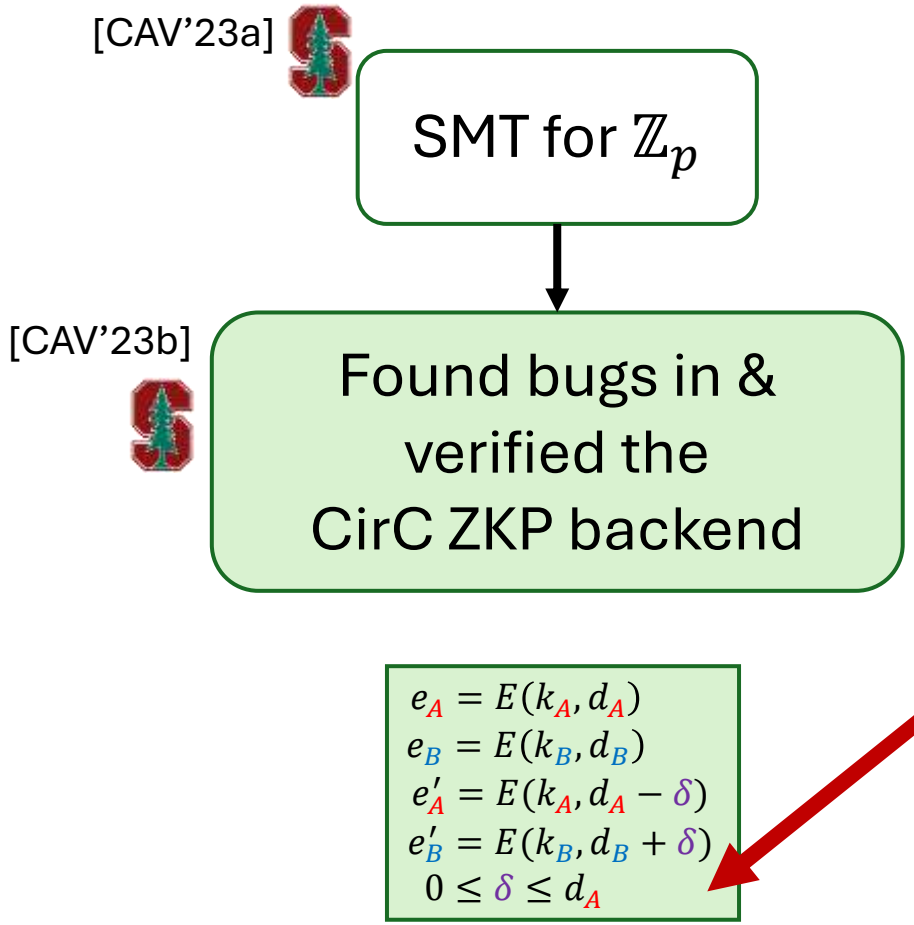
Experiment: translation validation

Verifying compilations of random* Boolean programs.



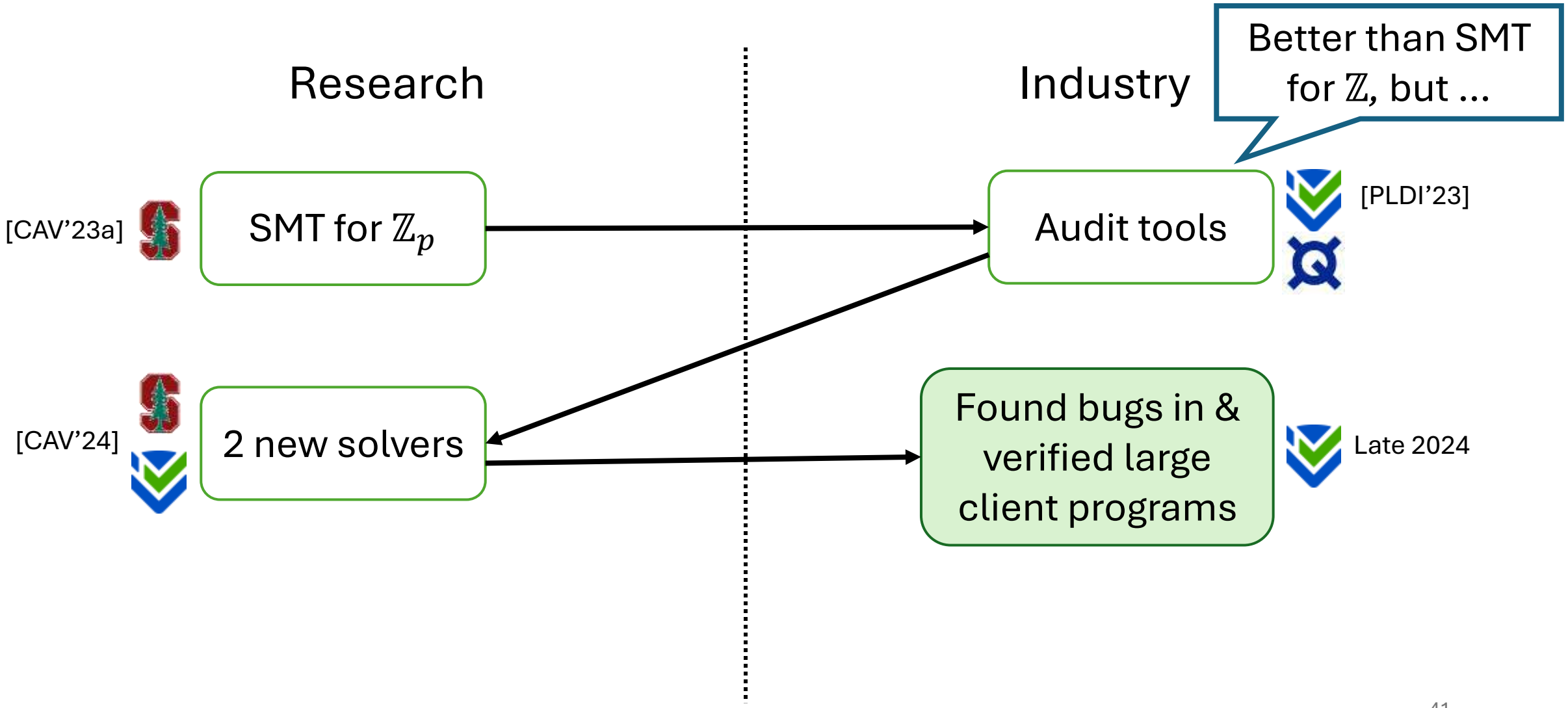
* With 2-12 input variables and 16-128 intermediate operators.

Verifying a compiler backend



- 1st ZKP compiler correctness definition
  - **compilers** + **cryptography**
- 1st verified ZKP compiler pass
  - **compilers** + **verification**
- **Verification failed!?**
  - A miscompilation in our example
- After patches, **verification passed**

Industrial use



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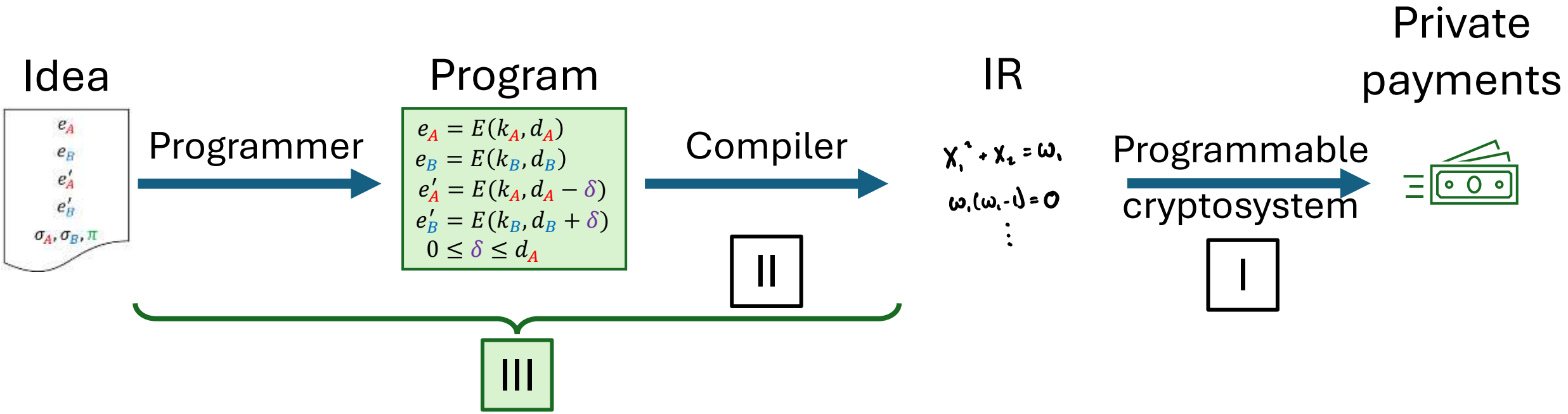

Part I: Collaborative ZKPs

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[**Ozdemir**, Kremer, Tinelli, Barrett; CAV '23]

Part IV:
What’s next?

Now is the right time

Encryption is ubiquitous

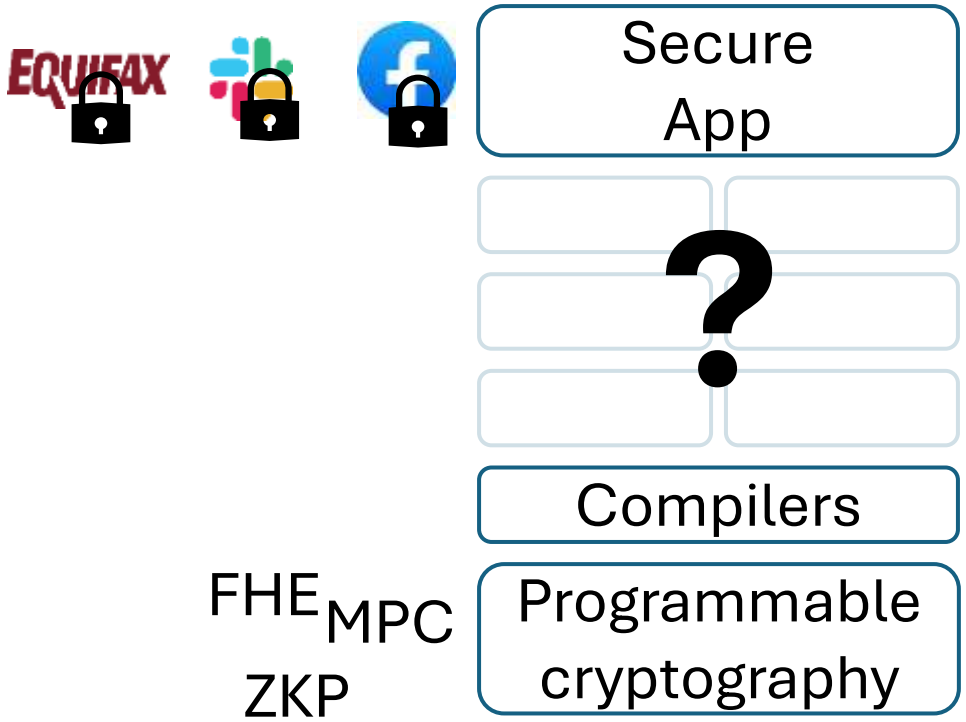
- **Good news:** people want privacy
- **Next frontier:** **data in use**

Programmable cryptography is becoming practical

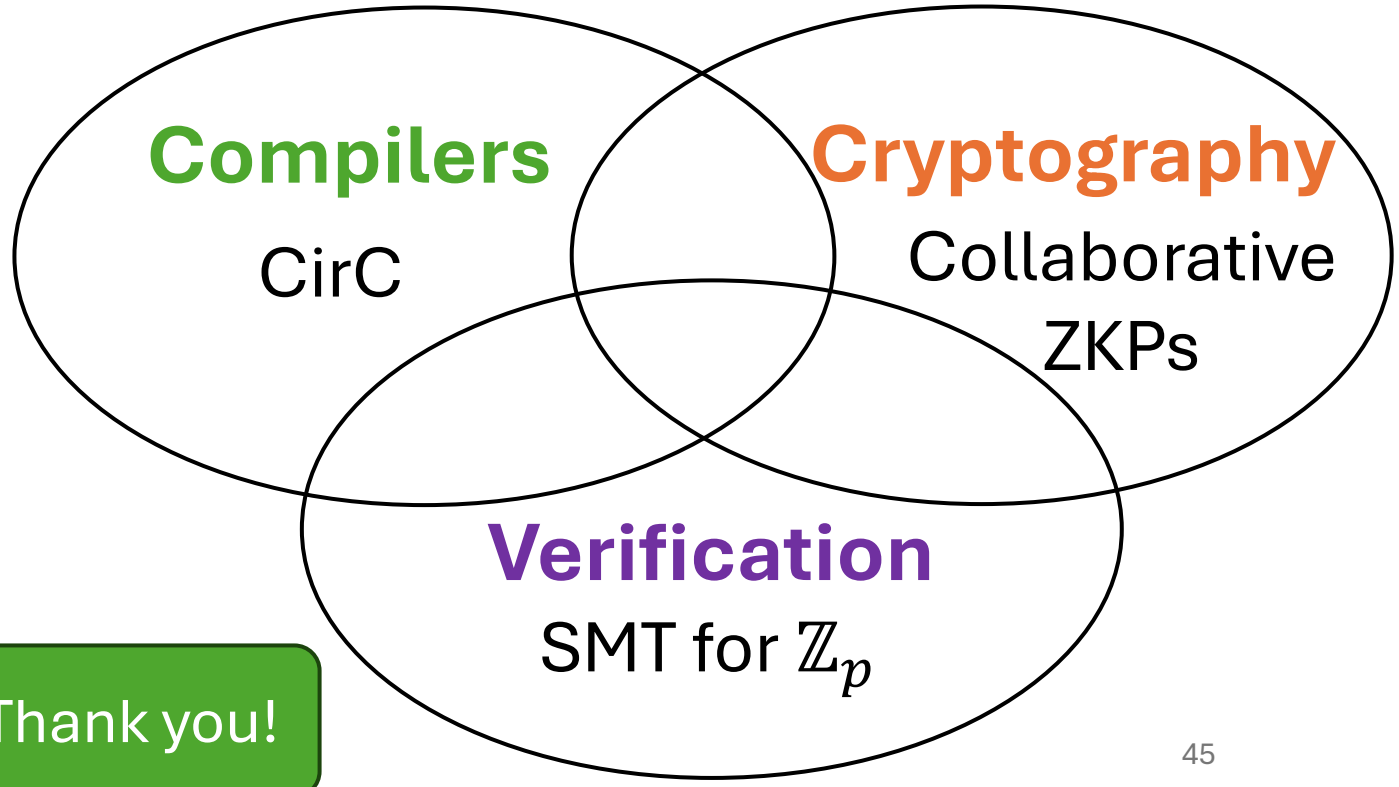
- Deployments: payments (Monero/Zcash), statistics (iOS/Android), …
- **Good news:** efficient *enough*
- **Bad news:** so far, it requires **application & cryptographic** expertise

A stack that **separates concerns** will unlock more applications

My vision:
The **full stack** of programmable cryptography



My PhD work:



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Committee

Mentors







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Officemates







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Mathias Preiner





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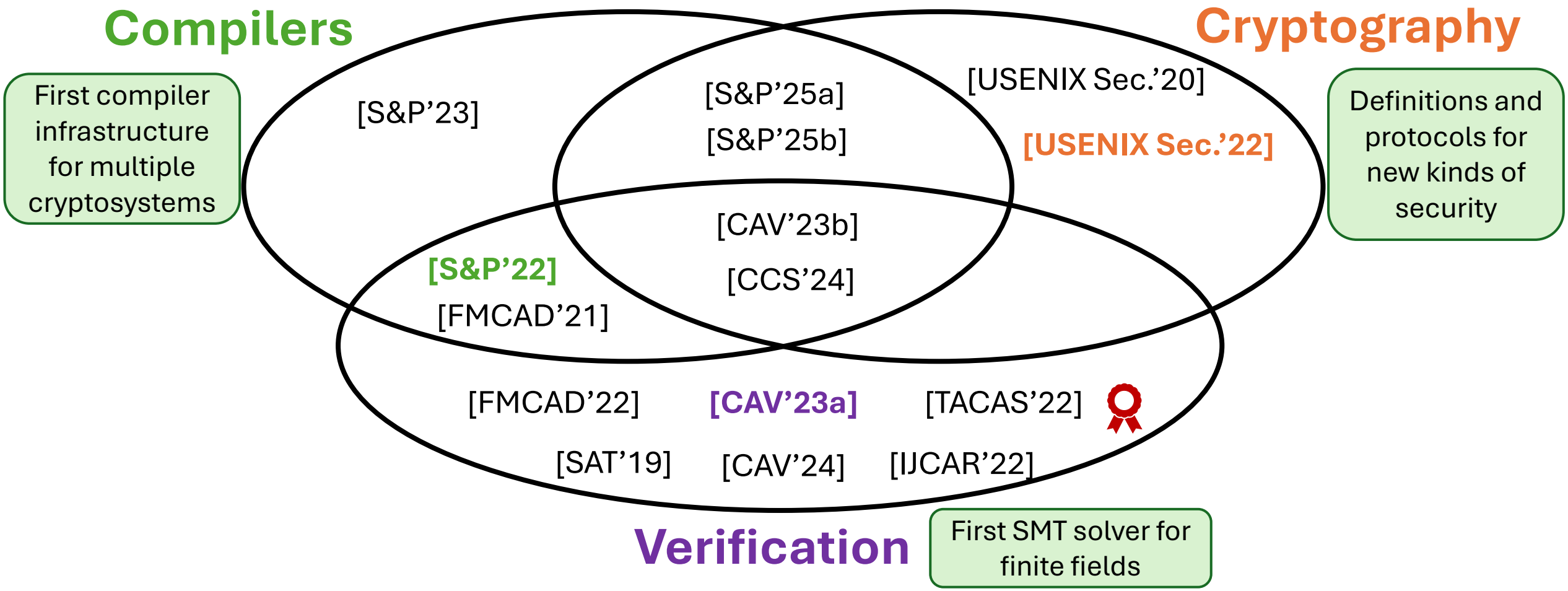
Compilers
Cryptography
First compiler infrastructure for multiple cryptosystems
Definitions and protocols for new kinds of security
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[S&P'25a]
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Verification
First SMT solver for finite fields