Fundamental Problems about Mass

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Abstract

Inertial mass is traditionally regarded as an intrinsic property of matter, independent of external influences. In contrast, we propose that inertia arises from the cumulative gravitational influence of all mass distributed throughout the universe. Under this framework, gravitational and inertial mass would share a common origin in gravity itself, providing a natural explanation for their observed equivalence, as articulated in Einstein’s equivalence principle. This perspective also challenges the necessity of the Higgs mechanism as the source of inertial mass within the Standard Model.

Introduction

Mass, one of the seven fundamental quantities in physics, plays a central role in countless physical laws and principles. It is a cornerstone of science and technology, with its influence extending into nearly every aspect of daily life. Yet, how well do we truly understand what mass is? In physics, the term can refer to gravitational mass, inertial mass, or effective mass. Why do these distinctions exist? Do they all reflect the same underlying physical reality? These are profound, foundational questions at the heart of physics—each complex enough to merit a Nobel Prize.

This article explores some of these questions, with a particular focus on the inconsistencies between the Theory of Relativity and the Standard Model. The Theory of Relativity, encompassing both special and general relativity, is a well-established framework that has been extensively validated by experimental evidence. The Standard Model, meanwhile, is the most successful theory for describing elementary particles and their interactions. However, a fundamental disparity exists between these two frameworks, particularly in their treatment of mass, revealing a gap in our understanding of its underlying mechanism.

Gravitational Mass

Gravity manifests as the sensation of weight, a force that a bird can instinctively sense and manage with ease. Yet it took humanity thousands of years to understand its true nature. In ancient times, Aristotle believed that objects like rocks fell to the ground because it was in their nature to seek their “natural place”.[1] Nearly two millennia later, Galileo Galilei challenged this view through systematic experiments, carefully observing the motion of falling and rolling objects.[2] Around the same time, Johannes Kepler studied the motion of celestial bodies and formulated his three laws of planetary motion, revealing consistent patterns in gravitational interactions across the cosmos.[3]

Building on these insights, Isaac Newton proposed that the same principles governing planetary orbits must also apply to the Moon's motion, and, indeed, to all objects on Earth.[4] By combining his laws of motion with rigorous mathematical reasoning, Newton established a theoretical framework that explained and extended Kepler's empirical laws. This work culminated in the formulation of his law of universal gravitation in 1687: any two bodies attract each other with a force proportional to the product of their masses and inversely proportional to the square of the distance between them, expressed mathematically as:

(1) $$F = G\frac{m_1 m_2}{d^2}$$

In this equation, F denotes the gravitational force between two masses m_1 and m_2 separated by a distance d, and G is the Newtonian constant of gravitation (6.67×10^{-11} $m^3/kg \cdot s^2$).

This marked a major milestone in humanity's understanding of gravity. We now know that weight arises from the gravitational attraction between an object's mass and the Earth. But this leads to a deeper and more fundamental question: why do masses attract each other in the first place? Despite being the most familiar of the four fundamental forces, gravity remains the most enigmatic. Unlike the other three forces, gravity is not covered in the Standard Model of particle physics. The graviton, a hypothetical quantum particle proposed to mediate gravitational interactions, has not been verified.[5] Attempts to reconcile Einstein's general relativity, the current prevailing theory of gravity, with quantum mechanics have led to deep theoretical inconsistencies.[6] String theory has been proposed as a possible unifying framework; however, it remains speculative, lacking direct experimental evidence so far.[7-9]

Inertial Mass

Intuitively, heavier objects are harder to move. This resistance to acceleration is a result of inertial mass. Inertial mass measures how much an object resists changes in motion when a force is applied. This is the form of mass that appears in Newton's second law of motion:[4]

(2) $$F = ma$$

In this equation, m denotes the inertial mass of a body, F represents the applied force, and a is the resulting acceleration. To explain the origin of inertial mass, the Standard Model of particle physics introduces the Higgs boson, an elementary particle associated with the Higgs field, which is a theoretical energy field that permeates the universe.[10] As particles interact with this field, they acquire mass in the form of inertia, or resistance to acceleration. The Higgs field is often likened to a kind of "cosmic molasses", impeding the motion of particles and thereby endowing them with inertial mass. In contrast, gravitational mass determines the strength of an object's gravitational attraction. Thus, inertial mass and gravitational mass represent fundamentally distinct physical properties: inertial mass is an intrinsic, internal property of a body, independent of external objects, while gravitational mass is an external property that depends on the body's interaction with other objects.

Although inertial mass and gravitational mass represent fundamentally different physical properties, it is consistently observed that heavier objects are harder to move. Indeed, all known experiments demonstrate that these two forms of mass are numerically identical. But why are they equal or so closely correlated? Could this be a mere coincidence?

Einstein believed the correlation between inertial and gravitational mass was no coincidence. He proposed that it reflected a deep and fundamental principle of nature. This bold idea, known as the equivalence principle, became the cornerstone of his theory of general relativity.[11-12]

In Einstein's famous elevator thought experiment, imagine a person inside a sealed elevator. If the elevator is in free fall, the person would experience weightlessness, unable to tell whether they are falling under the influence of gravity or drifting freely in deep space. Conversely, if the elevator is either stationary on Earth or accelerating upward in space at exactly 9.8 m/s^2 (Earth's gravitational acceleration), the person would feel a force pressing them against the floor. In both cases, the physical sensations would be indistinguishable; There would be no way to determine, from within the elevator alone, whether the force is due to gravity or acceleration. This equivalence principle lies at the core of Einstein's theory of general relativity.

General relativity has been confirmed by numerous experiments and astronomical observations, including the bending of light by gravity and the precession of Mercury's orbit, providing strong empirical support for the theory. If general relativity is correct, then the proposed equivalence between gravitational and inertial mass must point to a deeper physical connection. This raises the intriguing possibility that both types of mass arise from the same underlying mechanism.

Effective Mass

Adding another layer of complexity is the concept of effective mass. Unlike gravitational and inertial mass, which manifest through mechanical effects, effective mass represents the energy content of a system. It is this form of mass that appears in Einstein's mass-energy equivalence equation, a consequence of special relativity:[13-14]

(3) $$E = mc^2$$

In this equation, *E* represents the total energy of a system, *m* denotes its effective mass, and *c* is the speed of light. Mathematically, this equation is equivalent to:

(4) $$m = \frac{E}{c^2}$$

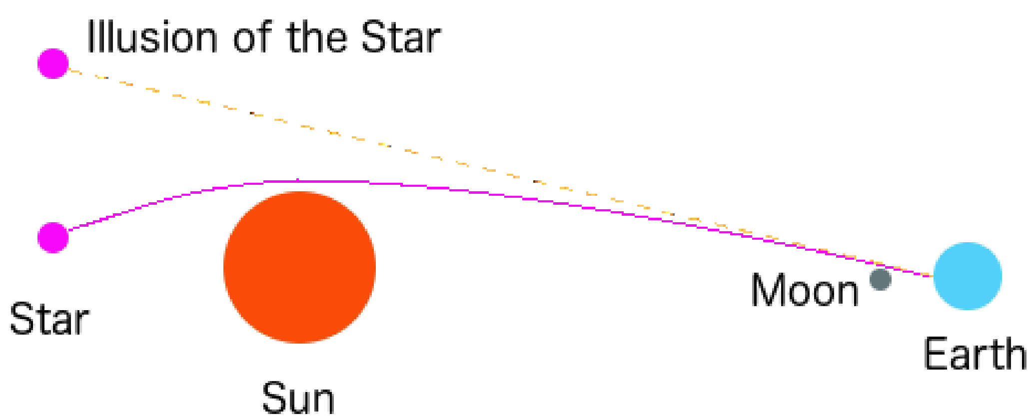
This implies that a system's effective mass is entirely determined by its total energy. But how deeply does this equivalence resonate in physics? At its core, it reveals a fundamental truth: mass and energy are two manifestations of the same physical quantity. As a result, the total effective mass of a system includes not only the rest mass of its constituent particles but also contributions from kinetic energy, potential energy, and even radiative energy. For instance, a fully charged battery with additional potential energy in its chemical bonds should, in principle, be slightly heavier than when it is discharged. However, this mass difference is minuscule, as the energy change is divided by the square of the speed of light, a factor so large that the effect is undetectable without extremely sensitive instruments. In contrast, the binding energy that holds nucleons together within an atomic nucleus is substantial, making the resulting mass difference before and after nuclear reactions measurable. This effect has been confirmed experimentally.

Moreover, mass-energy equivalence also implies that, under certain conditions, mass and energy can be converted into one another. For example, during hydrogen fusion in the Sun, a small amount of mass is lost and released as a significant amount of energy. In electron-positron annihilation, the entire mass of both particles is transformed into pure radiation. More generally, any fundamental particle can annihilate with its corresponding antiparticle, releasing energy equal to their combined mass. Conversely, in pair production, high-energy photons interacting near a nucleus can generate an electron and a positron, demonstrating the creation of matter from energy. These phenomena affirm mass-energy equivalence in two fundamental aspects: mass and energy are interconvertible, and they reflect the same property of physical systems.

Gravitational Mass of Light

At a deeper level, mass-energy equivalence also extends to gravitational interactions. Specifically, the effective mass associated with energy also contributes to gravitational effects. Just as the path of an object is deflected when it passes near another massive body, so too should the trajectory of a moving concentration of energy. Since light carries energy, it should have an effective mass and thus be subject to gravitational influence. This raises a fascinating question: Can light not only be affected by gravity but also exert a gravitational force? While we typically envision light as traveling in straight lines, the mass-energy equivalence implies that its path can bend in the presence of a strong gravitational field. Once a surprising and counterintuitive idea, this prediction was later confirmed by astronomical observations.

Astronomers once mapped the positions of stars relative to the Sun and Earth. In one remarkable instance, a star was located directly behind the Sun from Earth's perspective, as shown in the figure below. If the star's light traveled in a straight line, it would have been blocked by the Sun and would have remained invisible from Earth. However, during a solar eclipse, when the Sun's intense glare was briefly obscured, the star became visible. Surprisingly, its observed position appeared offset to the side, along the path marked by the dashed line in the figure. This displacement reveals that the star's light path was curved by the gravitational field of the Sun.[15-16]



Indeed, the bending of light by gravity around massive objects acts like an optical lens, a phenomenon known as gravitational lensing.[17-18] This effect is now a common feature in images captured by modern telescopes, such as the James Webb Space Telescope. In more extreme cases, near immensely massive objects such as black holes, gravity can curve the path of light so dramatically that it becomes completely trapped. As a result, no light can escape, rendering the object invisible and giving rise to the term black hole.[19]

One might argue that the bending of light around massive objects is solely a consequence of spacetime curvature, as described by Einstein's general relativity. However, this reasoning risks using the effect to explain the cause. The concept of spacetime curvature in general relativity is built upon the equivalence principle, which assumes the equivalence of mass and energy, as well as inertial and gravitational mass. The idea that energy has a gravitational effect is a fundamental aspect of this principle—it is not itself a consequence of spacetime curvature, but rather a foundational assumption that helps give rise to it.

Inertial Mass of Light

Extending mass-energy equivalence to gravitational effects naturally raises another question: Does this equivalence also apply to inertia? If so, then the effective mass associated with energy should also exhibit inertial properties, meaning that energy contributes not only to gravitational mass but also to inertial mass.

Just as mass-energy equivalence arises from special relativity, so too does the concept of relativistic mass. As an object's speed increases and its kinetic energy rises, its relativistic mass increases, making it more resistant to acceleration, an expression of increased inertial mass. This relationship can be derived directly from the mass-energy equivalence Equation (3):

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \tag{5}$$

In this equation, *m* denotes the relativistic mass of an object with velocity *v* and rest mass m_0, which is measured when the object is not moving relative to the observer. According to this equation, as an object's velocity increases, so does its relativistic mass. Near the speed of light, this mass approaches infinity, meaning that the force required for further acceleration also becomes infinite. In effect, the object's inertial mass becomes unbounded. In this context, relativistic mass represents the effective mass of a moving object. This demonstrates that kinetic energy contributes to an object's resistance to acceleration, in other words, its inertia, highlighting that mass-energy equivalence extends to inertial effects through the contribution of kinetic energy.

Now, consider the total effective mass of the solar system. In addition to the rest masses of the Sun and planets, the system contains kinetic energy from planetary motions, such as Earth's orbit around the Sun and its rotation on its axis, as well as gravitational potential energy arising from the mutual attractions among celestial bodies, such as the interaction between Earth and the Sun. These energies increase the system's total effective mass, making it greater than the simple sum of the individual rest masses. As a result, the gravitational and inertial mass of the solar system, regarded as a single-bound system moving through the Milky Way, is determined not only by the masses of its components but also by the internal kinetic and potential energies.

This principle also applies to a gas enclosed in a container, where every form of energy contributes to the system's effective mass. As the system heats, molecular motion increases, raising the kinetic energy; additional energy is stored as potential energy in chemical bonds, and radiative energy within the system also rises. Together, these forms of energy add to the system's total energy content. As a result, the effective mass increases, influencing both the gravitational and inertial mass of the system. The same concept extends to solids, where internal particle vibrations, binding energies, and radiation contribute similarly. In essence, the total effective mass of any system includes not only the rest mass of its components but also energy contributions from kinetic, potential, and radiative sources.

For example, the energy stored in chemical bonds contributes a small amount to an object's total mass. As a result, a fully charged battery is theoretically slightly heavier than when it is discharged. However, because the energy-to-mass conversion described by Equation (4), which divides energy by the square of the speed of light, yields such a small value, the resulting mass difference is minuscule and undetectable without extremely sensitive instruments. In contrast, the potential energy between nucleons, arising from the strong nuclear force, makes a significant contribution to the mass and is experimentally measurable. This effect is confirmed by observations of nuclear reactions, where differences in binding energy correspond to observable changes in mass. In essence, this bound energy becomes an inseparable part of matter, contributing to both gravitational attraction and resistance to acceleration, manifesting as gravitational and inertial mass.

Essentially, the stored potential energy in matter contributes to a system's total mass, including its inertial mass. To illustrate this, consider a piece of solid helium and iron at absolute zero. At this temperature, their radiative energy, molecular potential, and kinetic energy are at a minimum. Now, suppose each contains the same number of fundamental particles, such as protons and neutrons. According to the Standard Model, which attributes inertial mass to interactions with the Higgs field, both systems should have the same inertial mass. However, due to differences in nuclear binding energy, the nucleons in solid helium are less tightly bound than those in iron, resulting in higher potential energy. By the principle of mass-energy equivalence, this additional energy implies that solid helium should possess a slightly greater inertial mass than iron, despite having the same particle content. This discrepancy highlights a tension between the Standard Model's explanation of mass generation and the relativistic view of mass as a form of energy.

A similar issue arises with light: as a carrier of radiative energy, it should exhibit inertial behavior and contribute to inertial mass, according to the mass-energy equivalence. To illustrate this, consider a one-kilogram container with perfectly reflective interior walls, filled with light carrying energy equivalent to 10 kilograms of mass. According to mass-energy equivalence, the trapped light contributes an additional 10 kilograms to the system's effective mass. Consequently, accelerating the container at 1 m/s^2 would require a force of 11 newtons, one newton for the container's rest mass, and 10 newtons for the energy content of the light, indicating a total inertial mass of 11 kilograms. This example illustrates how energy, even in the form of light, contributes to inertial mass under the mass-energy equivalence.

However, light is composed of photons, which, under the Standard Model, do not interact with the Higgs field and therefore do not acquire mass via the Higgs mechanism. According to this framework, the container and its contents, photons included, should possess only one kilogram of inertial mass corresponding to the container's rest mass. To accelerate the same container at the same speed requires only 1 newton of force instead of 11 newtons. This creates an apparent contradiction: mass-energy equivalence asserts that all energy, including radiative energy carried by photons, contributes to inertial mass, while the Standard Model offers no mechanism for photons to acquire mass. This tension highlights a deeper conceptual gap between the relativistic understanding of mass-energy equivalence and the Standard Model's explanation of mass generation.

Discussions

Inconsistencies in the concept of mass emerge when both the Standard Model and the framework of relativity are assumed valid, each strongly supported by experimental evidence. These tensions expose unresolved gaps in our fundamental understanding of mass. General relativity, which posits the equivalence of gravitational and inertial mass, has achieved remarkable success. However, this success neither proves nor disproves that gravity and inertia share a common origin.

In classical physics, inertial mass is considered an intrinsic property of matter, unaffected by external objects and independent of gravity. From this perspective, Newton formulated his concept of absolute space, exemplified by his famous bucket experiment. As the bucket spins relative to the observer, the surface of the water becomes concave, indicating that the water is rotating, even though it is momentarily at rest relative to the bucket. This concavity, according to Newton, reveals rotation relative not to surrounding objects but to absolute space itself.

However, philosopher Ernst Mach challenged Newton's concept of absolute space, arguing that rotation and inertial effects, such as centrifugal force, should result from an object's motion relative to the stars of the universe, rather than to an absolute frame.[20] His ideas gave rise to what is now known as Mach's principle, which has been interpreted in various, and often loosely defined, ways, including the popular expression: "mass out there influences inertia here". A broader formulation states that local physical laws are determined by the large-scale structure of the universe.

Due to its vagueness, Mach's principle has inspired numerous distinct and sometimes contradictory interpretations, some of which are demonstrably false. While influential, it has never been developed into a precise, quantitative theory that explains how distant masses could produce inertial effects. Even Mach himself never clearly articulated a definitive version of the principle.

Mach's ideas played a significant role in guiding Einstein's development of general relativity. Einstein recognized that the overall distribution of matter in the universe influences the spacetime geometry, specifically, the metric tensor, which determines inertial frames and identifies which frame is stationary to rotation. Although Einstein was deeply intrigued and inspired by Mach's principle, it did not become a formal or foundational assumption of general relativity. In contrast, the equivalence of gravitational and inertial mass is a fundamental cornerstone of the theory.

In contrast to the prevailing view in physics, which treats inertial mass as an intrinsic property of an object, we propose that inertia arises from the cumulative gravitational influence of all mass distributed throughout the universe. If correct, this hypothesis would have profound implications for our understanding of fundamental physics. Gravitational and inertial mass would share a common origin in gravity itself, providing a natural explanation for their observed equivalence, as expressed in Einstein's equivalence principle. This perspective also challenges the necessity of the Higgs mechanism as the source of inertial mass within the Standard Model.

Furthermore, the hypothesis predicts that in a universe devoid of other masses, unlike our own, a massive object could, in principle, reach speeds as high as that of light, since the inertial resistance arising from gravitational interactions would be absent. Although this prediction is difficult to test directly, any observable influence of distant mass on local inertia would suggest that inertial mass is not entirely an intrinsic property, but rather shaped, at least in part, by external mass distributions.

Revision History

- 07/05/2025: Initial Post on Stanford Site
- 11/02/2025: Published on Zenodo
- 12/17/2025: Adding Links to Summaries of Related Articles

Links to Summaries of Related Articles

- https://cs.stanford.edu/people/zjl/abstract.html, PDF
- https://sites.google.com/view/zjl/abstracts, PDF
- https://xenon.stanford.edu/~zjl/abstract.html, PDF
- https://doi.org/10.5281/zenodo.17967154, PDF

Further Literature

- Misconceptions in Thermodynamics (PDF: DOI) (中文: DOI)
- The Mechanism Driving Crookes Radiometers (PDF: DOI) (中文: DOI)
- The Cause of Brownian Motion (PDF: DOI) (中文: DOI)
- Can Temperature Represent Average Kinetic Energy? (PDF: DOI) (中文: DOI)
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- The Triangle of Energy Transformation (PDF: DOI) (中文: DOI)
- Is Thermal Expansion Due to Particle Vibration? (PDF: DOI) (中文: DOI)
- Superfluids Are Not Fluids (PDF: DOI) (中文: DOI)
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- The Easiest Way to Grasp Entropy (PDF: DOI) (中文: DOI)
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- The Restoration Principle (PDF: DOI) (中文: DOI)
- Is There a Sea of Free Electrons in Metals? (PDF: DOI) (中文: DOI)
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- Superconductor Origin of Earth's Magnetic Field (PDF: DOI) (中文: DOI)
- Fundamental Problems about Mass (PDF: DOI) (中文: DOI)
- The Evolution from the Law of Gravitation to General Relativity (PDF: DOI) (中文: DOI)
- The Simplest Derivation of $E = mc^2$ (PDF: DOI) (中文: DOI)
- How to Understand Relativity (PDF: DOI) (中文: DOI)

- Mathematics Is Not Science (PDF: DOI) (中文: DOI)
- Tidal Energy Is Not Renewable (PDF: DOI) (中文: DOI)
- AI Contamination (PDF) (中文)
- DeepSeek pk ChatGPT (PDF) (中文)

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