

# EDGE DETECTION USING MORPHOLOGICAL AMOEBAS IN NOISY IMAGES

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## ABSTRACT

Edge detection is a significant step in image processing. Morphological edge detectors developed until now used a fixed structuring element (SE) on all the image pixels; however, they cannot consider the local features of an image due to the fixed SE and we should choose an appropriate SE by lots of experiments. In this paper, new morphological edge detectors using amoebas, dynamic structuring elements which adapt their shapes to image contours, are proposed. The experimental results show that amoeba-based edge detectors have better performance than corresponding classic edge detectors. The proposed methods have less sensitivity to noise while detecting more details of image than other morphological edge detectors with a fixed SE.

**Index Terms**— edge detection, morphological amoeba, structuring element, mathematical morphology

## 1. INTRODUCTION

Edge detection is one of the most significant works in image processing and computer vision. Classic edge detectors such as Sobel, Prewitt and Canny algorithms [1,2] used differential operator but they are pretty sensitive to noise because both noise and edge belong to the scope of high frequency.

To overcome the shortcomings of classic algorithms, edge detectors using mathematical morphology has been researched [3]. Mathematical morphology based on set theory is a new mathematical theory used for analyzing and processing images. Lee, *et al.* [4] suggested blur-minimization (BM) edge detector considering both noise and edge detection, and Feehs and Arce [5] proposed  $\alpha$ -trimmed morphological (ATM) edge detector. Zhao, *et al.* [6] suggested reduced noise morphological (RNM) edge detector for edge detection in corrupted medical images.

Mathematical morphology uses structuring elements which have certain features to process image. Conventional morphological edge detectors perform operations by using a fixed, space-invariant structuring element (SE) on all the image pixels. This causes problem since the local features of an image may not be identical everywhere and a fixed SE may not fit to the image. How to optimize the SE is a

difficult and heated research field of mathematical morphology. "Does a structuring element that changes its shape according to image features exist?" If so, it would not matter what kind of structuring elements were used for edge detection and one would not have to take the local properties of input image pixels into account. The answer to this question is, yes, such a structuring element does exist: the "amoeba". In fact, the term amoeba was first used by Lerallut *et al.* [7] who proposed morphological operators with amoebas for noise reduction. Bouaynaya *et al.* [8,9] examined a theory of spatially variant binary and functional mathematical morphology where the structuring elements can vary both in size and shape. They, however, have an evident lack of efficient implementations on grayscale images needs for practical applications.

In this paper, we propose morphological edge detectors with amoebas which consider the image contour variations to adapt their shape. Note that the proposed methods are the first morphological edge detectors using a dynamic SE.

## 2. STRUCTURING ELEMENTS IN MATHEMATICAL MORPHOLOGY

### 2.1. Classic structuring elements

Until now, the shapes and the sizes of structuring elements have been determined by lots of experiments by trial and error. For example, as structuring elements with various kinds of shapes are applied, we know that circular structuring elements are commonly appropriate to medical images not having straight lines. We also usually use square structuring elements in the images having lots of straight lines like air photographs of cities.

In determining size of structuring elements, when its size is large, big features in images are preserved and when its size is small, fine features are well preserved. Hence, normally big structuring elements are applied when we reduce noise in that if it is too small, noise also can be recognized as features of images. However, if we use too large structuring elements, ability of image restoration can be dropped since details of images are removed.

As we mentioned above, there have been no morphological algorithms with special structuring elements which show good performance in all images in that different

structuring elements should be applied to the images having different features. Although an algorithm has excellent performance in one image, if it is applied to the other image, the result is usually bad.

## 2.2. Amoebas: dynamic structuring elements

An amoeba is a sort of protozoans, having no definite forms; thus, the term amoeba is sometimes used to refer to something with an indefinite, changeable shape.

In this paper, we will call the ‘‘amoeba’’ as a dynamic SE which changes its shape to adapt to the image contour variations so the shape of the amoeba must be computed for each pixel around which it is centered. Fig. 1 shows the shape of the amoeba depending on the position of its center. In flat areas such as the center of the disc or the background, the amoeba is maximally stretched while it is shrunk in boundary areas.

The shape of amoeba is calculated by ‘‘amoeba distance’’ which is defined as follows. Let  $d_{pixel}(x, y)$  be a difference of intensities between pixel  $x$  and  $y$  and  $\sigma = (x = x_0, x_1, \dots, x_{n-1}, x_n = y)$  be a path between  $x$  and  $y$ . Then, the length of path  $\sigma$  is defined as

$$L(\sigma) = \sum_{i=0}^{n-1} [1 + \lambda \cdot d_{pixel}(x_i, x_{i+1})],$$

where  $\lambda$  is a real number. Then, amoeba distance  $d_\lambda$  is defined with parameter  $\lambda$  as

$$\begin{cases} d_\lambda(x, x) = 0 \\ d_\lambda(x, y) = \min_\sigma L(\sigma). \end{cases}$$

This distance performs the required coupling between the geometric distance and the grayscale distance, with the possibility of adjusting the parameter  $\lambda$  to take more or less into account the gradient information. Consequently, we can define an amoeba at pixel  $x$  like a following expression using amoeba distance.

$$Amoeba(x) = \{y | d_\lambda(x, y) \leq r\}, \quad (1)$$

where  $r$  denotes a radius of amoeba as a real number. If  $\lambda = 0$ , the shape of amoeba would be square.

## 3. EXISTING MORPHOLOGICAL EDGE DETECTORS

In this section, we review some existing morphological edge detectors with fixed SE. Before reviewing these detectors, we introduce several fundamental morphological operations. The dilation and erosion, the most basic morphological operations, of a gray-scale image  $f$  by a SE  $B$  are defined respectively as

$$\begin{aligned} (f \oplus B)(x) &= \max\{f(z) | z \in B(x)\}, \\ (f \ominus B)(x) &= \min\{f(z) | z \in B(x)\}. \end{aligned}$$

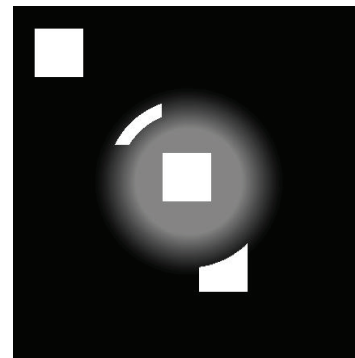


Fig. 1. The shapes of amoebas at various positions on an image

Based on these, the opening and closing of a gray-scale image  $f$  by a SE  $B$  are defined respectively as

$$\begin{aligned} f \circ B &= (f \ominus B) \oplus B, \\ f \bullet B &= (f \oplus B) \ominus B. \end{aligned}$$

We will briefly describe three morphological edge detectors.

### 3.1. BM edge detector

The BM edge detector developed by Lee, Haralick and Shapiro [4] is defined by

$$f_o(z) = \min\{f_{av}(z) - (f_{av} \ominus B)(z), (f_{av} \oplus B)(z) - f_{av}(z)\}, \quad (2)$$

where  $f_{av}(z)$  is the input image blurred with mean filter,  $f_o(z)$  is the output image, and  $B$  is the SE.

### 3.2. ATM edge detector

The ATM edge detector proposed by Feehs and Arce [5] replaced the mean filter of (2) with the  $\alpha$ -trimmed mean filter [10], and is defined by

$$f_o(z) = \min\{(f_\alpha \circ B)(z) - (f_\alpha \ominus B)(z), (f_\alpha \oplus B)(z) - (f_\alpha \bullet B)(z)\}, \quad (3)$$

where  $f_\alpha(z)$  is the input image blurred with  $\alpha$ -trimmed mean filter.

### 3.3. RNM edge detector

Zhao, *et al.* [6] used opening-closing operation to remove noise which is defined by

$$M(z) = ((f \bullet B) \circ B)(z). \quad (4)$$

Then they smoothed the image by applying closing. Finally, the RNM edge detector [6] is defined by

$$f_o(z) = ((M \bullet B) \oplus B - M \bullet B)(z). \quad (5)$$



Fig. 2. Original Cameraman image



Fig. 3. Cameraman image corrupted by Gaussian noise



Fig. 4. Edge detection result by BM detector

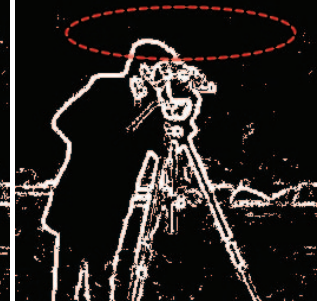


Fig. 5. Edge detection result by Amoeba BM detector

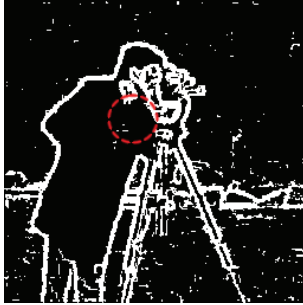


Fig. 6. Edge detection result by ATM detector

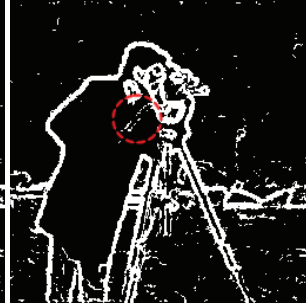


Fig. 7. Edge detection result by Amoeba ATM detector



Fig. 8. Edge detection result by RNM detector

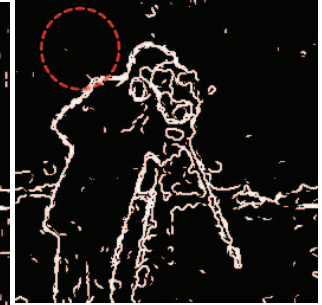


Fig. 9. Edge detection result by Amoeba RNM detector

#### 4. PROPOSED EDGE DETECTORS USING MORPHOLOGICAL AMOEBAS

The dilation, erosion, opening and closing on amoebas will be called respectively "Amoeba dilation", "Amoeba erosion", "Amoeba opening" and "Amoeba closing", and be defined as follows. The Amoeba dilation and Amoeba erosion of a gray-scale image  $f$  is defined respectively as

$$\begin{aligned} (f \oplus \tilde{A})(x) &= \max\{f(z) | z \in \text{Amoeba}(x)\}, \\ (f \ominus \tilde{A})(x) &= \min\{f(z) | z \in \text{Amoeba}(x)\}. \end{aligned}$$

where  $\text{Amoeba}(x)$  denotes an amoeba at pixel  $x$  in (1). The Amoeba opening and Amoeba closing of a gray-scale image  $f$  are also defined respectively as

$$\begin{aligned} (f \circ \tilde{A})(x) &= (f \ominus \tilde{A}) \oplus \tilde{A}, \\ (f \bullet \tilde{A})(x) &= (f \oplus \tilde{A}) \ominus \tilde{A}. \end{aligned}$$

Applying these amoeba operations to edge detectors discussed in Section 3, we now propose new edge detectors.

##### 4.1. Amoeba BM edge detector

The Amoeba BM edge detector corresponding to (2) is defined as

$$f_o(z) = \min\{f_{av}(z) - (f_{av} \ominus \tilde{A})(z), (f_{av} \oplus \tilde{A})(z) - f_{av}(z)\}.$$

##### 4.2. Amoeba ATM edge detector

The Amoeba ATM edge detector corresponding to (3) is defined as

$$f_o(z) = \min\{(f_\alpha \circ \tilde{A})(z) - (f_\alpha \ominus \tilde{A})(z), (f_\alpha \oplus \tilde{A})(z) - (f_\alpha \bullet \tilde{A})(z)\}.$$

##### 4.3. Amoeba RNM edge detector

The Amoeba RNM edge detector corresponding to (5) is defined as

$$f_o(z) = ((\tilde{M} \bullet \tilde{A}) \oplus \tilde{A} - \tilde{M} \bullet \tilde{A})(z).$$

where  $\tilde{M}(z) = ((f \bullet \tilde{A}) \circ \tilde{A})(z)$  corresponding to (4) represents the filtered image from amoeba opening-closing operation.

#### 5. EXPERIMENTAL RESULTS AND ANALYSIS

In this section, we compare amoeba-based morphological edge detectors with classic morphological edge detectors. Fig. 2 is the original 8-bit Cameraman image of  $256 \times 256$  pixels and Fig. 3 is the noisy Cameraman image corrupted by Gaussian noise with  $\sigma = 35$ . Fig. 4, Fig. 6 and Fig. 8 are the results of BM, ATM and RNM edge detectors on fixed SE for the Cameraman image, respectively. Fig. 5, Fig. 7

and Fig. 9 are the results of Amoeba BM, Amoeba ATM and Amoeba RNM edge detectors for the Cameraman image, respectively.

From Fig. 4 and Fig. 5, during preserving fine edges well, Amoeba BM edge detector is less sensitive to noise than classic BM edge detector, especially at the grass and the sky.

Fig. 6 and Fig. 7 also show that Amoeba ATM edge detector catches more small details of edges while filtering noise better than ATM edge detector. Especially, Amoeba ATM edge detector detects the handle of the camera which is not detected by classic ATM edge detector.

As shown in Fig. 8 and Fig. 9, Amoeba RNM edge detector, which is proposed in this paper, also has better performance in noise suppression and edge detection than RNM edge detector on fixed SE.

Pratt's figure of merit (FOM) [11] was used as performance measures for quantitative evaluation and comparison to the other edge detectors. The FOM is defined as

$$\text{FOM} = \frac{1}{\max\{I_I, I_D\}} \sum_{i=1}^{I_D} \frac{1}{\beta d_i^2},$$

where  $I_I$  and  $I_D$  are the number of ideal and detected edge points, respectively, and  $d_i$  is the distance between  $i$ th detected edge point and an ideal edge. The scaling constant  $\beta (> 0)$  provides a relative tradeoff among smearing, isolation, and the edge offset, and was set to  $\beta = 1/9$ . To calculate FOM we used a square image with size  $256 \times 256$ , which contained a  $128 \times 128$  pixels square object. The dark and light square had a grayscale value of 100 and 150, respectively. Tables 1 shows the measured values of FOM used to compare amoeba edge detection to classic edge detection for by Gaussian noise with  $\sigma = 5, 15, 25, 35, 45$ . This table shows amoeba-based edge detection had better performance than classic edge detection for standard deviations over the entire range.

## 6. CONCLUSIONS

In edge detection using mathematical morphology, SE plays a very important role but, until now, morphological edge detectors use a fixed SE on the image. Thus, some features of the image could be ignored in that the local properties of input image may not be identical.

In this paper, we have presented morphological edge detectors using non-fixed structuring elements, or amoebas, which consider the image contour variations to adapt their shape. The experimental results show that the proposed edge detectors built on amoebas have better performance than corresponding classic edge detectors. Amoeba-based edge detectors are not only less sensitive to noise but also detecting small details of image better than conventional edge detector using fixed structuring elements.

Table 1. FOM for morphological edge detectors

Edge Detectors	$\sigma$				
	5	15	25	35	45
BM	0.9472	0.9305	0.9017	0.7589	0.3931
Amoeba BM	0.9477	0.9326	0.9091	0.8060	0.4506
ATM	0.9506	0.9387	0.9054	0.7731	0.4816
Amoeba ATM	0.9507	0.9409	0.9136	0.7841	0.4866
RNM	0.7519	0.7795	0.7511	0.5110	0.2406
Amoeba RNM	0.7897	0.8100	0.7821	0.6062	0.3003

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