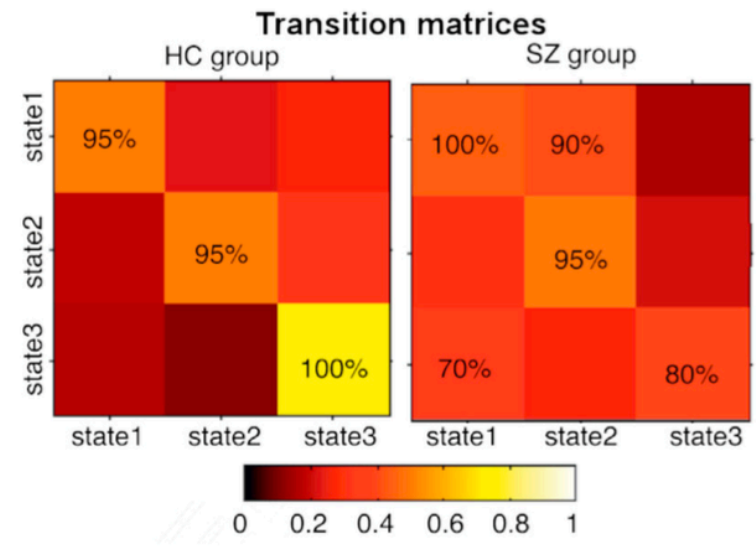
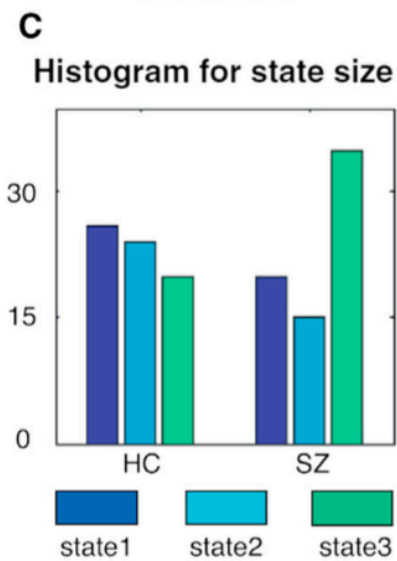
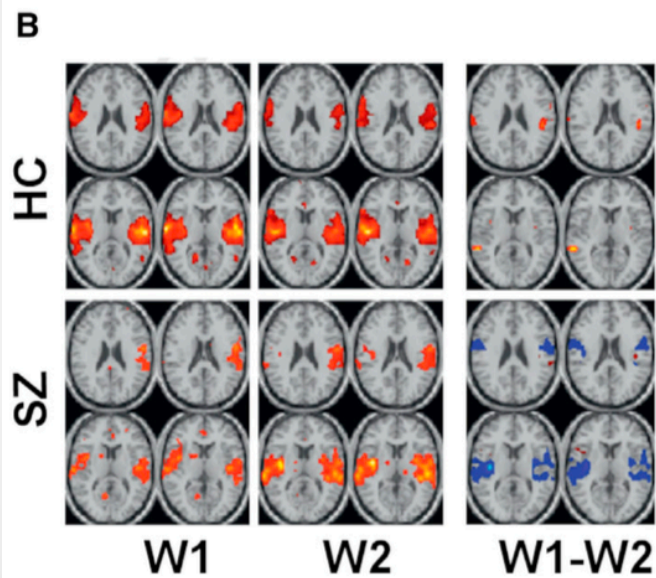


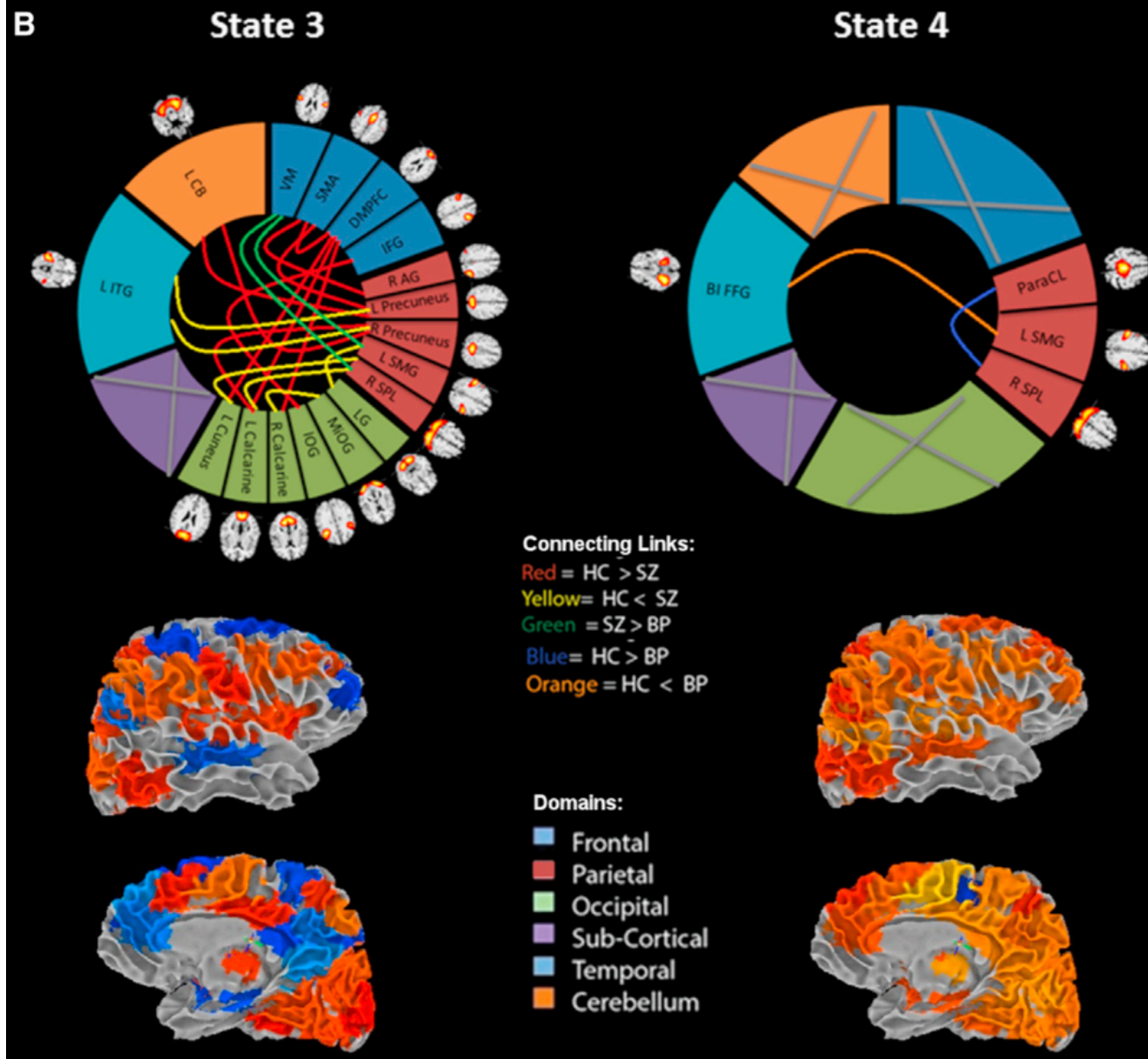
# Time-Varying Functional Connectivity

Sanmi Koyejo

Stanford University & University of Illinois



Dynamics of Schizophrenia vs. healthy controls  
(Calhoun et. al., 2015)



Dynamics of Schizophrenia vs. Bipolar vs. healthy controls (Calhoun et. al., 2015)

# Motivating Questions

- How are the regions of the brain functionally connected?
- How do these connections change over time?
- ...
- How are the changing connections related to behavior, disease, etc.?



# Main Steps

## Node Extraction

- Voxels
- ROI
- ICA

## Connectivity Measure

- Correlation
- Precision
- Mutual information
- MTD

## Time-Varying Evolution

- Nonparametric
- Parametric

## Estimation & Summary

- State estimation & description
- Cartographic profiling

## Inference

- Parametric
- Nonparametric (e.g. VAR)

# Highlights

- Estimation of time-varying functional connectivity
  - Parametric vs. non-parametric techniques
- Techniques for summarizing results
- Techniques for inference
  
- Will not cover:
  - Node extraction: Voxels vs. ICA vs. ROI
  - Techniques for selecting model hyper-parameters
  - Selecting the connectivity measure
  - Some signal processing techniques e.g. IVA (Ma et. al., 2014)

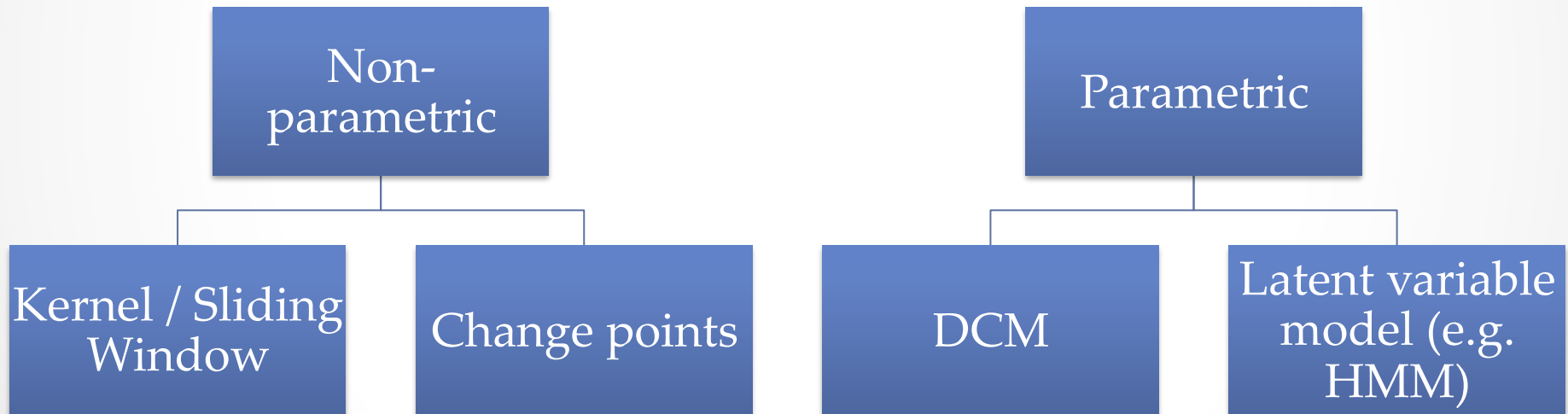
# Important to Remember

"All models are wrong but some are useful"

- George Box

- In general, models are statistical summaries, and are useful to the extent that they elucidate important properties of the brain
- Thus, these techniques are not “how the brain works” i.e. none of these models are “correct”

# Estimating Time-Varying Functional Connectivity



# Outline

- ~~Introduction~~
- **Non-parametric temporal evolution**
- Parametric temporal evolution
- Summary measures
- Inference
- Summary

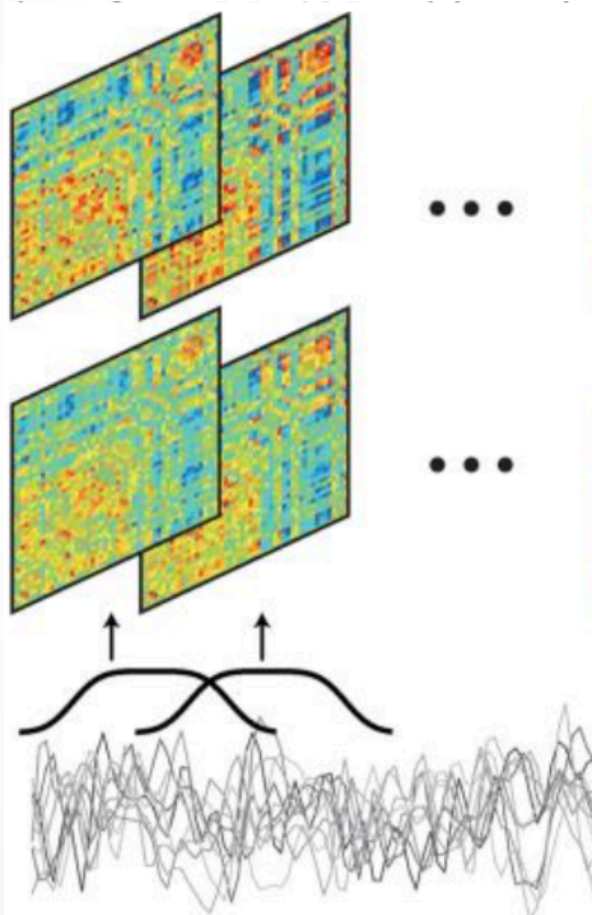
# Part 1

...

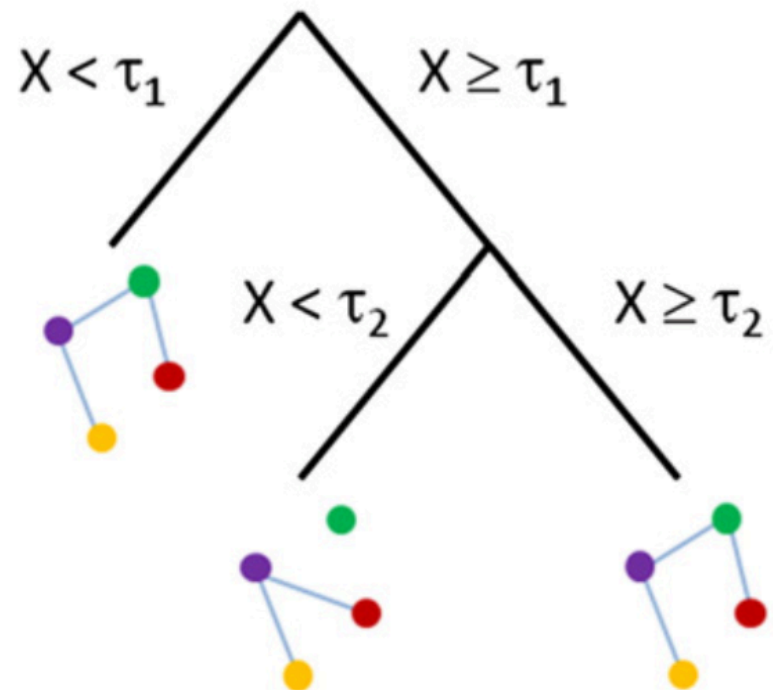
Non-parametric temporal variation



# Nonparametric Approach for Temporal Evolution



Rashid et. al. (2014)



Cribben et. al. (2012)

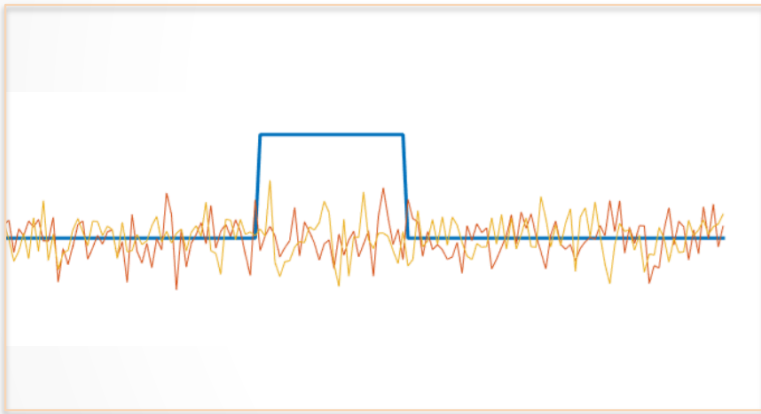
# (Kernel) Sliding Window

$$C^{i,j}(n) = FC(y_{n-m:n}^i, y_{n-m:n}^j)$$

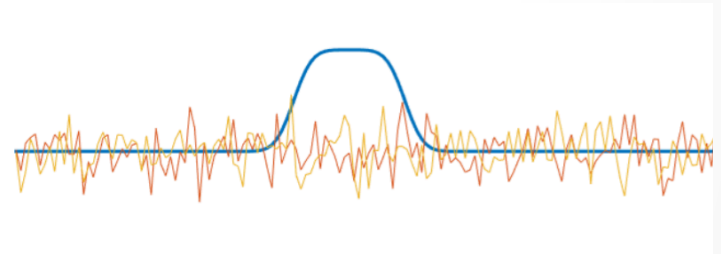
- FC = measure of functional connectivity
- $C(n)$  = connectivity estimate at each time point “n”
- E.g. kernel smoothed sliding window correlation (after subtracting mean)

$$C^{i,j}(n) = \frac{\sum_{s=n-m}^n k(s-n) y_s^i y_s^j}{\sqrt{\left(\sum_{s=n-m}^n k(s-n) y_s^i\right) \left(\sum_{s=n-m}^n k(s-n) y_s^j\right)}}$$

# Sliding Window Kernels



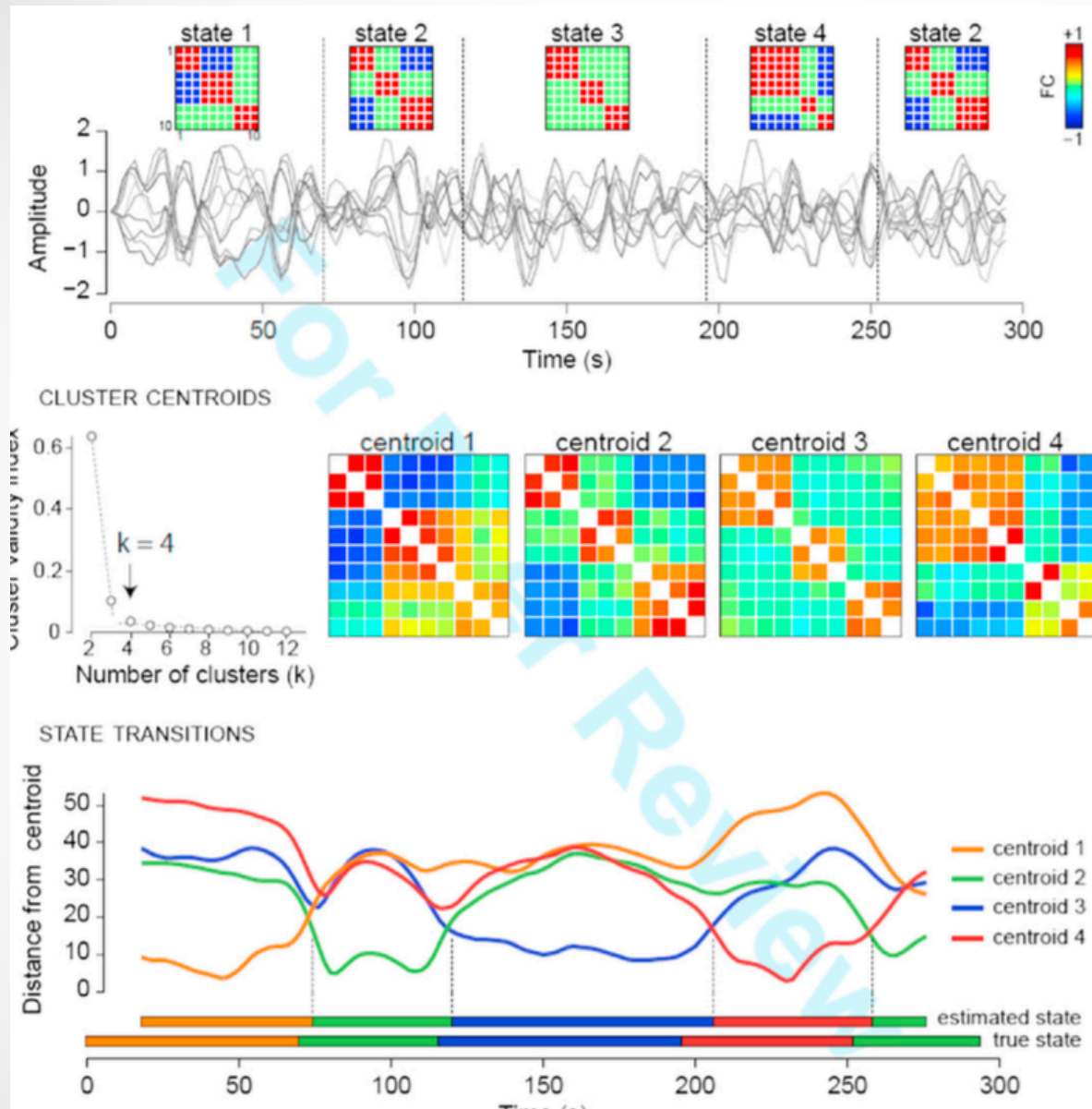
Uniform kernel



Gaussian kernel

Lindquist (2014)

# Simulated time-varying Connectivity (SimTB toolbox)

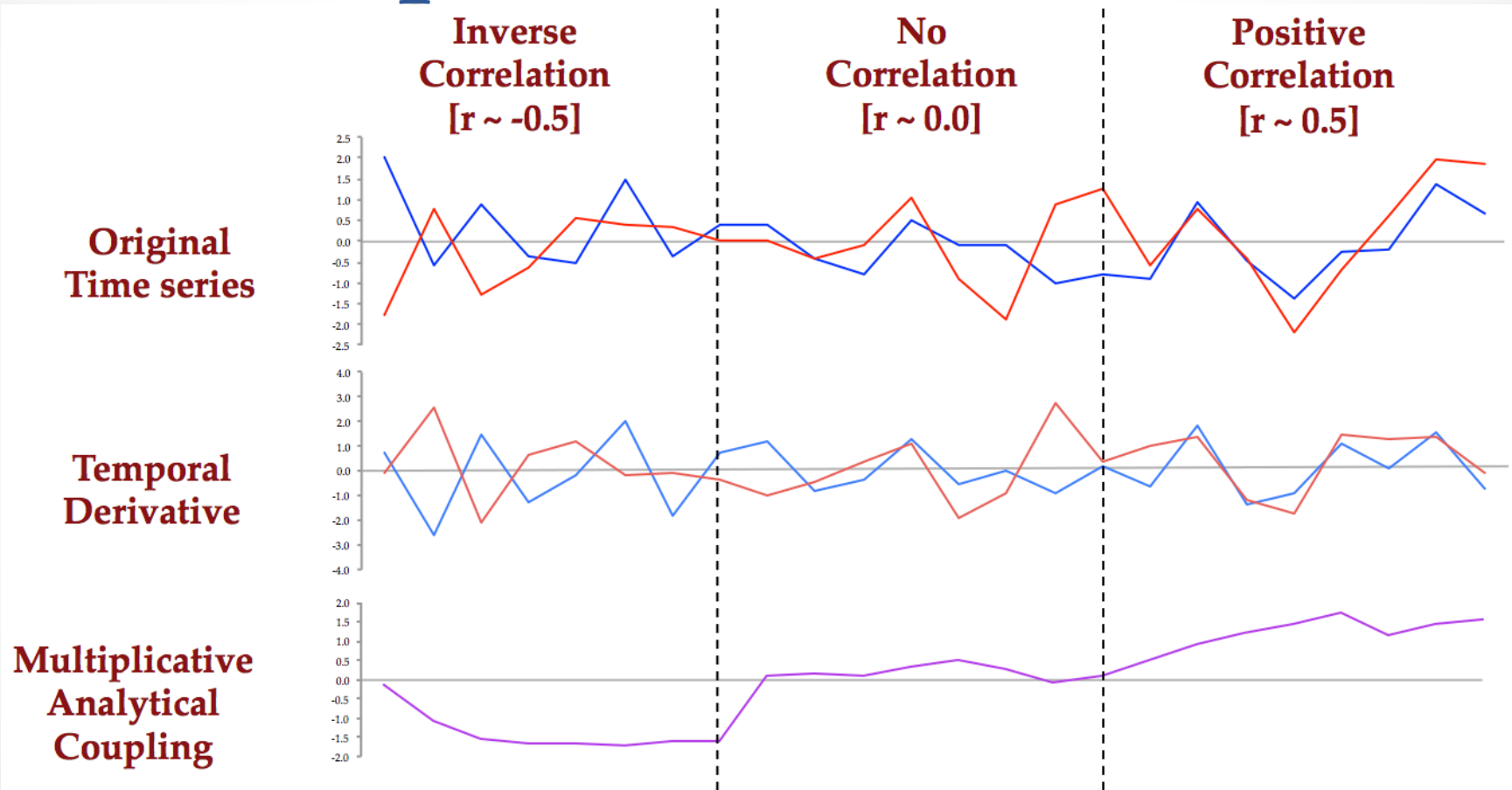


# Connectivity Measures

- Pearson / Spearman Correlation
- Partial correlation
- Mutual information
- Multiplication of temporal derivatives (MTD)

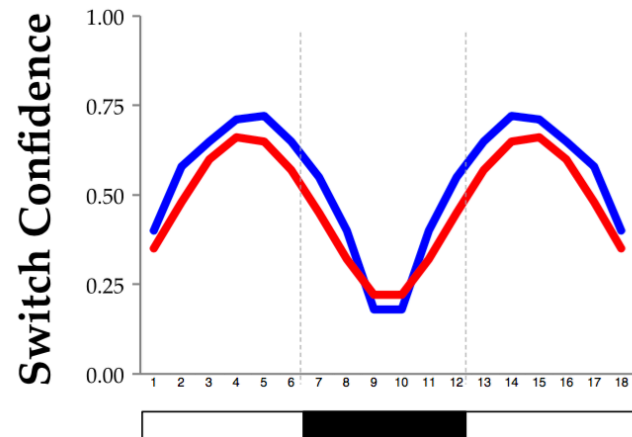
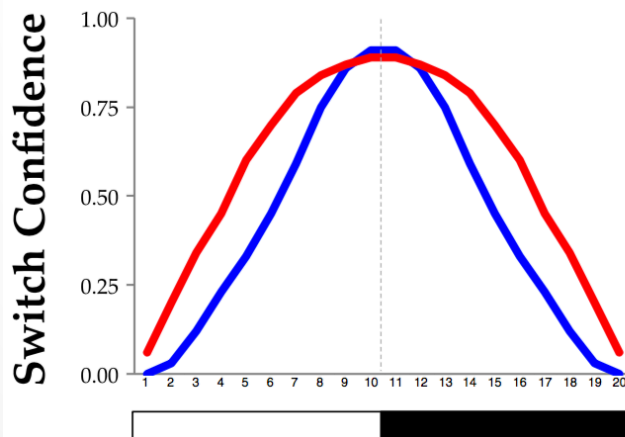
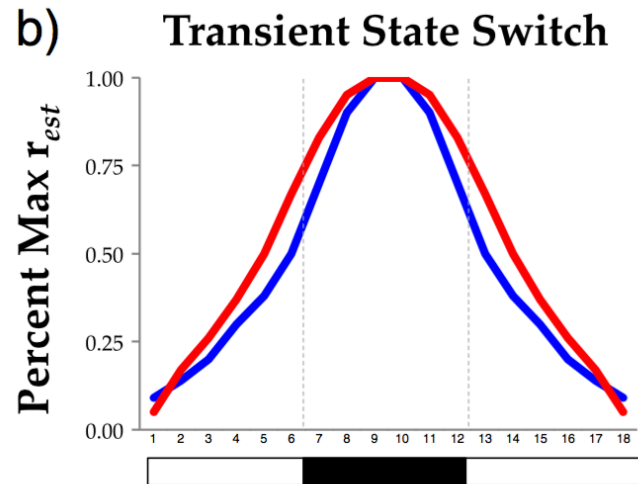
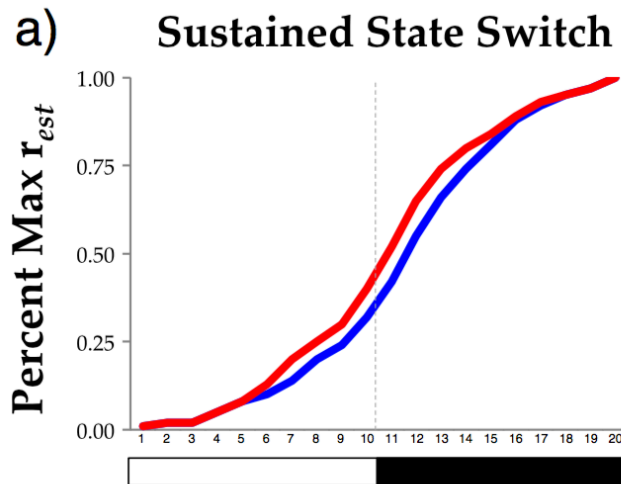
(plus regularized variations)

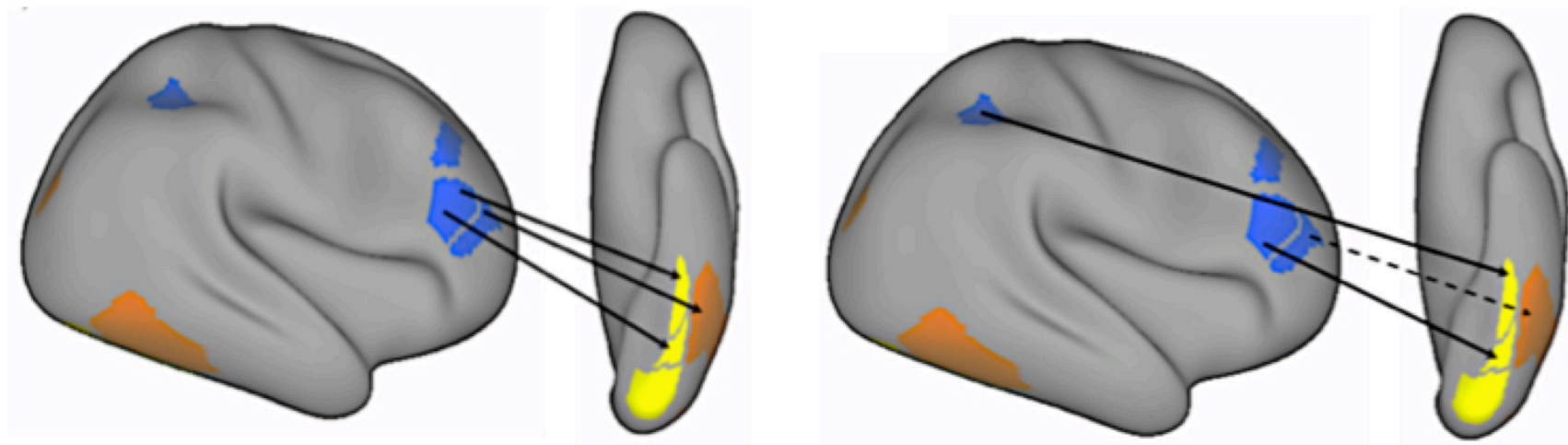
# MTD: Multiplication of Temporal Derivatives





# MTD vs. Pearson's





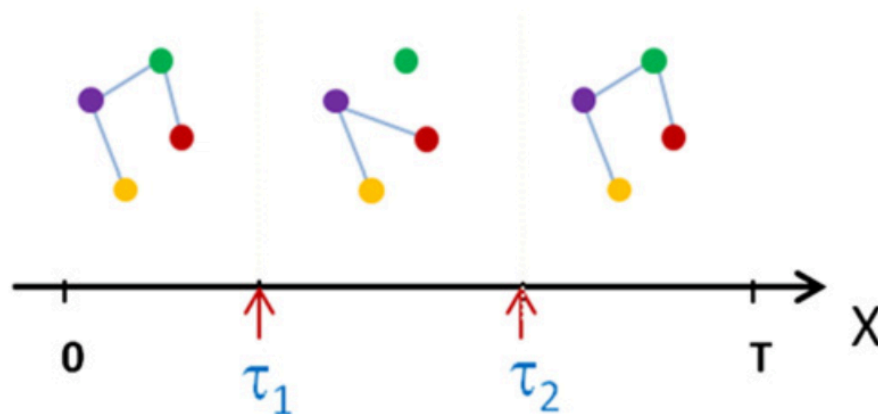
**Figure 4** Increased task-based **functional connectivity between frontoparietal and ventral visual cortical parcels**: left – during 2-back blocks compared to 0-back blocks; right – during face vs place identification ( $p < 0.001$ ; FDR 0.05).

- 40 subjects from the HCP, visual working memory
- 2-back vs. 0-back & Faces/Places/Tools/Body Parts
- MAC using Gordon ROI's

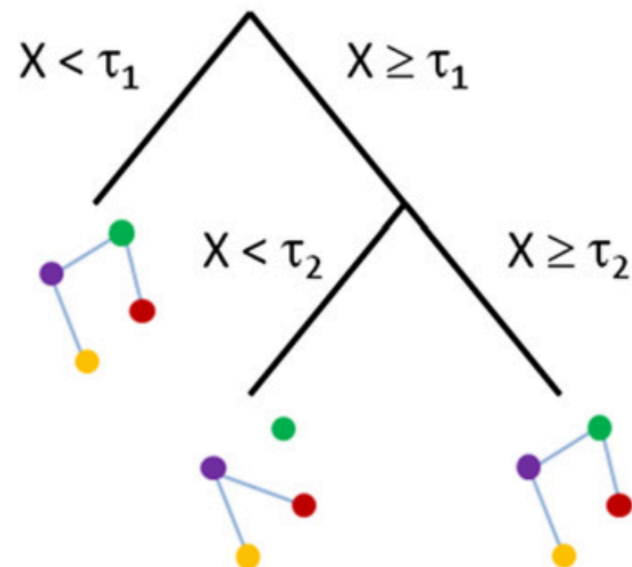
Shine et. al. (2015)

# Change-Point Detection

A. Dynamic Connectivity Regression



B. Regression Tree Diagram



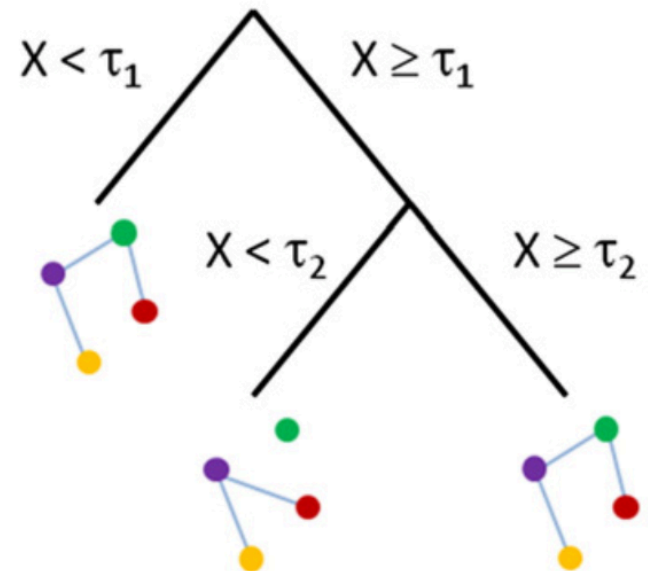
Cribben et. al. (2012)

# Main Steps of DCR

- Select a statistic for connectivity within each window e.g. sparse precision
- Select a criterion for splitting the time series which balanced model fit vs. complexity e.g. BIC

Algorithm:

- (Recursively) at each leaf:
  - At each time point “t” within block
  - Estimate model with/without the split @ “t”
  - Compute best split “t\*” within the block
  - Split the time series if it improves criterion

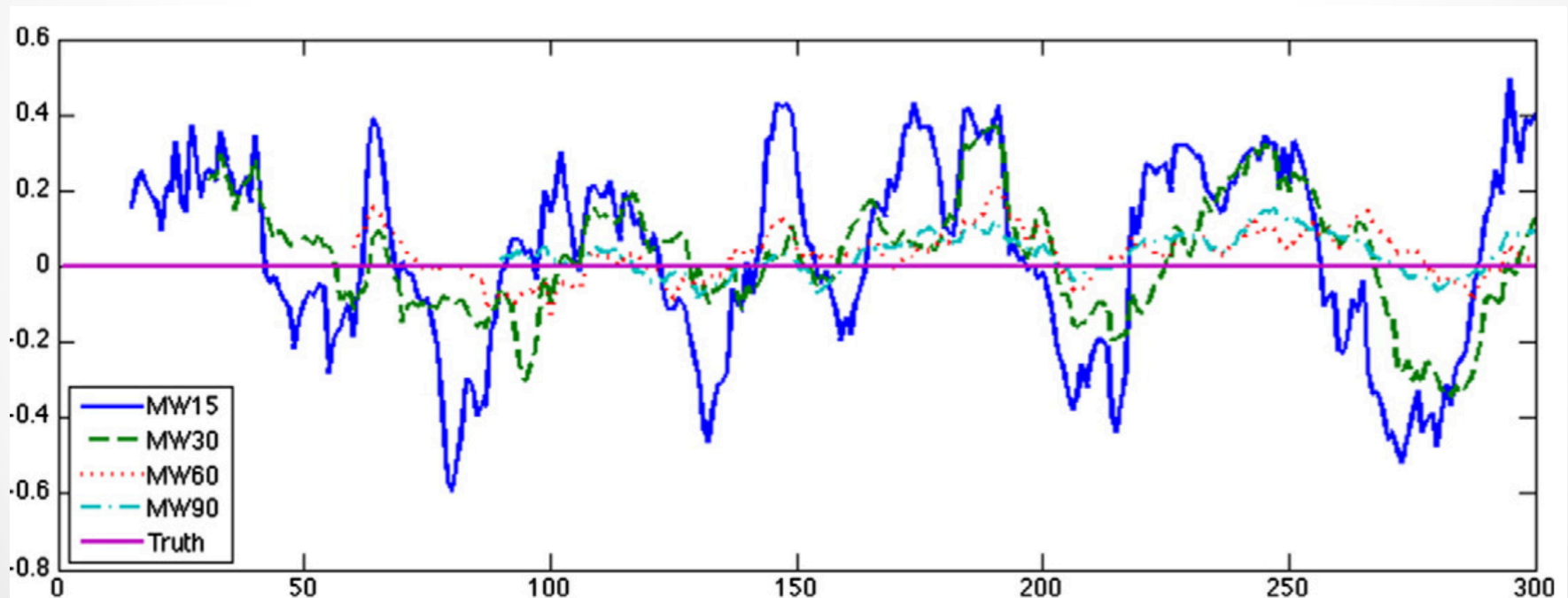


# Pros of Non-Parametric Temporal Model

- No need to hypothesize model for temporal variation
- Easy to plug-in new kinds of connectivity estimators i.e. (sparse) precision, mutual information, multiplication of temporal derivatives
- Convenient for quick prototyping

# Cons of Non-Parametric Temporal Model

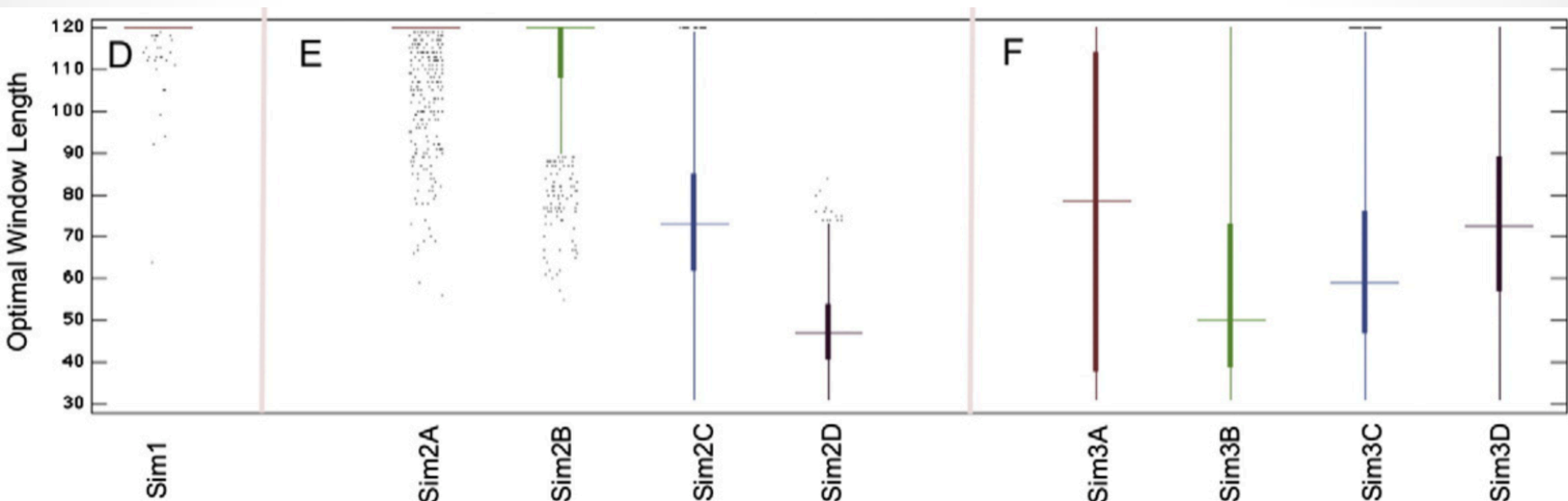
- Very limited data within each window, can lead to false positives





# Cons of Non-Parametric Temporal Model

- May be difficult to scale e.g. DCR requires an exponential number of model evaluations wrt. length of the sample in the worst case
- Often sensitive to hyper-parameters



# Outline

- ~~Introduction~~
- ~~Non-parametric temporal evolution~~
- **Parametric temporal evolution**
- Summary measures
- Inference
- Summary

# Part 2

...

Parametric temporal variation

# Univariate GARCH

- Popular for modeling financial time series
- Variance evolves following an ARMA-type model

$$y_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2 \quad \left. \vphantom{\sigma_t^2} \right\} \text{GARCH (1,1)}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i y_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \quad \left. \vphantom{\sigma_t^2} \right\} \text{GARCH (p,q)}$$

# Dynamic Conditional Correlation (DCC)

$$\sigma_{i,t}^2 = \omega_i + \alpha_i y_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 \quad \text{for } i = 1, 2$$

$$\mathbf{D}_t = \text{diag}\{\sigma_{1,t}, \sigma_{2,t}\}$$

Univariate GARCH

$$\epsilon_t = \mathbf{D}_t^{-1} \mathbf{e}_t$$

$$\mathbf{Q}_t = (1 - \theta_1 - \theta_2) \overline{\mathbf{Q}} + \theta_1 \epsilon_{t-1} \epsilon'_{t-1} + \theta_2 \mathbf{Q}_{t-1}$$

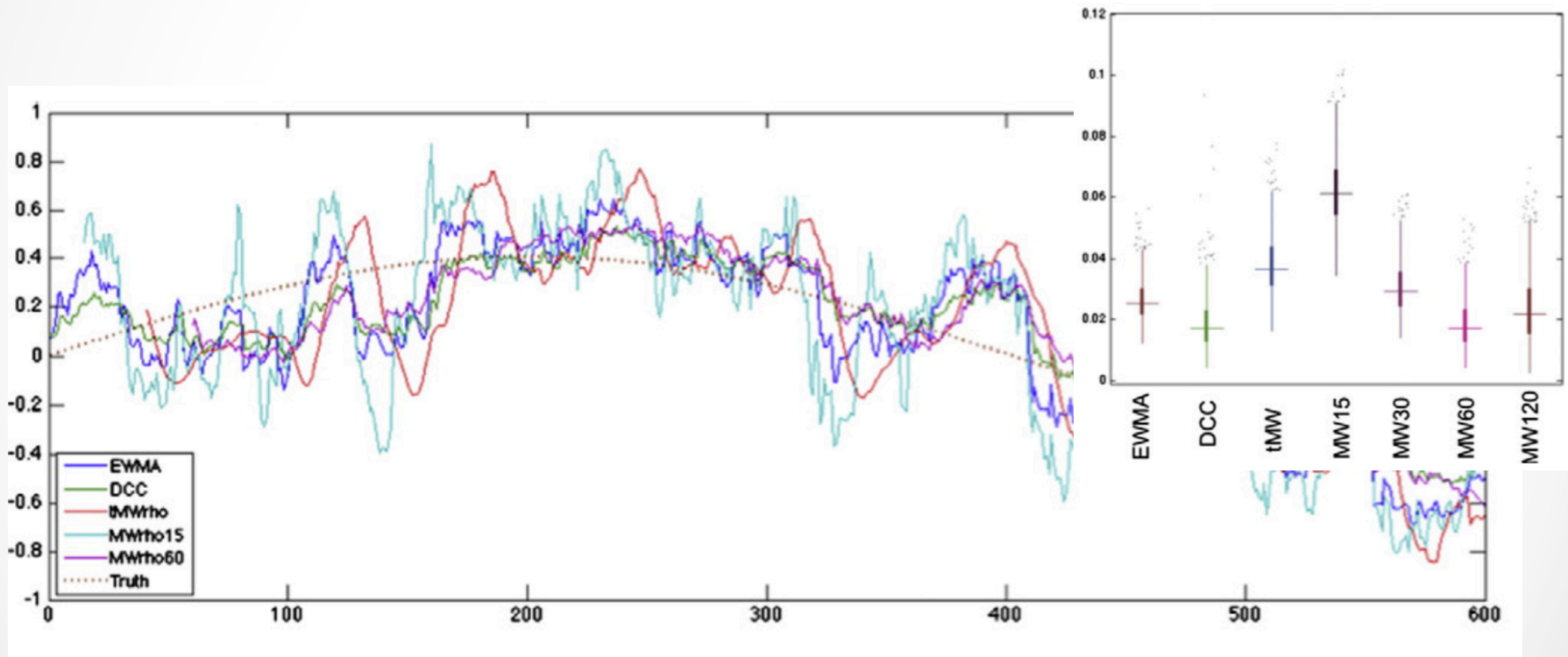
Cross-correlation

$$\mathbf{R}_t = \text{diag}\{\mathbf{Q}_t\}^{-1/2} \mathbf{Q}_t \text{diag}\{\mathbf{Q}_t\}^{-1/2}$$

$$\Sigma_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t.$$

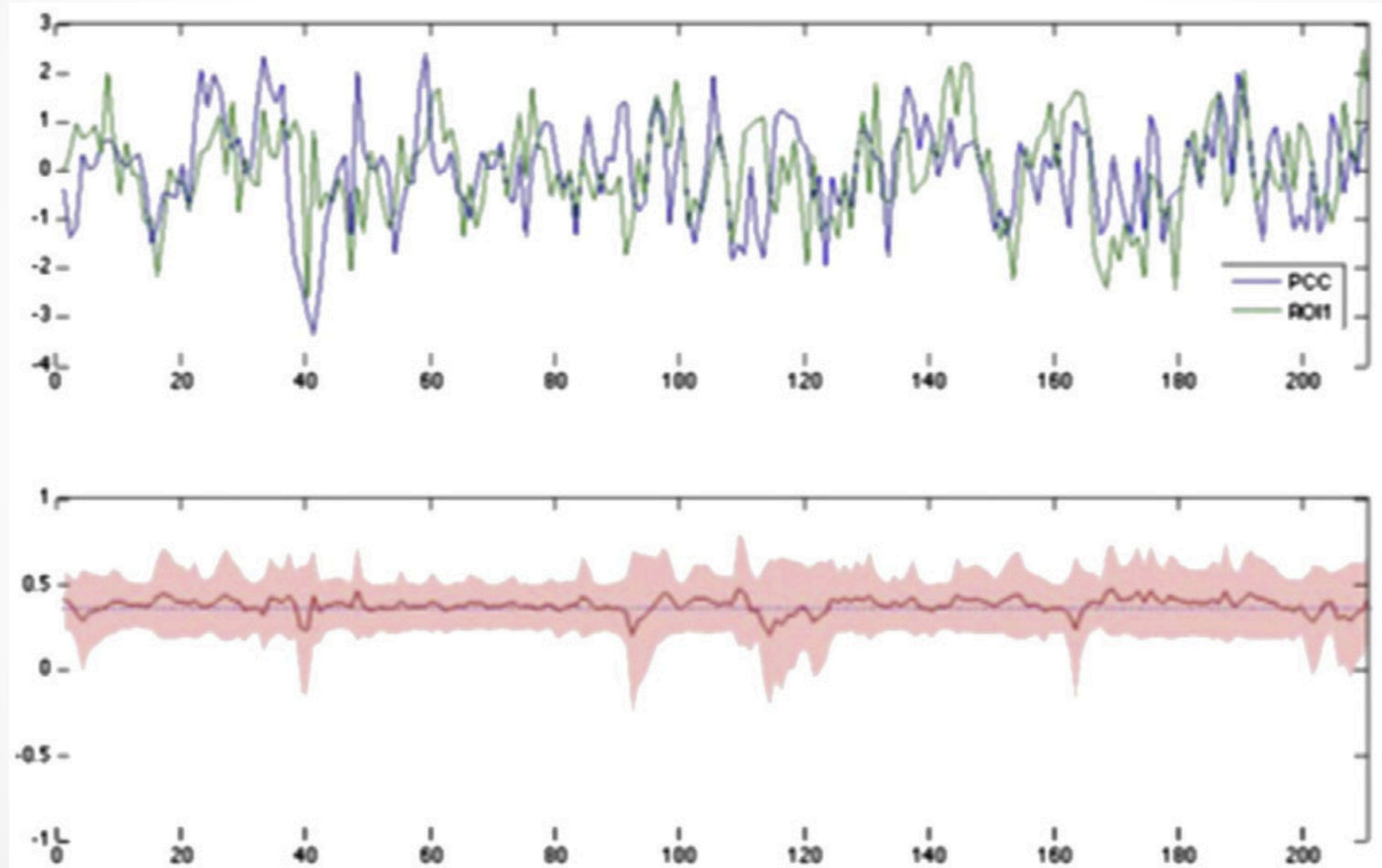
Combined covariance

# Sinusoidal Signal

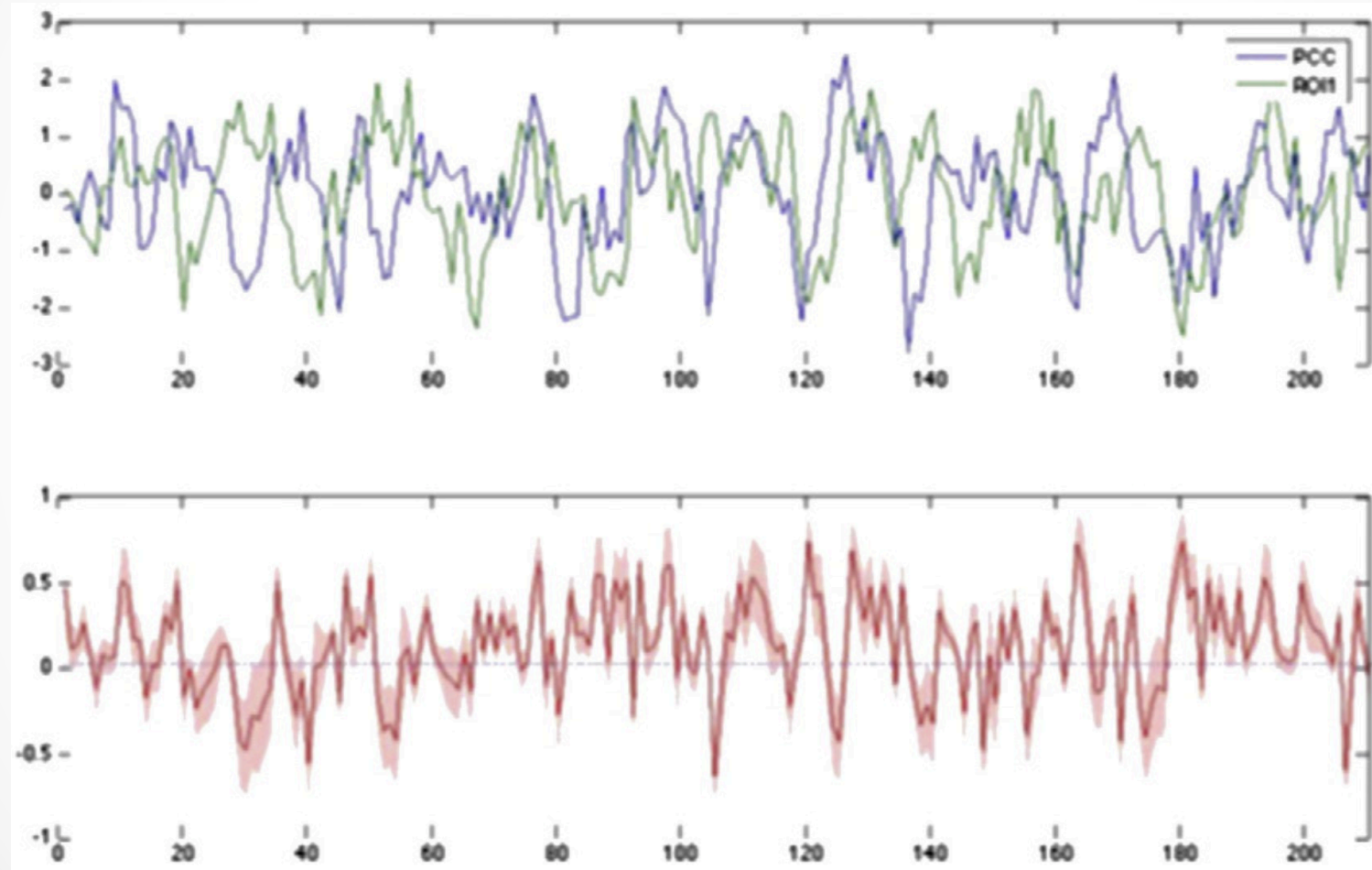




# Application to Kirby 21 Dataset

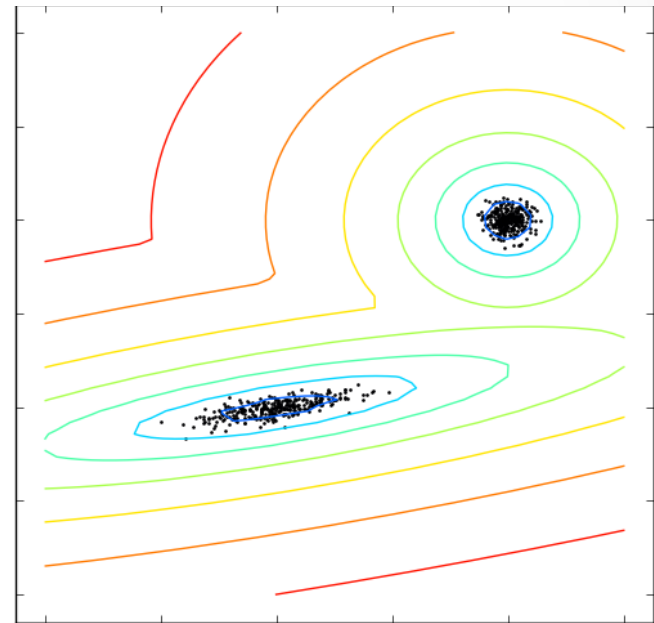
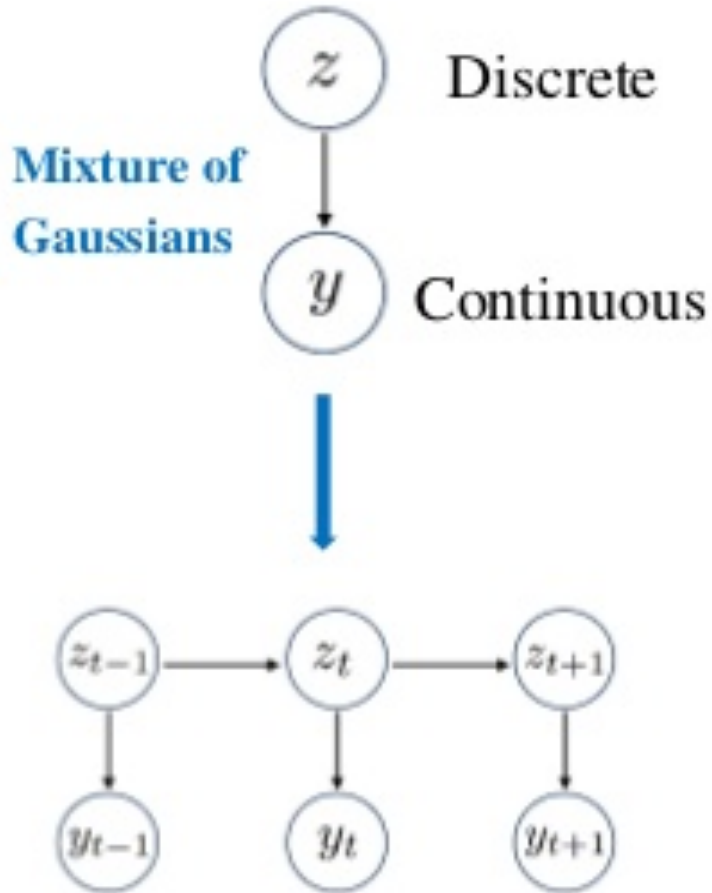


# Application to Kirby 21 Dataset



# Discrete State Hidden Markov Model

- Direct analogue to clustering

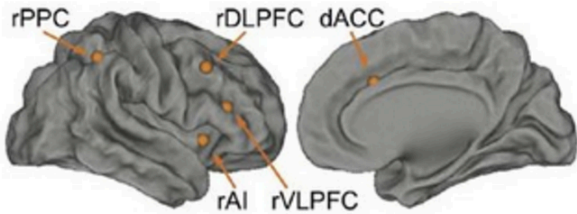


$$z_t \sim P(z_t | z_{t-1})$$

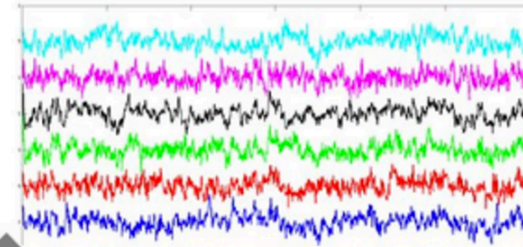
$$y_t \sim \mathcal{N}(\mu_{z_t}, \Sigma_{z_t})$$

Liu (2014)

## Regions of Interest

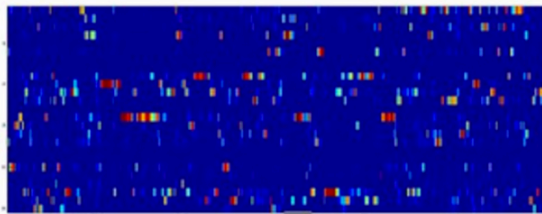


## Time Series

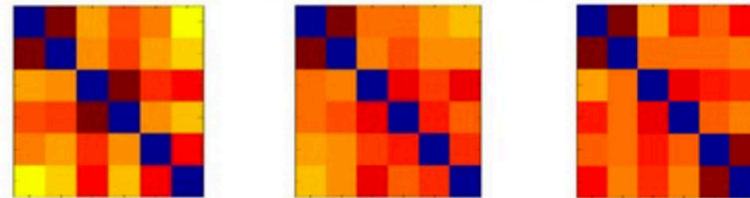


**VB-HMM**

Posterior probabilities of states



Partial Correlations of each state

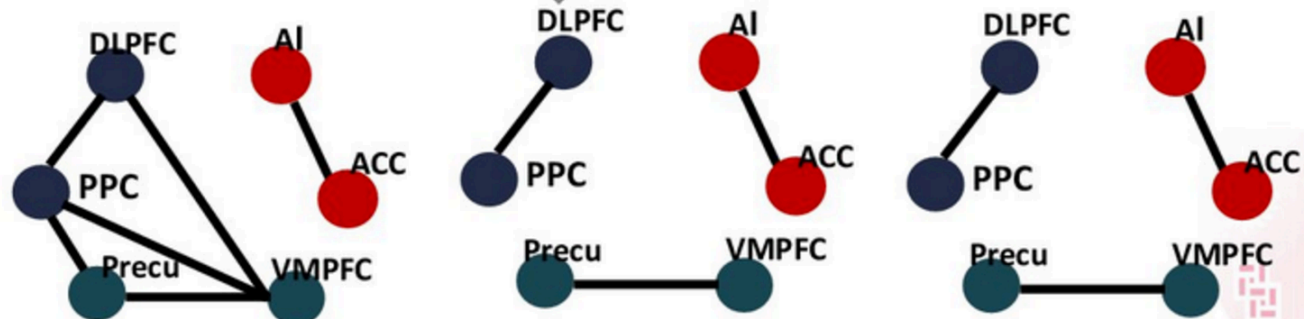


**Viterbi Decoding**

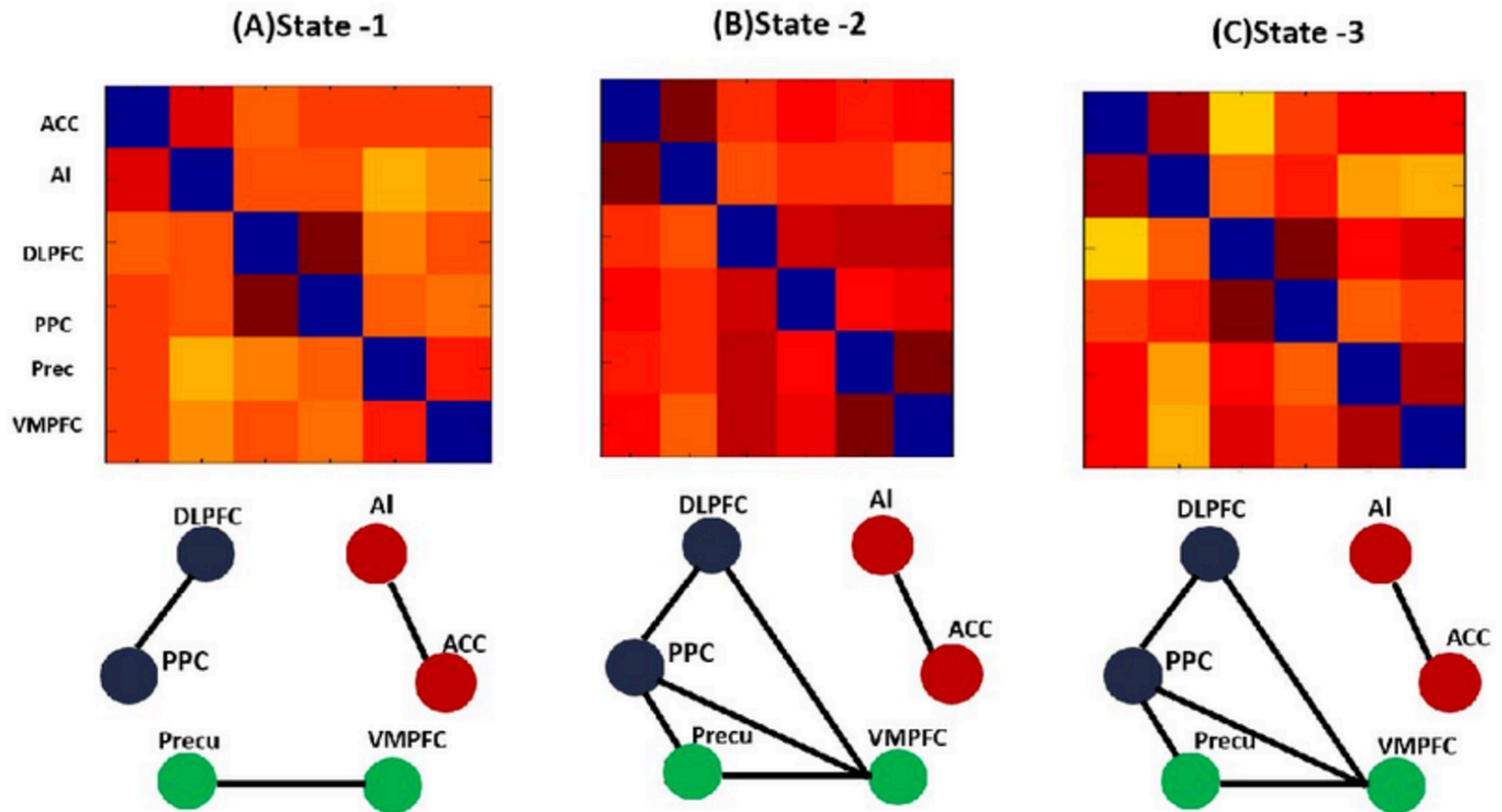
Percent Occupancy,  
Mean Life

Ryali et. al. (2015)

**Community detection**

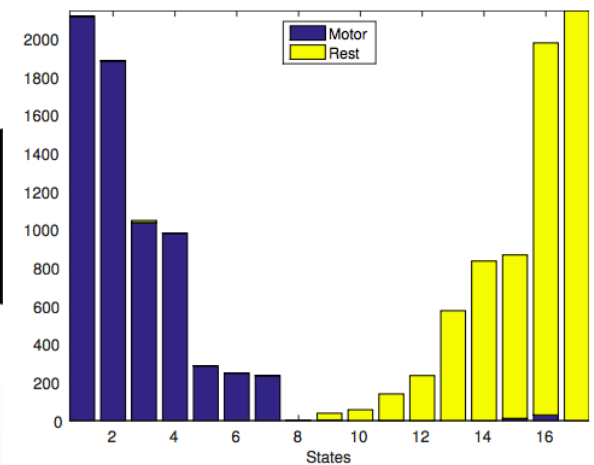
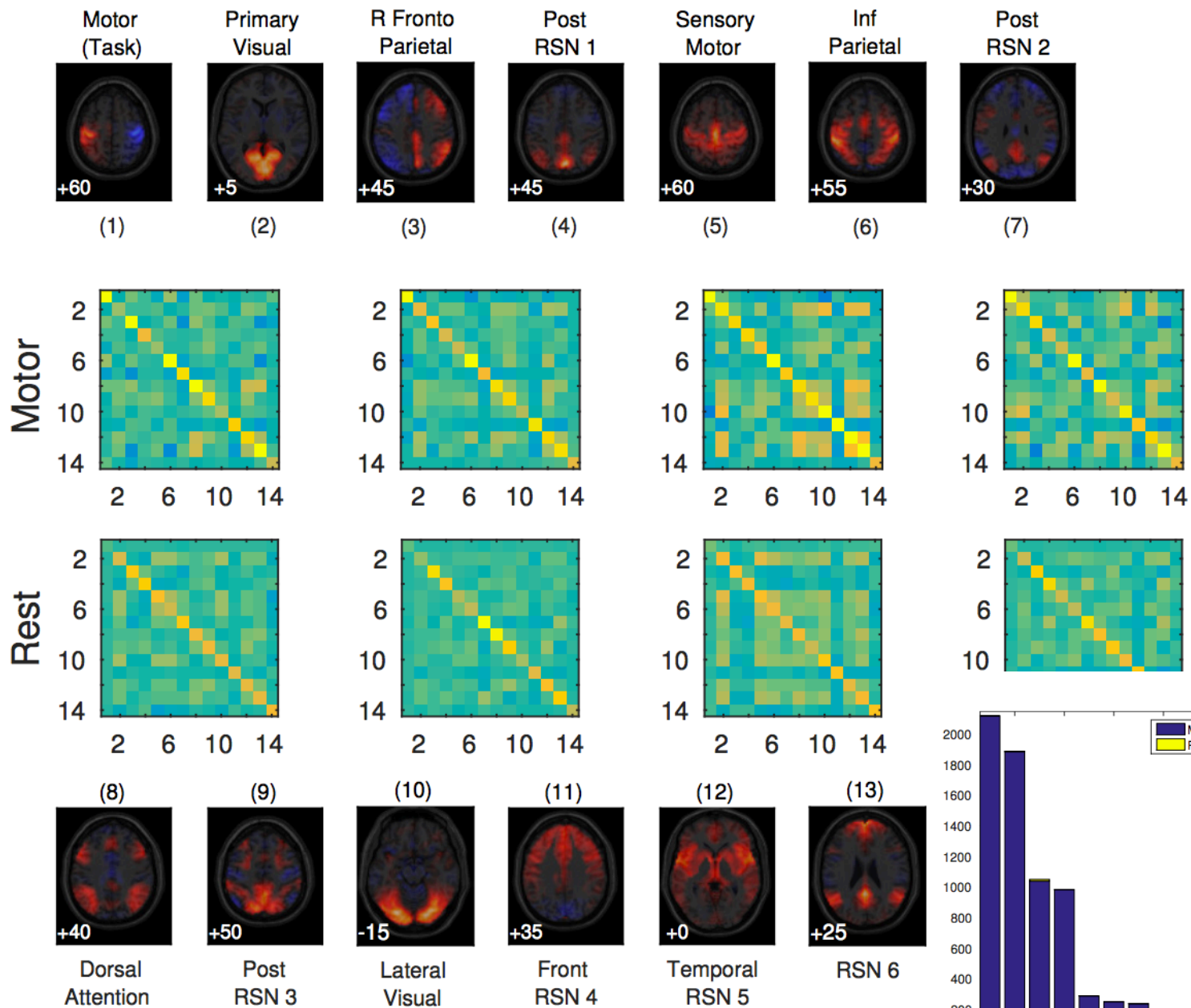


# Estimated Network States



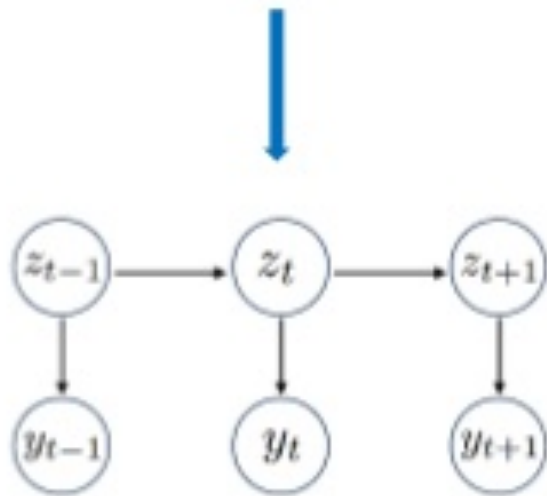
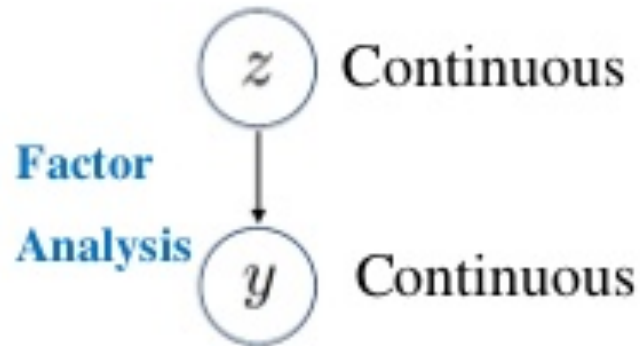
Ryali et. al. (2015)





- Nielsen et. al. (2016)

# Continuous State Hidden Markov Model



Liu (2014)

- Direct analogue to factor analysis

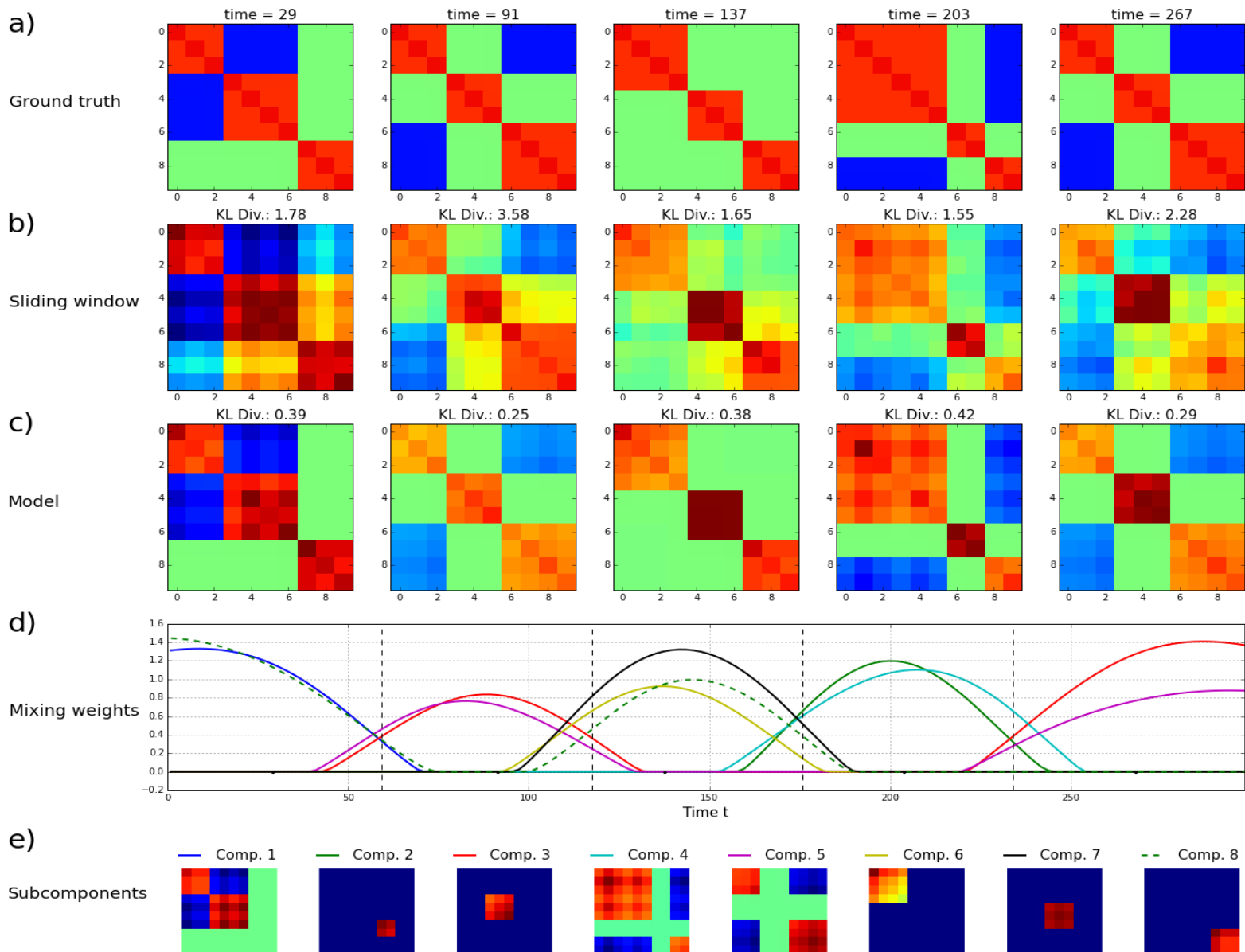
$$z_t \sim \mathcal{N}(0, S_t)$$

$$y_t \sim \mathcal{N}(V z_t, \sigma^2 I)$$

- Equivalent to evolving covariance model

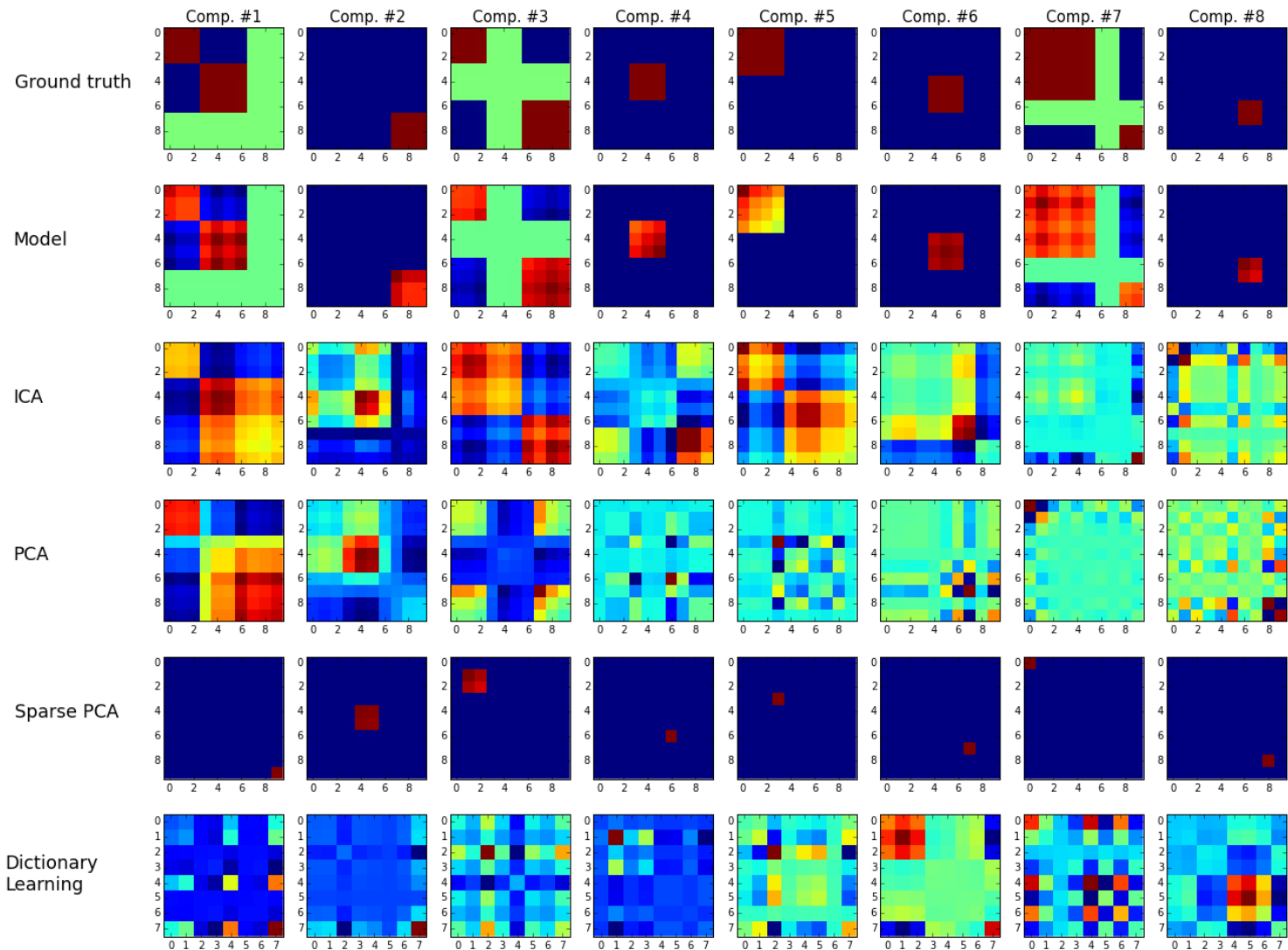
$$S_t \sim P(S_{t-1})$$

$$y_t \sim \mathcal{N}(0, V S_t V' + \sigma^2 I)$$





# Factor Model Visualization



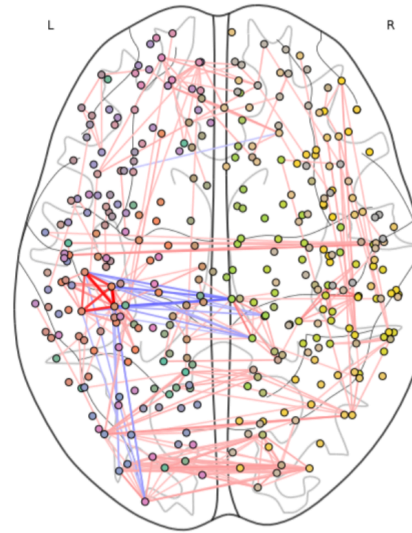
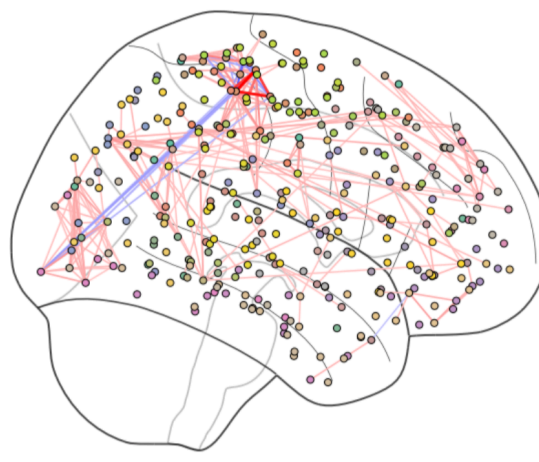
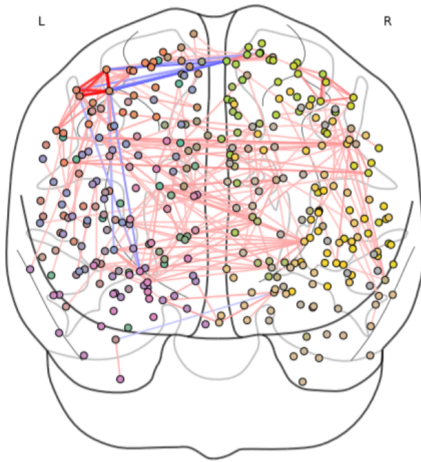
# Classification Accuracy

- HCP data, Gordon 333 atlas, Motor task
- Task block + motion regressed out, model the residual
- Train on 5 subjects, test on held out subjects using log likelihood

Task	Classification using model	Random guessing
Right Hand Tapping	0.784 (0.078)	0.167 (0.112)
Left Foot Tapping	0.523 (0.197)	0.169 (0.116)
Tongue Wagging	0.420 (0.136)	0.174 (0.121)
Right Foot Tapping	0.409 (0.208)	0.170 (0.112)
Left Hand Tapping	0.761 (0.136)	0.168 (0.116)
Rest	0.352 (0.132)	0.161 (0.111)

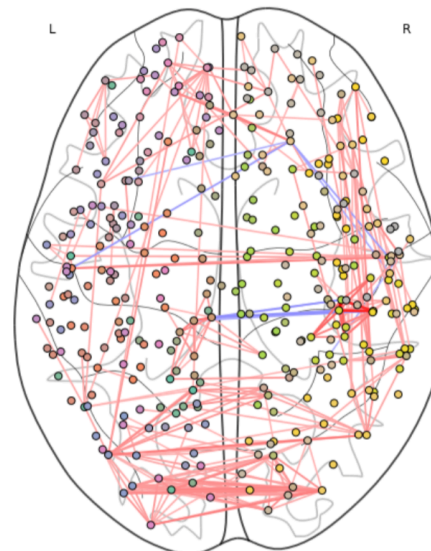
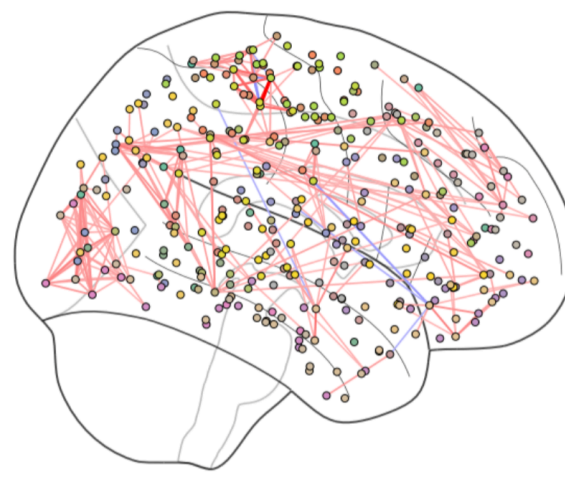
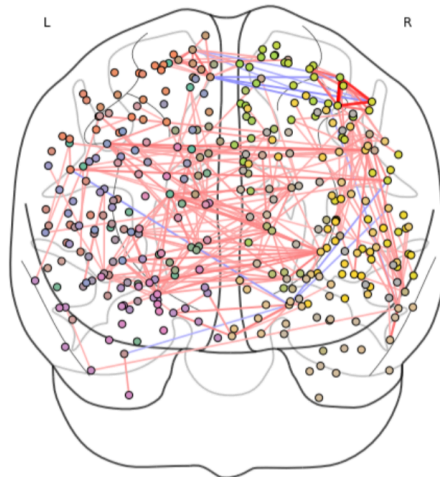
Andersen et. al. (2016), In prep.

Subset 1: Precision matrix for Right Hand Tapping

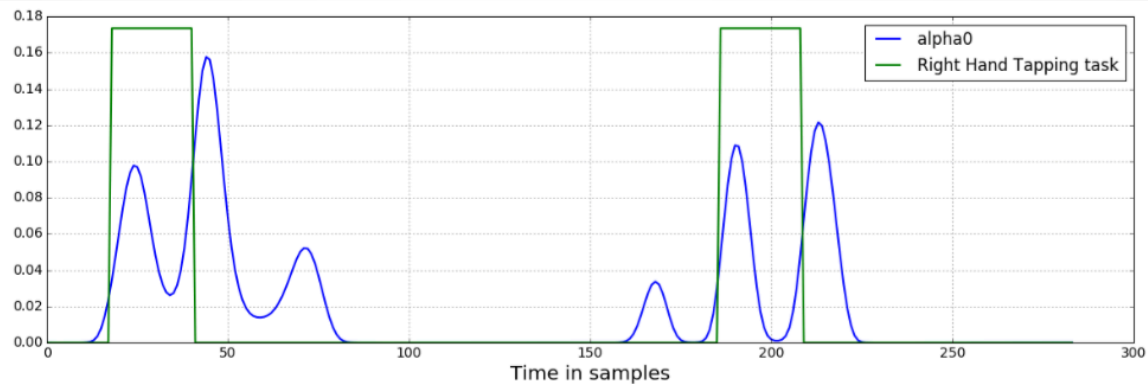


Right hand

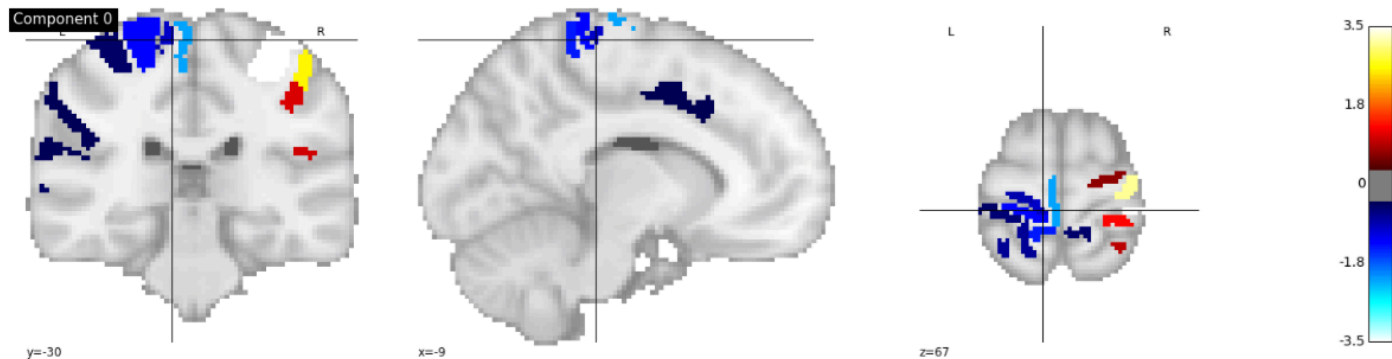
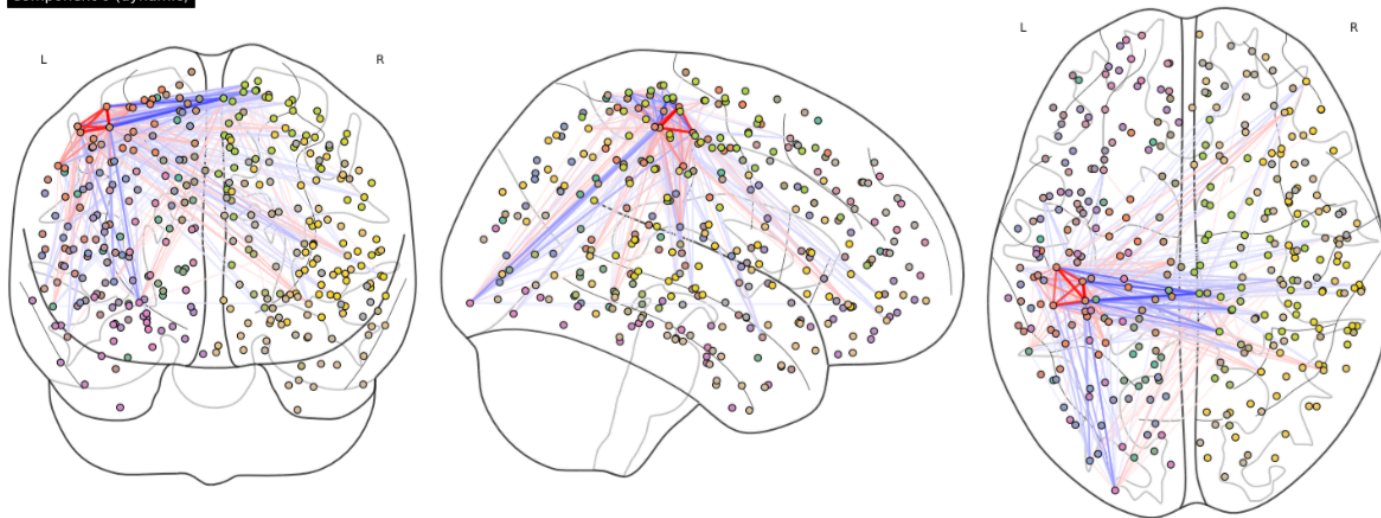
Subset 1: Precision matrix for Left Hand Tapping



Left hand



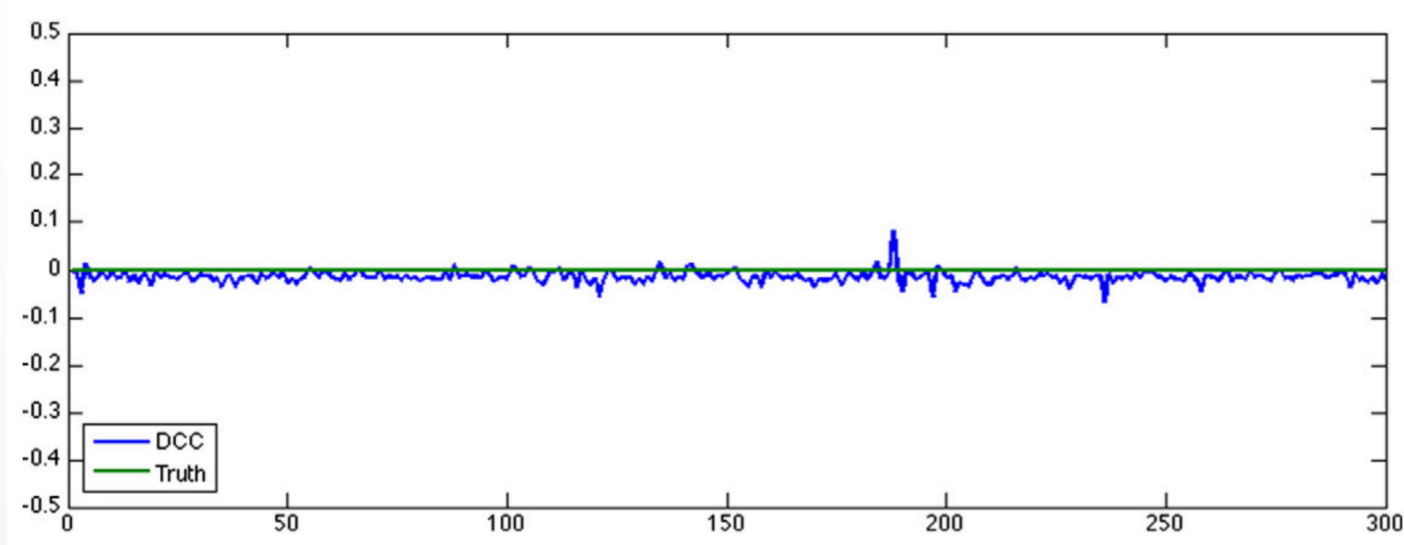
Component 0 (dynamic)



Andersen et. al. (2016), In prep.

# Pros of Parametric Temporal Model

- Very accurate when model structure is evident in the data
- Tends to be conservative when model structure is not a strong signal



# Pros of Parametric Temporal Model

- Explicit about underlying assumptions
- Model summaries are often built-in (discrete HMM)  
e.g. graph states, temporal variation
- Estimation can be faster than non-parametric approaches for simple models
- Certain parametric models have built-in inference



# Cons of Parametric Temporal Model

- Often requires expert knowledge to develop and fit the model e.g. variational inference, Viterbi decoding, ...
- May be computationally expensive, particularly when using complicated models with many parameters
- As in all models, some risk of false negative when model does not match data

# Ipython Notebook Example

...

Comparing sliding window to HMM model fit



# Outline

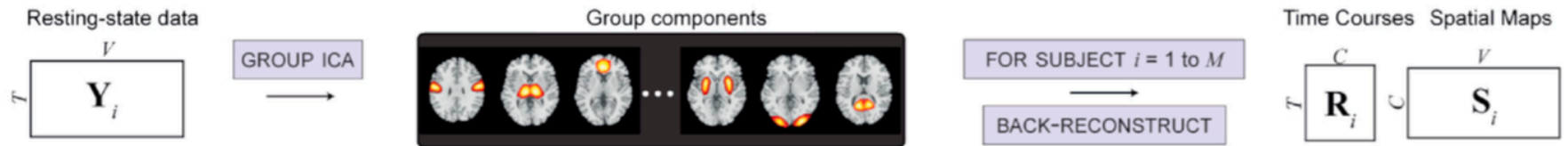
- ~~Introduction~~
- ~~Non-parametric temporal evolution~~
- ~~Parametric temporal evolution~~
- **Summary measures**
- Inference
- Summary

# Part 3

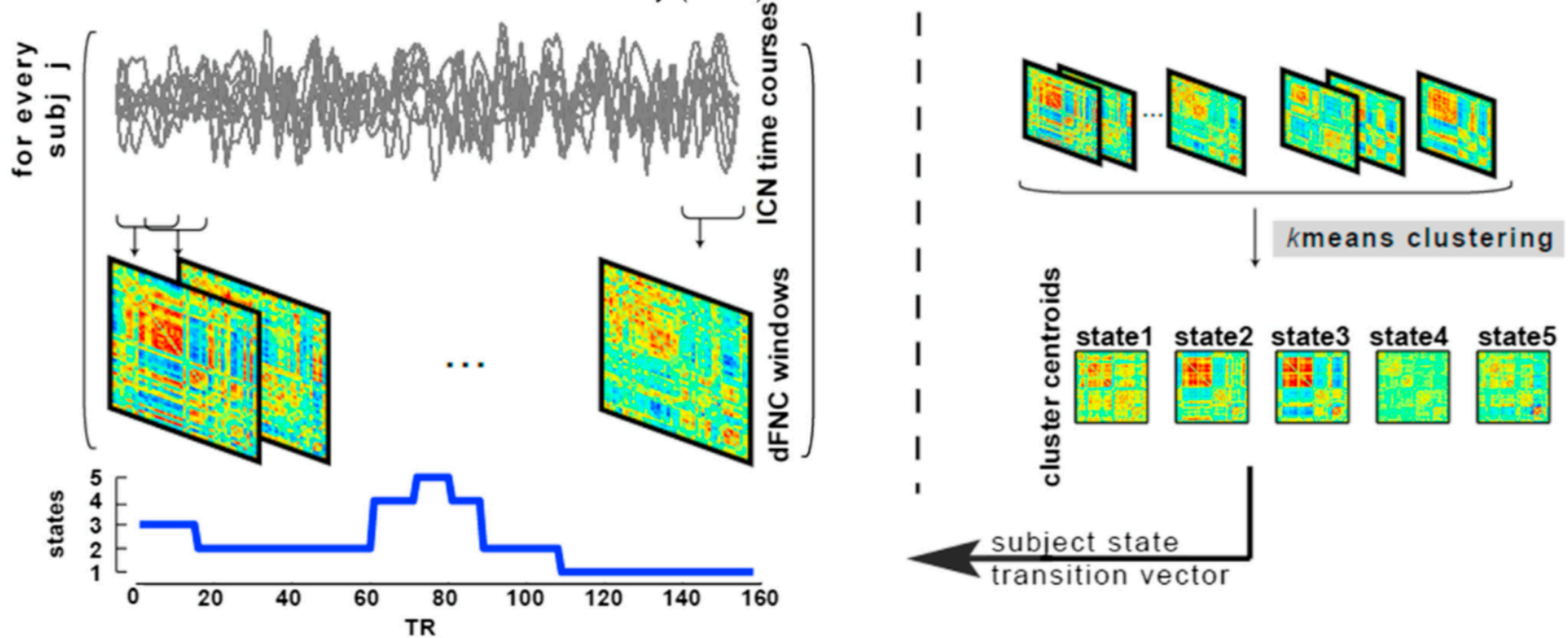
• • •

Summary Measures

## A Identification of intrinsic connectivity networks (ICNs)

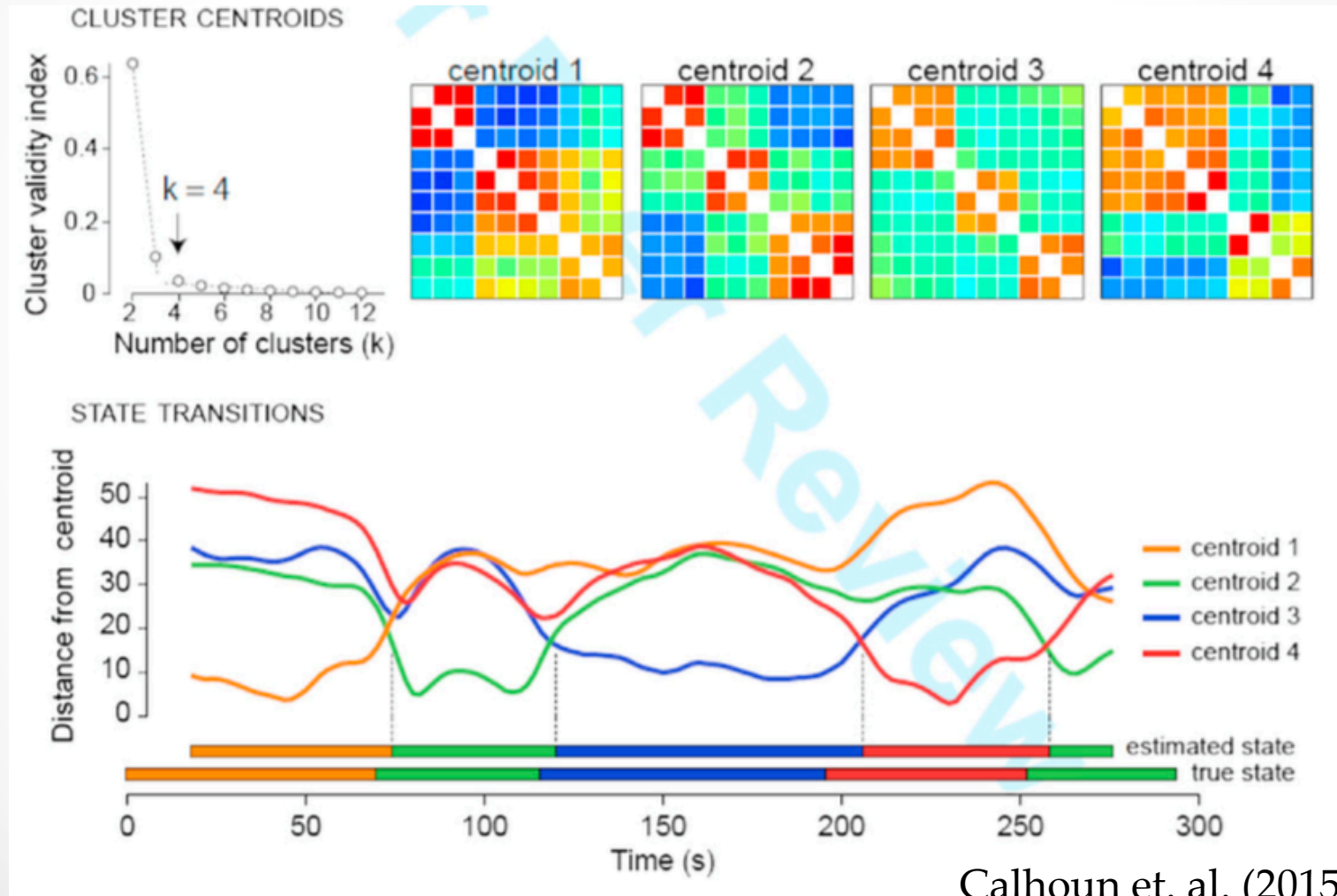


## B Assessment of functional network connectivity (FNC) between ICNs

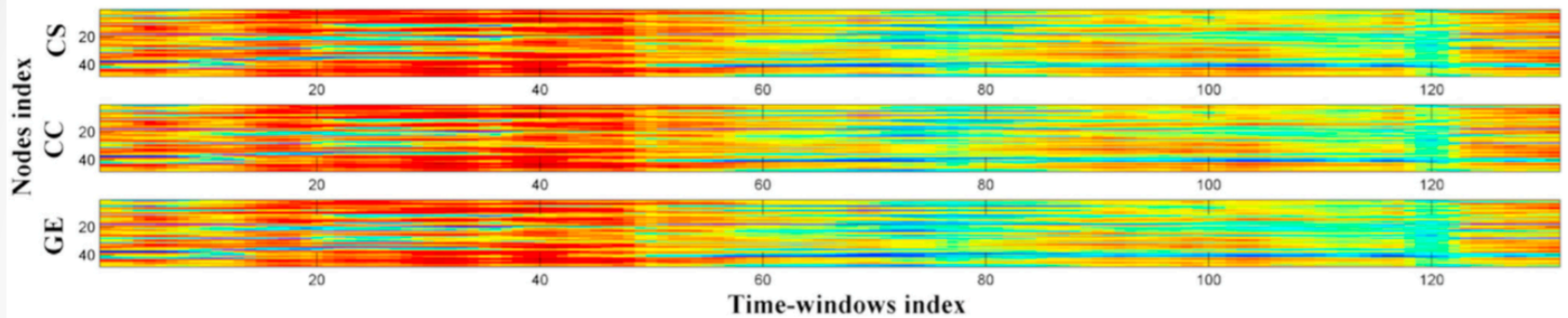


Calhoun et. al. (2015)

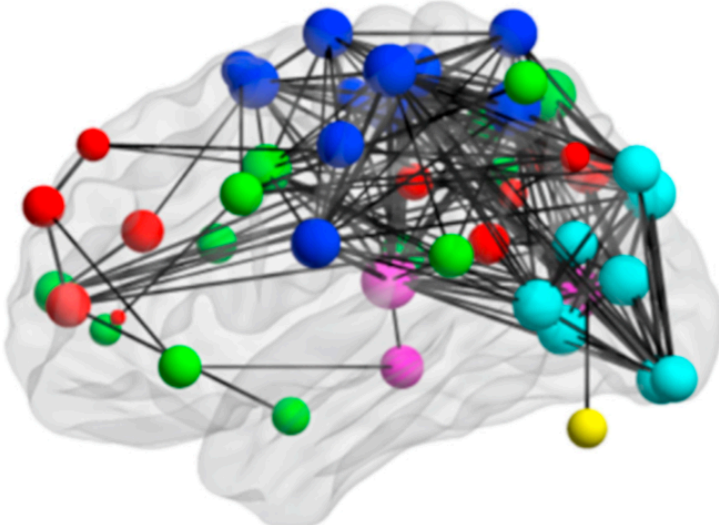
# Cluster & Estimate States



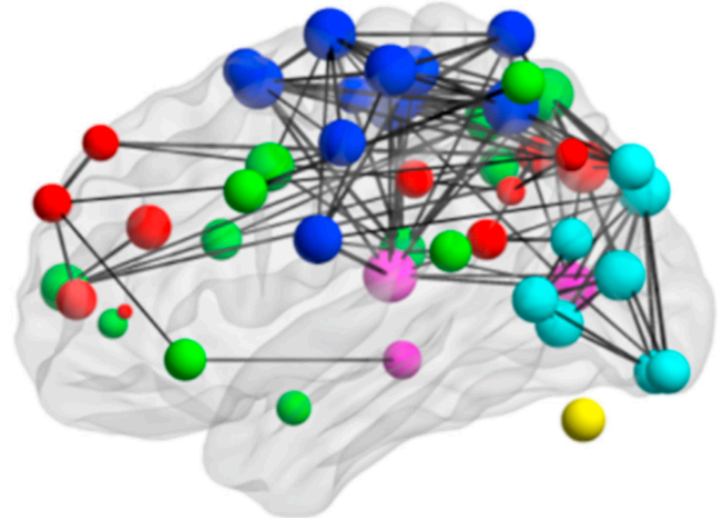
# Model State Transitions



HC

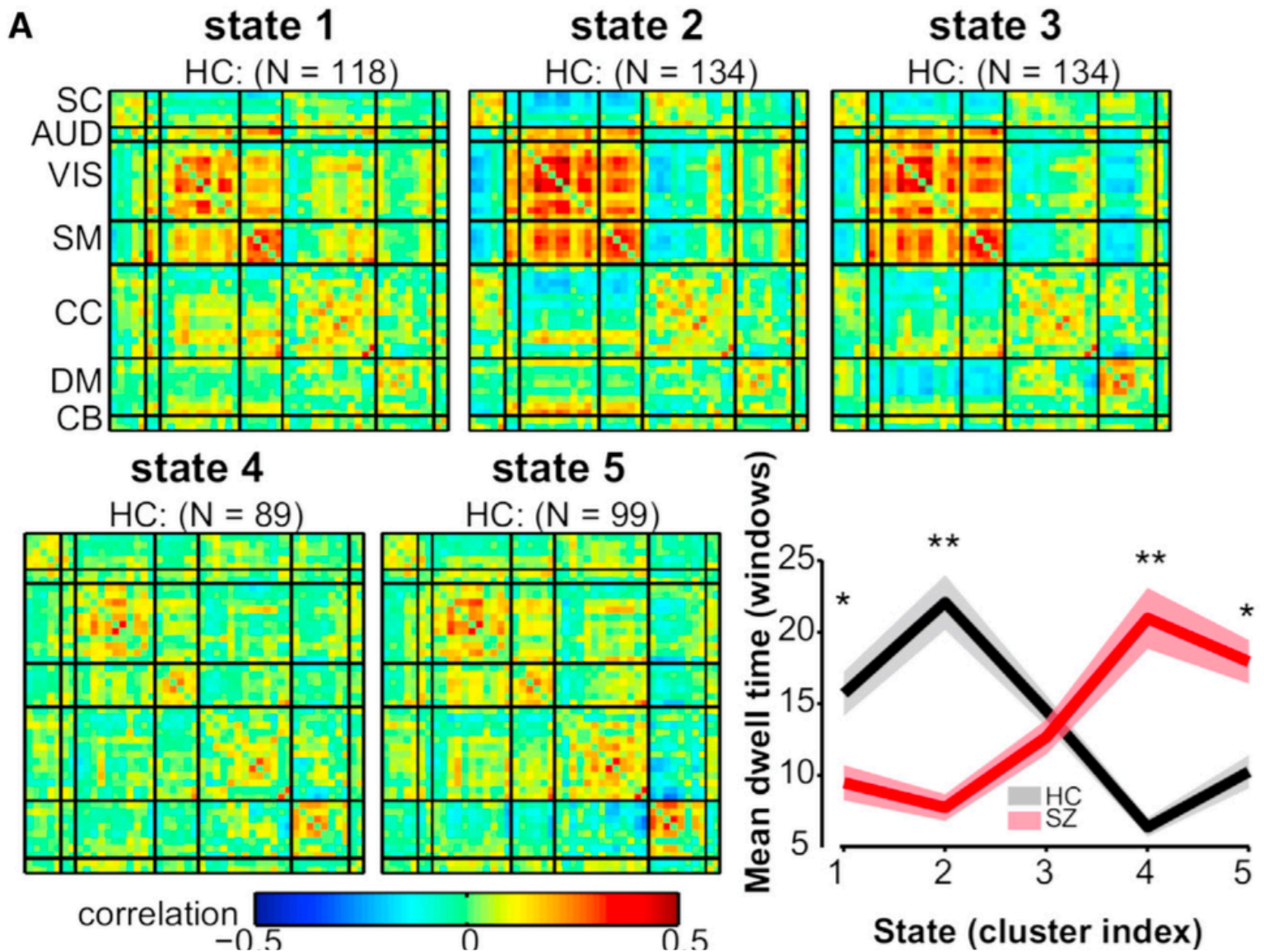


SZ



Calhoun et. al. (2015)



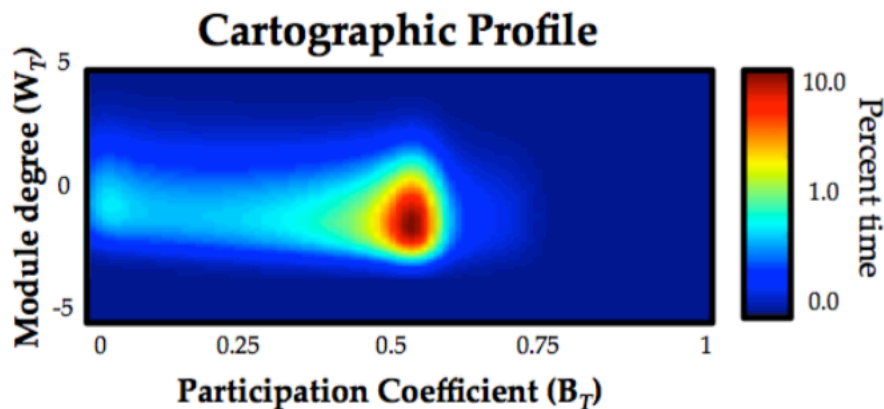


Dynamics of Schizophrenia vs. healthy controls  
(Calhoun et. al., 2015)

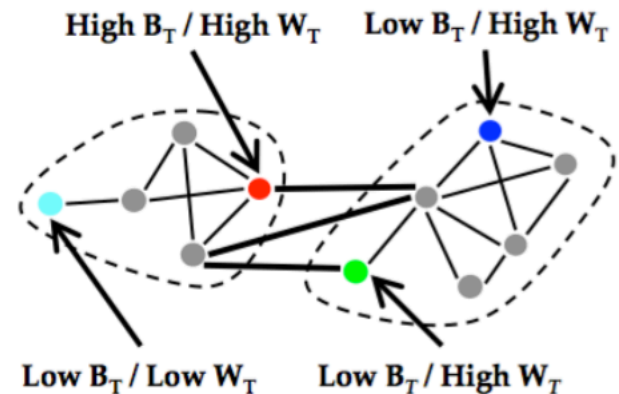
# Cartographic Profiling

- Estimate modules (clusters) between voxels/regions at each time point
- Compute graph statistics e.g. module degree, participation coefficient

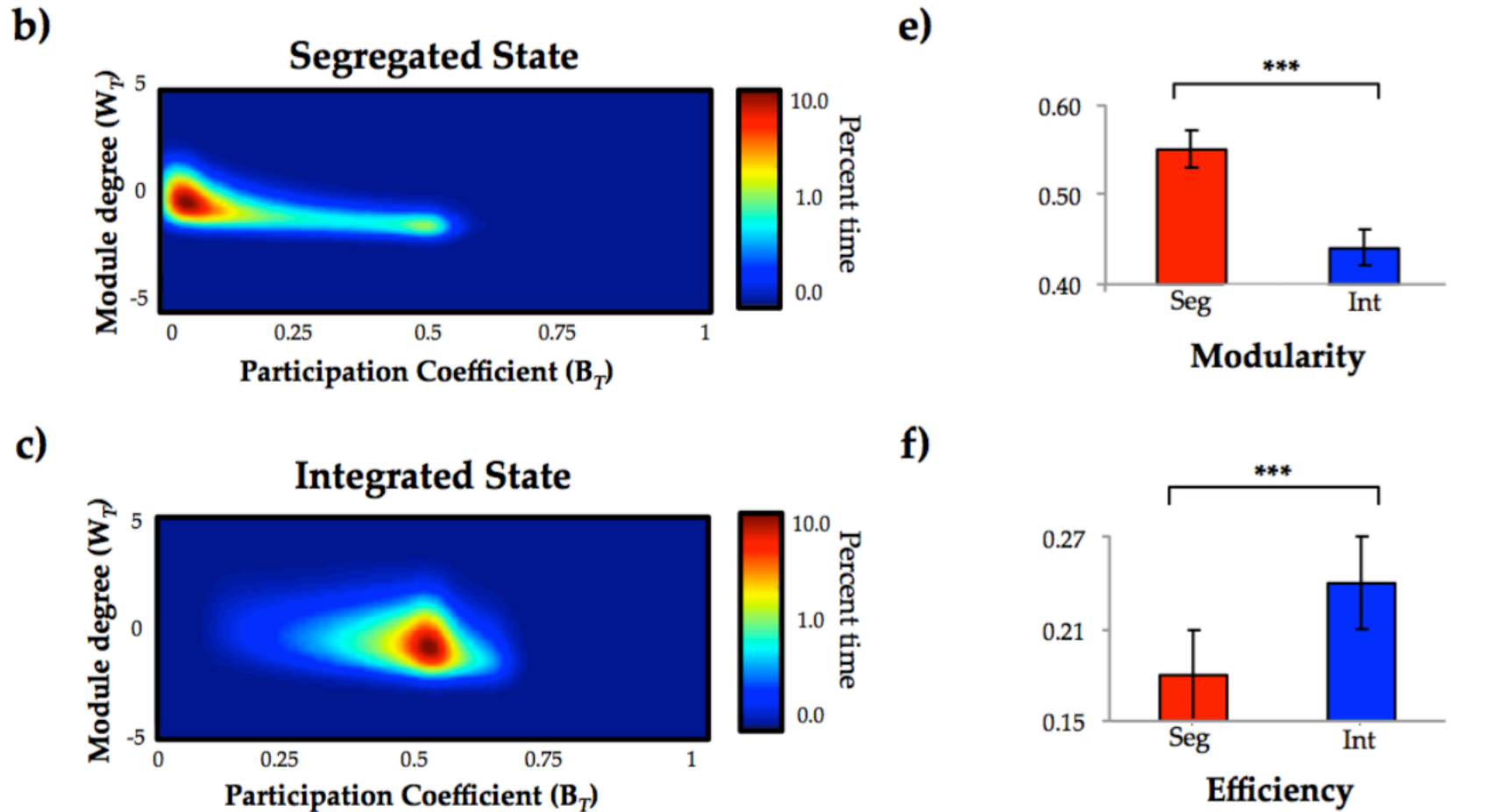
a)



d)

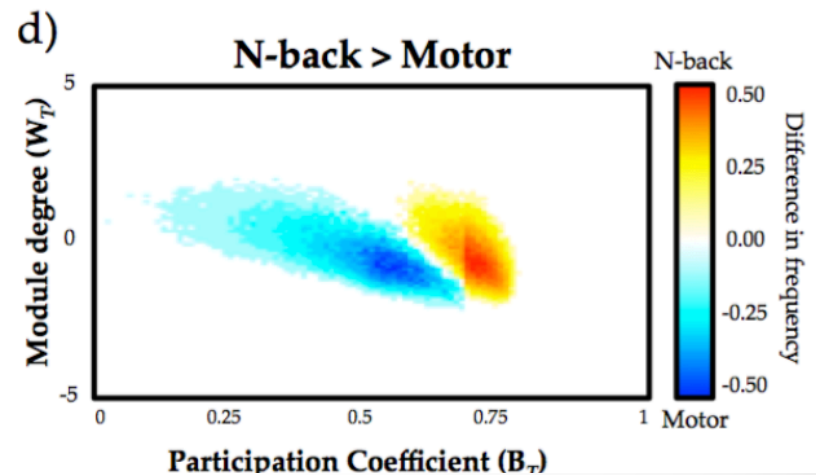
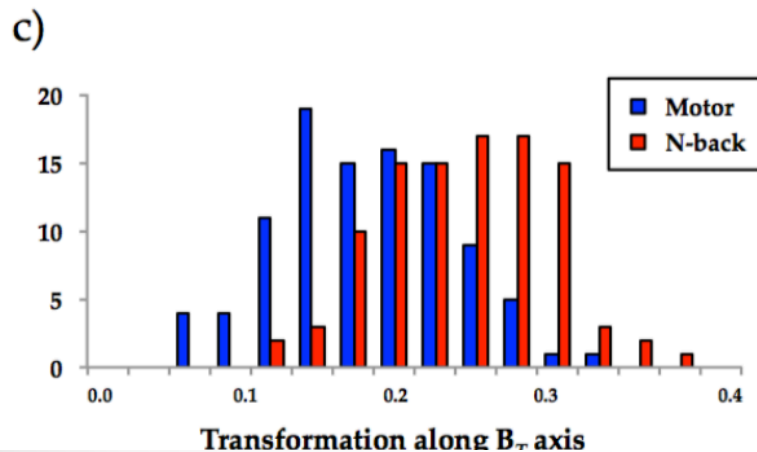
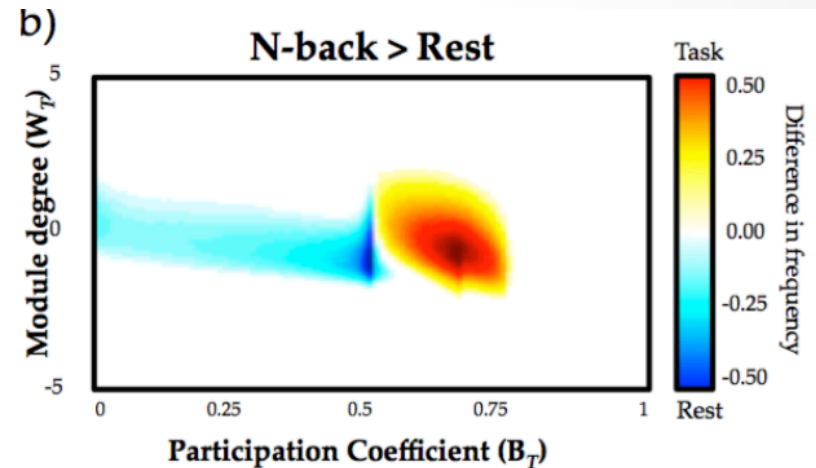
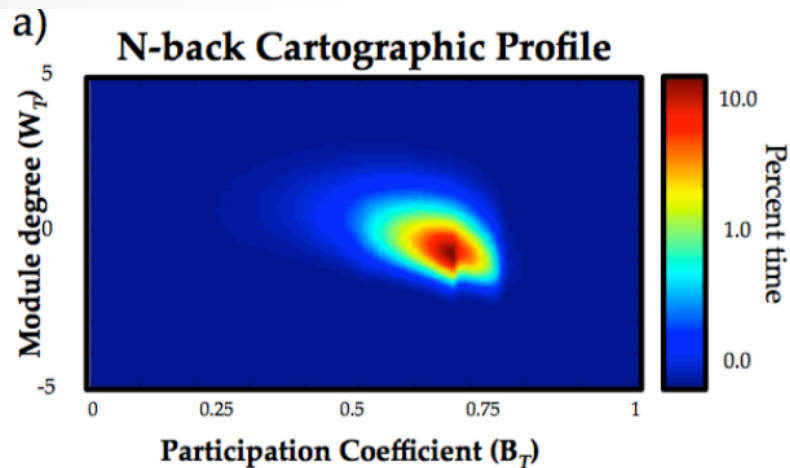


# Distinct Segregated and Integrated States





# Differences in Task vs. Rest



# Summaries for Parametric Temporal Evolution

- Parametric models often have “natural” interpretations e.g. Gaussian HMM automatically estimates “states”
- However, can be difficult to synthesize interpretation for large models
- Suggest to combine both parametric and non-parametric summaries to fully explore the results

# Outline

- ~~Introduction~~
- ~~Non-parametric temporal evolution~~
- ~~Parametric temporal evolution~~
- ~~Summary measures~~
- **Inference**
- Summary

# Part 4

...

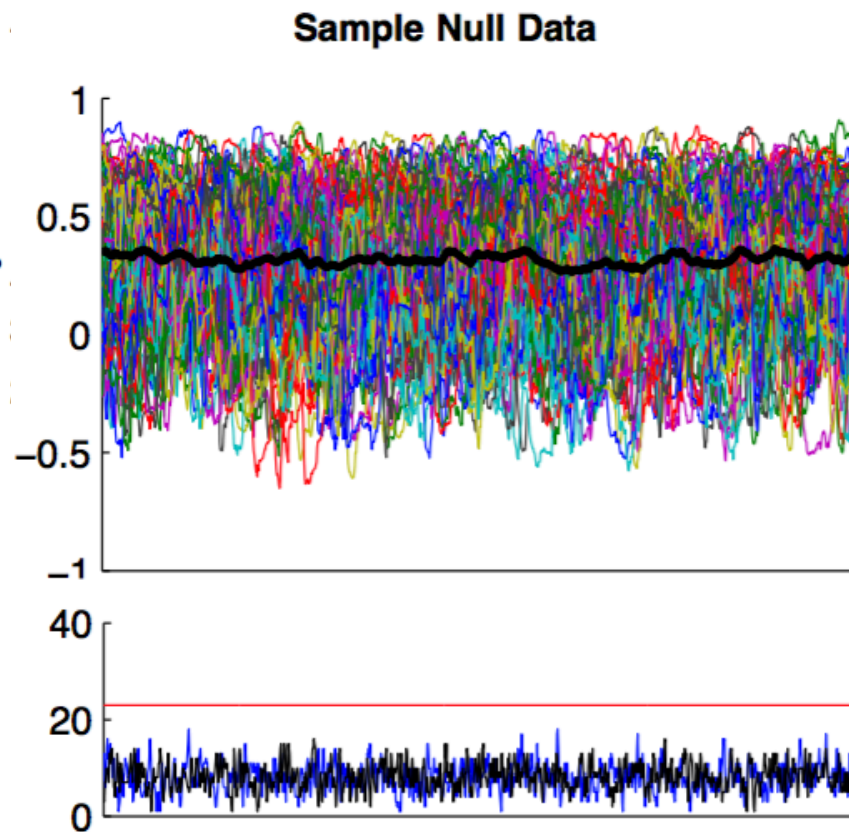
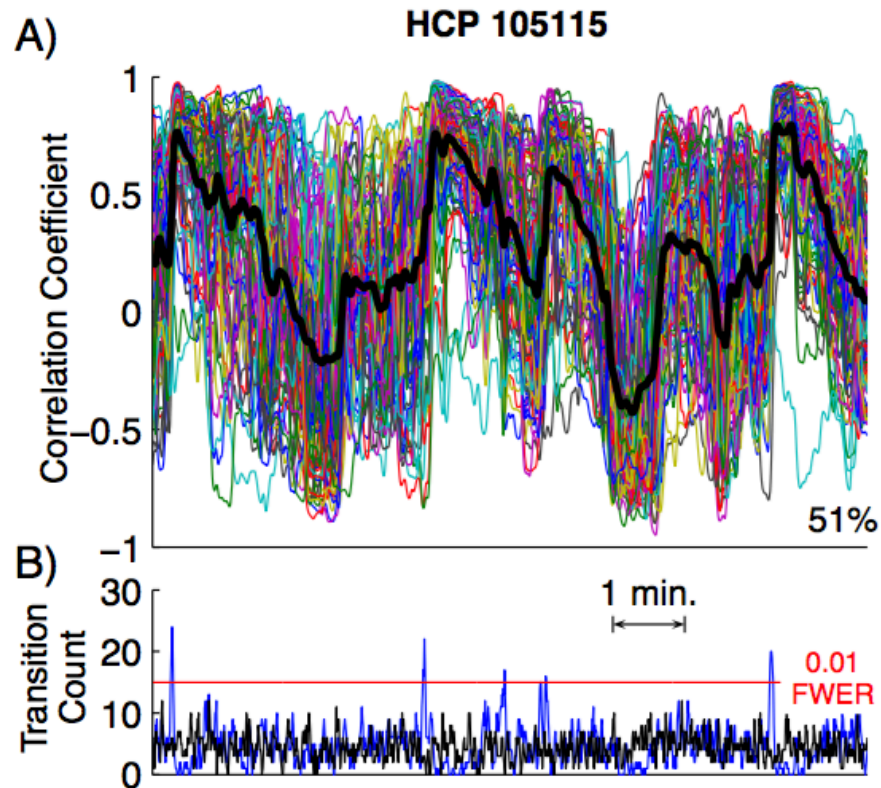
Inference

# Asymptotic Tests

- Often interested in rejecting the null hypothesis that non-zero graph edges are due to chance
- Asymptotic tests are not exact, but typically perform well in simulation tests
- There is a test statistic for DCC that is asymptotically normal (Engle & Sheppard., 2001)
- There is a test statistic for sliding window kernel (sparse) precision estimation that is asymptotically normal, even for high dimensional data (Wang & Kolar, 2014, Junwei et. al., 2015)

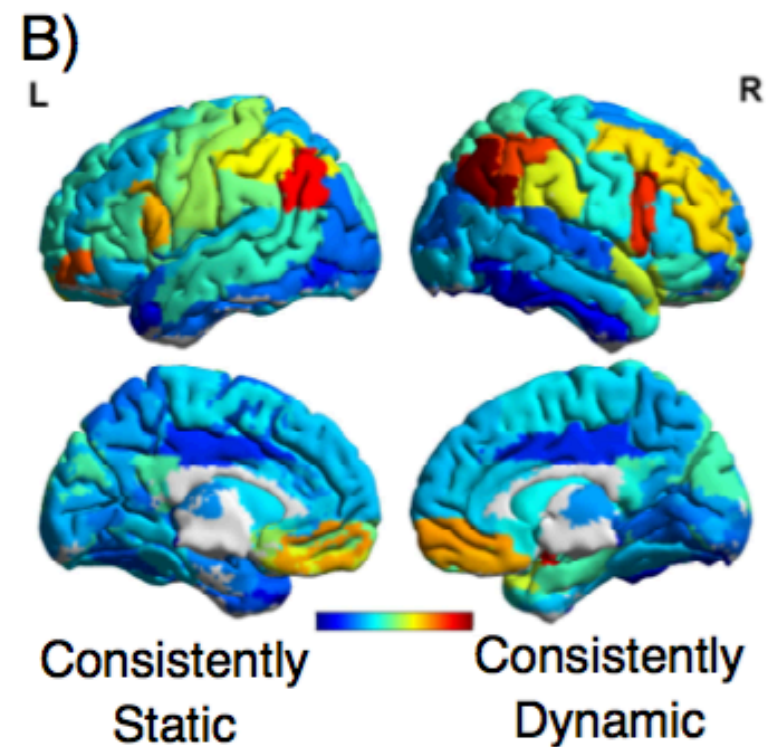
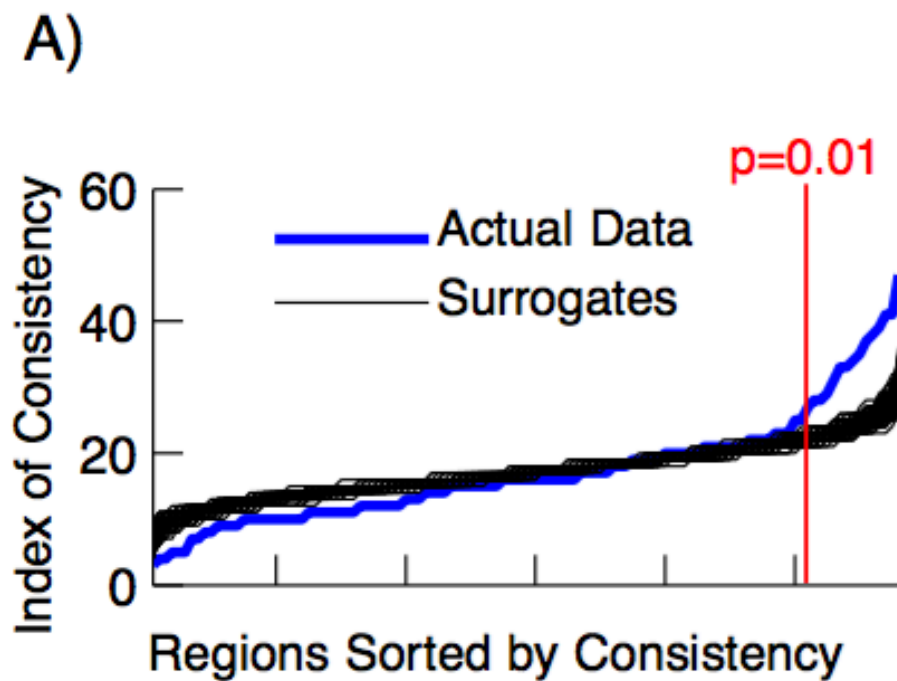
# Non-parametric test

- Parametric tests may not exist for interesting statistics such as summary measures
- **Non-parametric approach:** generate multiple synthetic time series that are matched to the time averaged connectivity e.g. from vector autoregressive (VAR) model with matched static connectivity
- Compare statistics from stationary model with statistics from the presumed dynamic model using standard non-parametric one-sample test



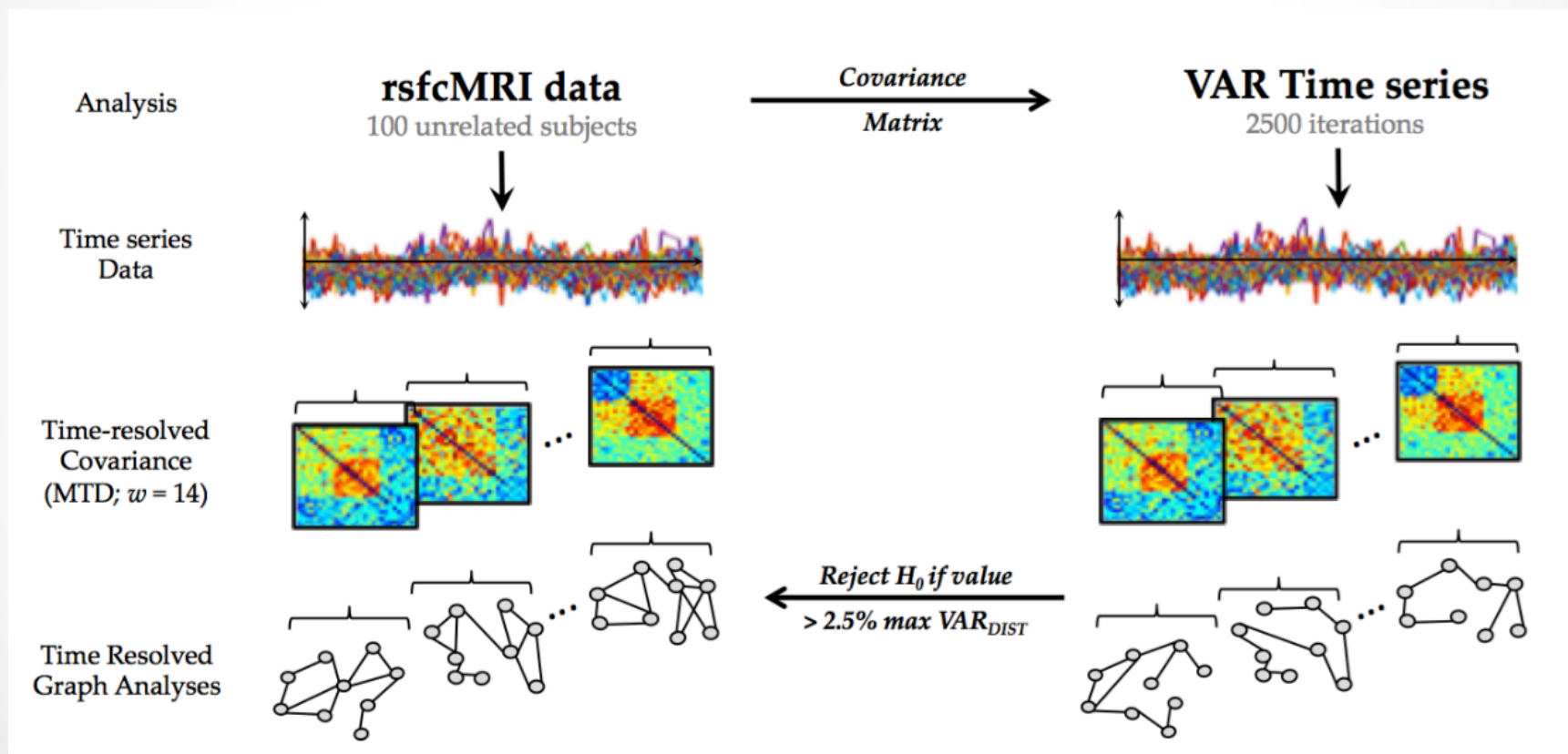
Example from Zalesky et. al. (2014)





Example from Zalesky et. al. (2014)





Example from Shine et. al. (2016), Submitted

# Conclusion

- Discussed parametric vs. nonparametric approaches for modeling temporal variation
  - Standard tradeoffs between parametric vs. non-parametric estimators
- Discussed model summary using clustering and cartographic profiling
  - also useful for parametric evolution models
- Discussed inference using parametric techniques (in a few cases) or non-parametric techniques

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# Software

- Discrete Hidden Markov Models:  
<https://github.com/hmmlearn/hmmlearn>
- MTD: <https://github.com/macshine/coupling>
- DCC (Lindquist):  
[https://github.com/canlab/Lindquist\\_Dynamic\\_Correlation](https://github.com/canlab/Lindquist_Dynamic_Correlation)

# Thank You!!!

Questions?

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