Submodular maximization for random streams

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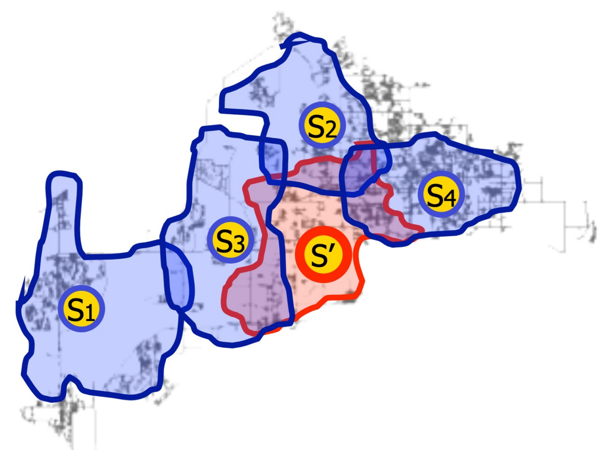
NEURAL INFORMATION PROCESSING SYSTEMS

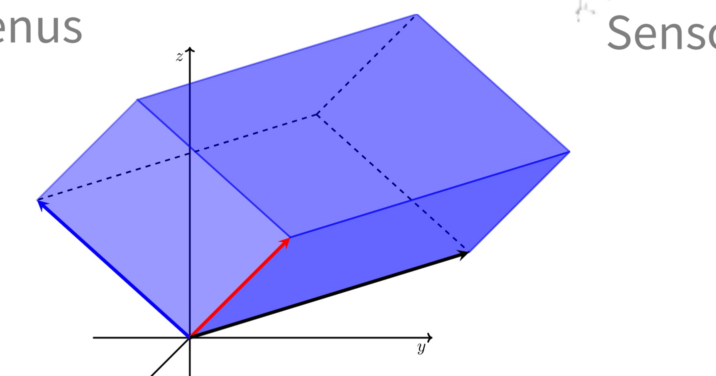
MOTIVATION

Are submodular functions in your backyard?

McDonald's Menus



Sensor placement



$f(S) = \log\det A_S^T A_S$

Determinantal point processes

Submodularity: $f(S\cup\{e\})-f(S) \geq f(T\cup\{e\})-f(T)$ when $S\subseteq T$.
Monotone: $f(S\cup\{e\}) \geq f(S)$
Typical application: $\max_{|S|\leq k} f(S)$ for a given k.

- "cardinality constrained submodular maximization"

REQUIREMENTS OF MODERN SYSTEMS

Modern data often comes in the form of a stream.

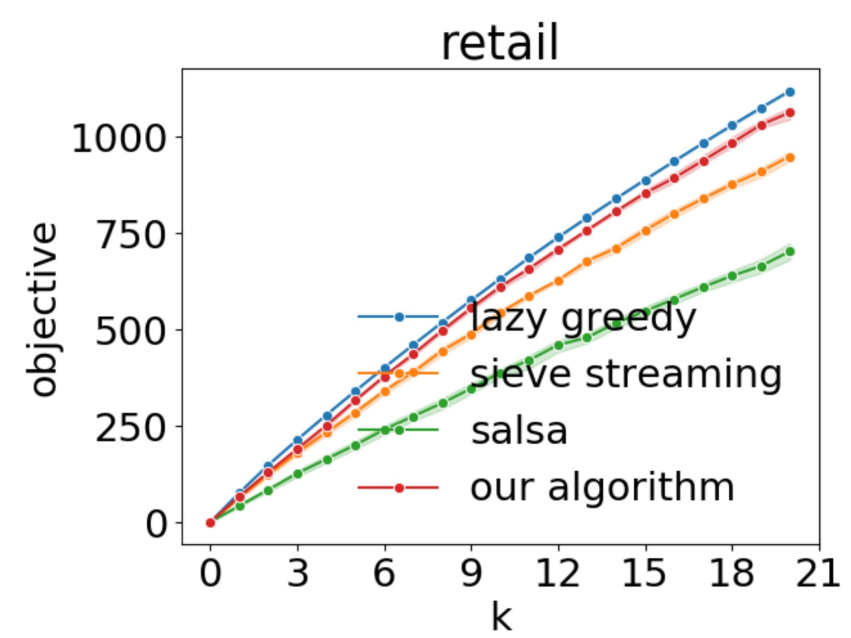
Characteristics of typical systems:

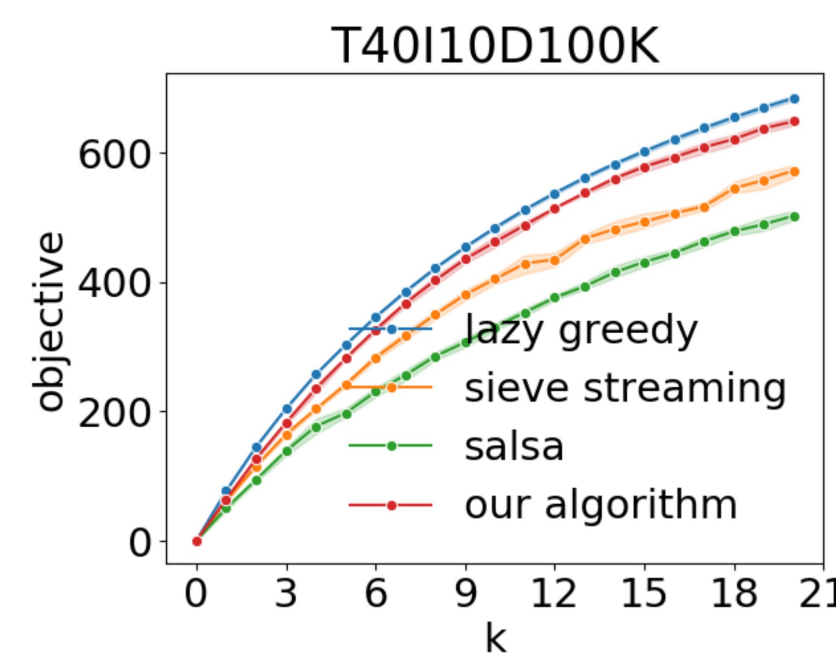
- Memory is limited
- Random access is not possible
- Data is sampled from some underlying distribution

Classical algorithms require storing the entire dataset. Can we do better?

OUR CONTRIBUTIONS

Our algorithms take exponentially less memory than current state-of-the-art while achieving better approximations in practice.





Our algorithm on real-world data. "Lazy greedy" is the best known *offline* solution.

Authors	Memory	Approx.	Notes
Sieve streaming, Badanidiyuru et al. '14	$O\left(\frac{k\log k}{\epsilon}\right)$	$\frac{1}{2}-\epsilon$	Adversarial order
Salsa, Norouzi-Fard et al. '18	$O(k\log k)$	$\frac{1}{2}+c$, $0<c \leq 10^{-13}$	Random order, also gives 1/2 l.b. for adversarial order
Agrawal et al. '19	$O(k\exp(\mathrm{poly}(1/\epsilon))$	$1-\frac{1}{e}-\epsilon$	Optimal approx.; constant of $> 2^{100}$ for $\epsilon < 0.2$.
Our paper	$O(k/\epsilon)$	$1-\frac{1}{e}-\epsilon$	Works in practice; also $\frac{1}{e}$ for non-mono.

THE COMPUTING MODEL

We only require black-box access to the submodular function and the ability to store data set elements.

Formally, we assume the following:

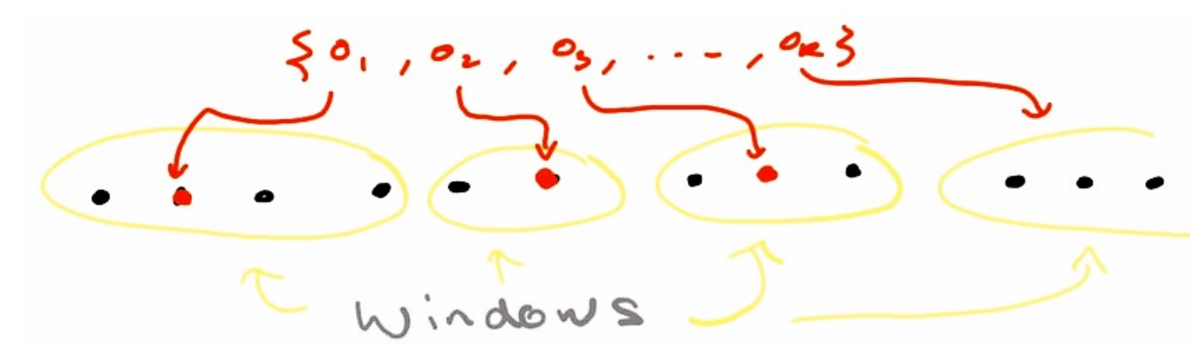
- Elements from a ground set T is streamed in to the algorithm, with order according to a uniformly random permutation.
- The algorithm chooses to store or throw away the element the moment it is seen.
- Only $\tilde{O}(k)$ elements can be stored.

ALGORITHM OUTLINE

Our algorithm relies on two ingredients: a *solution cascade* and a *random window partitioning*.

Random window partitioning:

- Randomly partition the stream into $O(k)$ windows.
- Most optimal elements go into their own window.
- In these windows we can make progress.



Solution cascade:

- A pyramid of solutions $\{L_1, L_2, \ldots, L_k\}$ with $|L_i| = i$.
- Greedily add one element to each level per window, choosing from window + elements added in previous windows.

Add e (and replace L_{i+1}) if $f(L_i \cup \{e\}) \geq f(L_{i+1})$.