Ternary Propagation-Based Local Search for more Bit-Precise Reasoning

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Satisfiability Modulo Theories (SMT)

Given a (quantifier-free) FOL formula and a combination of theories

$$((x \ll 01) \geq 00) \wedge (x < 01) \wedge ((\mathrm{read}(\mathrm{write}(a, x, x), x \cdot 10) = x + 01)$$

$x, 00, 01, 10 \ldots$ Bit-Vectors of size 2 $\qquad$ $a \ldots$ Array

is there an assignment to x such that this formula evaluates to **true**?

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$x, 00, 01, 10 \ldots$ Bit-Vectors of size 2 $\qquad$ $a \ldots$ Array

is there an assignment to x such that this formula evaluates to **true**? No

Here: Theory of Fixed-Size Bit-Vectors (without Quantifiers)

Theory of Fixed-Size Bit-Vectors

- constants, variables: $00000010,\ 2_{[8]},\ x_{[32]},\ y_{[2]}$
- bit-vector operators: $\approx,\ <,\ >,\ \sim,\ \&,\ \ll,\ \gg,\ \circ,\ [:],\ \ldots$
- arithmetic operators modulo 2^n (overflow semantics!)

Bit-Blasting

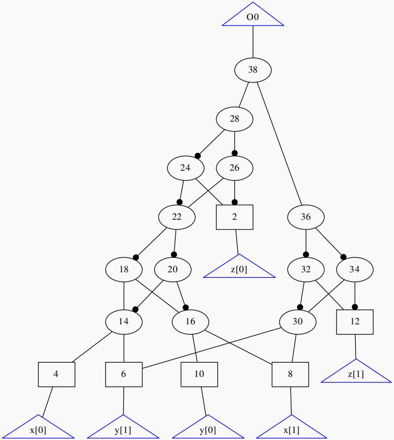
- current state-of-the-art for **quantifier-free** bit-vector formulas
- rewriting + simplifications + eager reduction to SAT

- ▶ efficient in practice
- ▶ may suffer from an exponential blow-up in the formula size

- ▶ may not scale well for increasing bit-widths

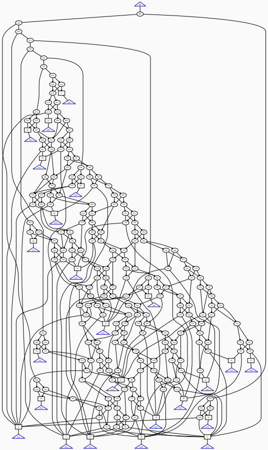
Bit-Blasting

Example $x_{[2]} * y_{[2]} = z_{[2]}$



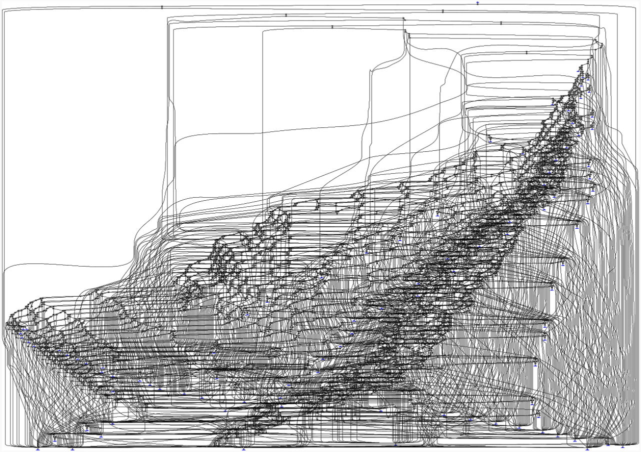
Bit-Blasting

Example $x_{[8]} * y_{[8]} = z_{[8]}$

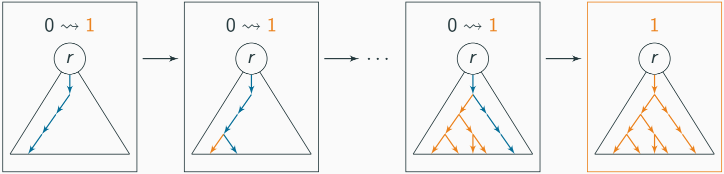


Bit-Blasting

Example $x_{[32]} * y_{[32]} = z_{[32]}$



Propagation-Based Local Search [CAV'16]



- ◦ assume satisfiability, start with **initial assignment**
- ◦ **propagate** target values towards inputs
- ◦ iteratively improve current state until **solution** is found

- **orthogonal approach**
- lifts concept of **backtracing** from ATPG to the **word-level**
- **without** bit-blasting, **no** SAT solver

- **not able** to determine **unsatisfiability**
- Probabilistically Approximately Complete (PAC) [Hoos, AAAI'99]
  - ▷ guaranteed to find a solution if there is one

Ternary Propagation-Based Local Search

Propagation-based Local Search [CAV'16]:

- **Main Weakness:** oblivious to **constant bits**
  - propagates target values that **can never be assumed**
  - **redundant work**

Ternary Propagation-based Local Search:

- **Generalization** with respect to **constant bits**
  - **symbolic conditions** to determine if value propagation is possible
- **Extension** with **bound-tightening heuristics** for inequalities

Ternary Propagation-Based Local Search

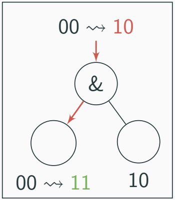
- **Non-deterministic algorithm**
  - **Main components: propagation path** and **value** selection
    - multiple possible paths and values
    - main sources of **non-determinism**
- **Down-propagation** of target values with respect to **constant bits**
  - constant bits are **precomputed** upfront
  - represented as **ternary bit-vectors**
    - **ternary bit-vector** $x = \langle x^{lo}, x^{hi} \rangle$ with $(\sim x^{lo} \mid x^{hi}) \approx \text{ones}$

      x^{lo} ... minimum (unsigned) value of x
      x^{hi} ... maximum (unsigned) value of x

      Example: $x_{[4]} = \bullet\bullet\bullet 0 = \langle 0000, 1110 \rangle$

Propagation Value Selection

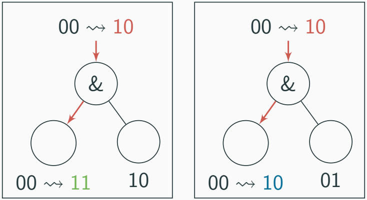
without changing other inputs



inverse value

Propagation Value Selection

without changing other inputs

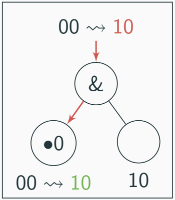


inverse value **consistent value**

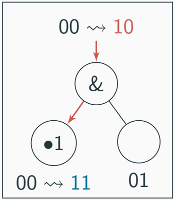
after changing other inputs

Propagation Value Selection

without changing other inputs



inverse value
wrt const bits



consistent value
wrt const bits

after changing other inputs

Propagation Value Selection

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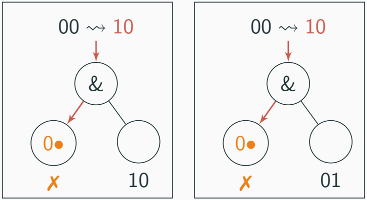


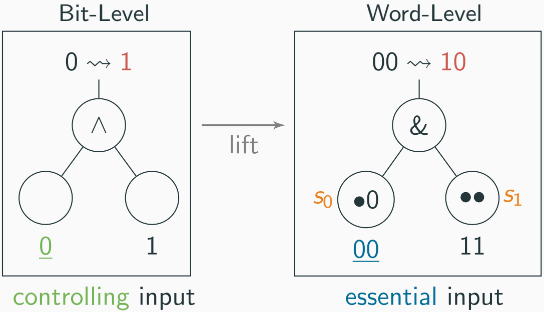
after changing other inputs

inverse value wrt const bits

consistent value wrt const bits

- inverse and consistent value **not always possible**
- **symbolic** invertibility/consistency conditions
- **break and restart** propagation if **consistency** condition **false**

Propagation Path Selection



- s_0 is essential if there **does not** exist an inverse value for s_1
- s_1 is essential if there **does not** exist an inverse value for s_0

Invertibility and Consistency Conditions

Given $x \diamond s \approx t$ with $x = \langle x^{lo}, x^{hi} \rangle$ **ternary**, and s, t **binary** bit-vectors.

$mcb(x, y)$ ... **true** if constant bits in x match corresponding bits in y

Invertibility Condition (IC).

$\forall x, s, t.\,(IC(x, s, t) \Leftrightarrow \exists y.\,(y \diamond s \approx t \wedge mcb(x, y)))$

Consistency Condition (CC).

$\forall x, t.\,(CC(x, t) \Leftrightarrow \exists y, s.\,(y \diamond s \approx t \wedge mcb(x, y)))$

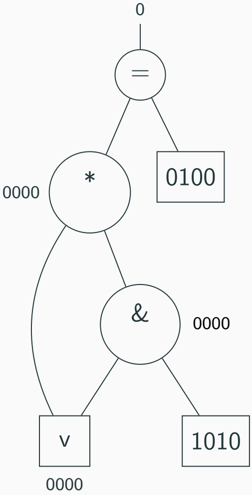
Example. $x \,\&\, s \approx t$

$IC(x \,\&\, s \approx t) = t \,\&\, s \approx t \wedge ((s \,\&\, x^{hi}) \,\&\, {\sim}(x^{lo} \oplus x^{hi})) \approx (t \,\&\, {\sim}(x^{lo} \oplus x^{hi}))$

$CC(x \,\&\, s \approx t) = t \,\&\, x^{hi} \approx t$

Ternary Propagation-Based Local Search

Example. $v_{[4]} \cdot (v_{[4]} \ \& \ 1010) \approx 0100$



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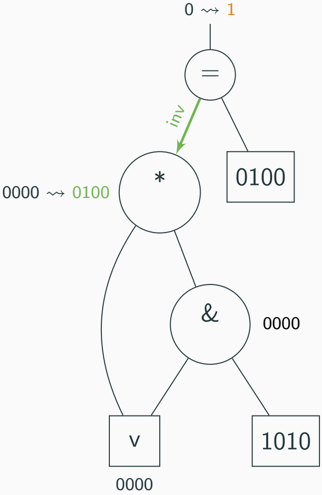

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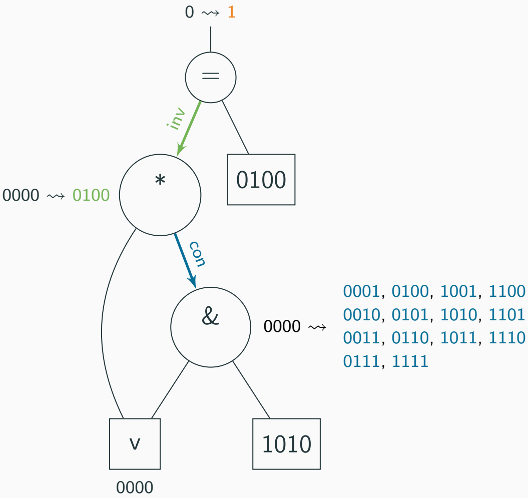

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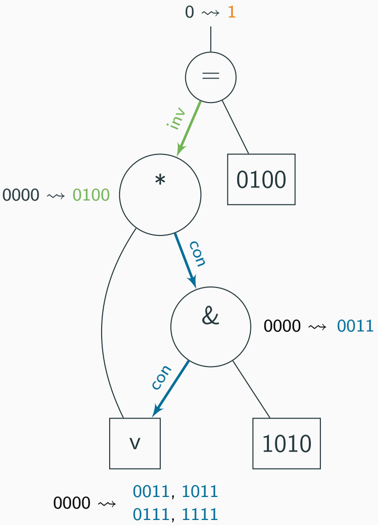

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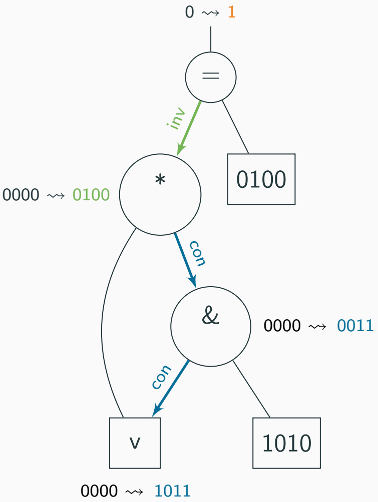

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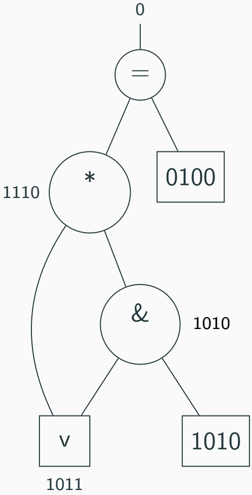

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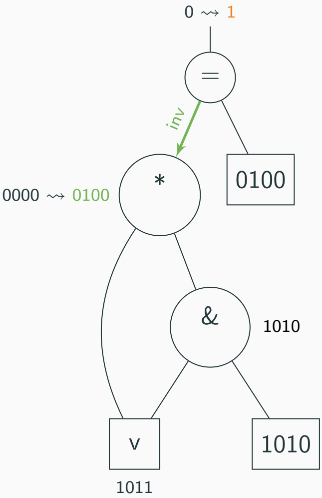

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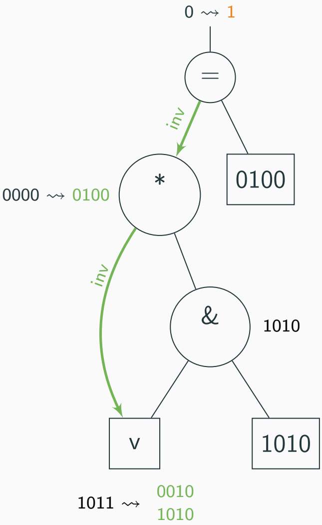

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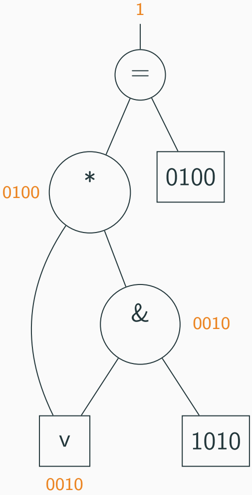

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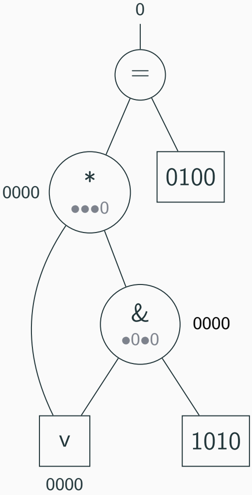

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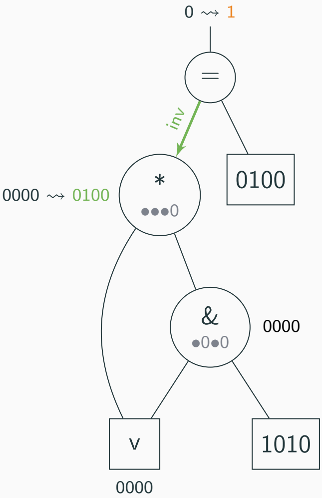

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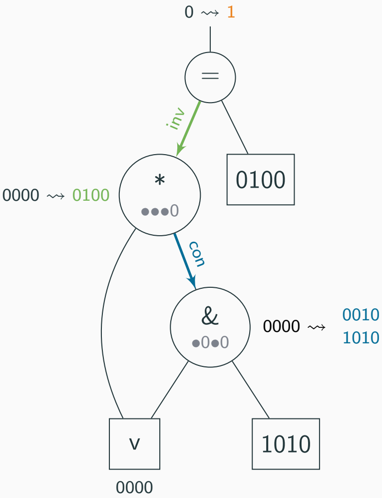

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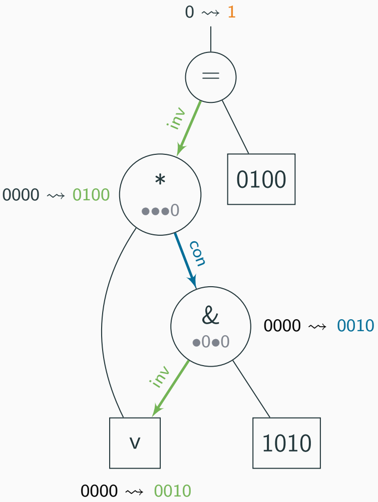

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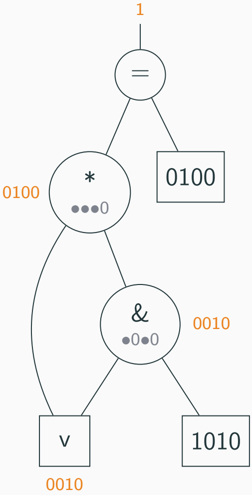


Extension: Bound Tightenting Heuristics

Example. $x \leq_u 0111 \wedge x >_u 0101$

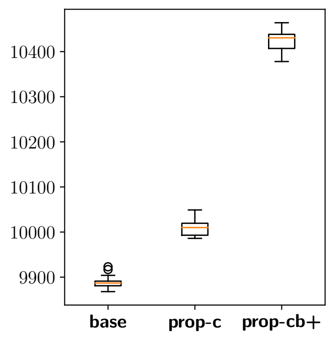
- $x \leq_u 0111$: $[0000, 0111]$
- $x >_u 0101$: $[0110, 1111]$
- actual: $[0110, 0111]$

- Heuristic for **inverse value** computation

- compute **bounds** for x in $x \diamond s$ ($s \diamond x$) over inequalities $\diamond \in \{<_s, <_u\}$
- based on **currently satisfied top-level inequalities** on x
  - **bounds invalid:** no inverse value for x
- **special handling** of $x\langle n \rangle <_u s$ and $x\langle n \rangle <_s s$
  - derive bounds based on prefix ($0_{[n]}$ or $\text{ones}_{[n]}$)

Results

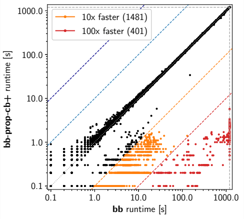
- implemented in our **new** SMT solver **Bitwuzla**



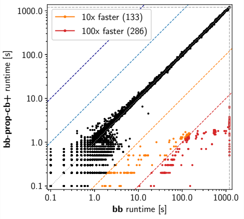
- **base** Propagation-based LS [CAV'16]
- **prop-c** **Ternary propagation-based LS**
  - + 120 (median) instances vs. **base**
  - 70% less moves
  - 63% less propagations
- **prop-cb+** **prop-c** with bound-tightening
  - + 400 (median) instances vs. **prop-c**

- 14,382 QF_BV **sat** instances in SMT-LIB
- 20 runs with different seeds for RNG
- 60s time limit, 8GB memory limit

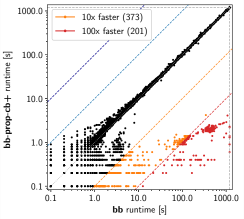
Results



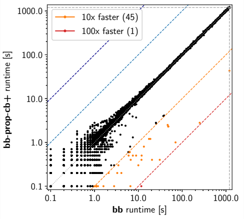
Lingeling SAT back end



CryptoMiniSat SAT back end



Kissat SAT back end
SAT competition winner 2020



CaDiCaL SAT back end

- **sequential portfolio** (first run LS, then fall back to bit-blasting)
- **all** 41,713 benchmarks in SMT-LIB QF_BV
- 1200s time limit, 8GB memory limit
- winner of division QF_BV in the SMT-COMP 2020

Conclusion

- **Generalization** of propagation-based local search **with respect to constant bits**
  - reduces redundant work
  - improves performance
- Determined **symbolic invertibility** and **consistency conditions**
  - for a representative set of bit-vector operators
  - verified up to size 65
  - **Future work:** proofs of correctness
- **Bound-Tightenting heuristic**
  - **Future Work:** apply in general, not only for inequality literals
- **Future work:** Hybrid approach
  - share information between bit-blasting and local search

References

- A. Niemetz, M. Preiner and A. Biere. Precise and Complete Propagation Based Local Search for Satisfiability Modulo Theories. In Proc. of CAV'16, pages 199-217, Springer, 2016.
- H. H. Hoos. *On the Run-time Behaviour of Stochastic Local Search Algorithms for SAT.* In Proc. of AAAI/IAAI'99, pages 661–666, AAAI Press / The MIT Press, 1999.