The Hints Promised in the Postscript to *Surreal Numbers*

9. To prove that $x \le x$ for all numbers x in the new system, first prove the auxiliary lemma

$$\text{if} \qquad x < Y_R \qquad \text{and} \qquad y \not\le x \qquad \text{then} \qquad x < y,$$

by induction on $d(x)$, for all fixed y.

19. Perhaps the simplest answer is $p = \bigl(\{(\{1\},\{1\})\},\{-1\}\bigr)$. Conway has proved in fact that we can solve the equation $p + p = x$ for any given x by taking $p = \bigl(\{q\},\{x - q\}\bigr)$ where

$$q = \bigl(X_{LL} \cup (x + x - X_{RR}) \cup (x + X_L - X_R), \emptyset\bigr).$$

22. Considering just the members of S created before $\aleph$ day, the unlike numbers are precisely g_0, g_1, g_2, g_3, $\ldots$, where

$$g_n = \bigl(\{g_0, \ldots, g_{n-1}\},\ \{g_0, \ldots, g_{n-1}\}\bigr)\,.$$

Moreover, the elements $\{g_0, \ldots, g_{n-1}\}$ form a finite field (under Conway's addition and multiplication operations), for $n = 2$, 4, 16, 256, 65536, $\ldots$, 2^{2^k}, $\ldots$!