✩ THE ART OF COMPUTER PROGRAMMING ✩

✩✩✩✩✩ ERRATA TO VOLUME 4 FASCICLE 7 ✩✩✩✩✩

This document is a transcript of the notes that I have been making in my personal copy of *The Art of Computer Programming*, Volume 4, Fascicle 7, since it was first printed in 2025.

Three levels of updates — "errors," "amendments," and "plans" — appear, indicated by three different typographic conventions:

▸**Page 666** line 1 ________________ 04 Jul 1776

Technical or typographical errors (aka bugs) are the most critical items, so they are flagged with a '▸' preceding the page number. The date on which I first was told about the bug is shown; this is the effective date on which I paid the finder's fee. The necessary corrections are indicated in a straightforward way. If, for example, the book says 'n' where it should have said '$n+1$', the change is shown thus:

$n \leadsto n+1$

Page 666 line 2 ________________ 14 Jul 1789

Amendments to the text appear in the same format as bugs, but without the '▸'. These are things I wish I had known about or thought of when I wrote the original text, so I added them later. The date is the date I drafted the new text.

Page 666 line 3 . . . . 20 Nov 1917

Plans for the future represent a third kind of item. In such notes I sketched my intentions about things that I wasn't ready to flesh out further when I wrote them down. You can identify these items because they're written in slanted type, and preceded by a bunch of dots '. . . . .' leading to the date on which I recorded the plan in my files.

My personal file of updates also includes a fourth category of items, not shown in this list. They are miscellaneous minor corrections or improvements that most readers don't want to know about, because they are so trivial. If you really want to see all of the gory details, you can download the full list from Internet webpage

`http://www-cs-faculty.stanford.edu/~knuth/taocp.html`

by selecting the "long form" of the errata.

The ultimate, glorious, future editions of Volumes 1–5 are works in progress. Please let me know of any improvements that you think I ought to make. Send your comments either by snail mail to D. E. Knuth, Computer Science, Gates Building 4B, Stanford University, Stanford CA 94305-9045, or by email to `taocp@cs.stanford.edu`*. (Use email for book suggestions only, please — all other correspondence is returned unread to the sender, or discarded, because I have no time to read ordinary email.) Although I'm working full time on*

Volume 4 these days, I will try to reply to all such messages within a year of receipt. Current news about The Art of Computer Programming is posted on

`http://www-cs-faculty.stanford.edu/~knuth/taocp.html`

and updated regularly.

— Don Knuth, January 2025

In spite of my respect for you, I am not convinced by all of your changes.
— ERNEST ANSERMET, letter to Igor Stravinsky (28 November 1929)

For the first time in many years, I've pulled out a copy
and read a few chapters to see how much my voice may have
changed over time. I confess to wincing every so often at a
poorly chosen word, a mangled sentence, . . .
I cannot honestly say, however, that the voice in this book is not mine.
— BARACK OBAMA, *Dreams From My Father* (2004)

☆☆☆☆☆☆☆ CONSTRAINT SATISFACTION ☆☆☆☆☆☆☆

junctions
half edges
boundary
line labeling

Last updated 28 December 2025

Page 37 3rd-from-last paragraph ———— 27 Dec 2025

1–13], $\rightsquigarrow$ 1–13, and several years later in *Artificial Intelligence* 104474 (25 December 2025), in press],

▸**Page 114** line 4 of exercise 22 ———— 18 Jan 2025

$(15, 20, 10, 12, 6) \rightsquigarrow (12, 20, 15, 10, 6)$

Page 115 new rating for exercise 28 ———— 11 Sep 2025

[*M27*] $\rightsquigarrow$ [*M37*]

Page 115 new rating for exercise 29 ———— 11 Sep 2025

[*24*] $\rightsquigarrow$ [*34*]

▸**Page 162** replacement for last seven lines of answer 27 ———— 11 Sep 2025

edge in common. A milder violation occurs when $a > b = c > d$, because six faces meet at vertex $(1, 1, b)$. Four faces meet at that vertex when $a > b = d > c$.

But the other cases are fine: *Case 1*, $a = b = c \ge d$. *Case 2*, $a = b > \max\{c, d\}$. *Case 3*, $a > b = c = d$. *Case 4*, $a > b > \max\{c, d\}$. When we take symmetry into account, these cases contribute respectively $\bigl(\binom{10}{1} + 4\binom{10}{2}, 4\binom{10}{2} + 8\binom{10}{3}, 4\binom{10}{2}, 8\binom{10}{3} + 16\binom{10}{4}\bigr)$ valid 3VPs, a total of 5830.

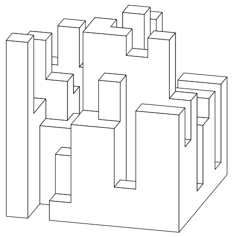
(And the B^4 histoscapes of 2×2 matrices with $a_{ij} < B$ yield $\frac{2}{3}B^4 + O(B^3)$ 3VPs.)

Page 163 replacement for final three lines of answer 28 ———— 12 Sep 2025

About 2.3 teramems of computation yield the desired number of 8×8 matrices: 573,350,756,580,609,962,448,490,328,484,980,268,982,588,520,613,841,558. (Those "mems" were however longer than usual, because 24 gigabytes of memory were needed.)

Page 163 replacement for main display and the rest of answer 29 ———— 14 Sep 2025

$$\begin{pmatrix} 0&6&4&4&8&4&5&5\\ 1&3&0&4&5&4&4&1\\ 6&8&6&8&9&7&5&5\\ 3&5&2&1&3&1&0&4\\ 8&6&4&9&6&5&0&1\\ 5&4&4&9&9&8&9&6\\ 5&0&3&5&7&1&3&4\\ 9&0&1&5&5&2&6&6 \end{pmatrix}$$



Incidentally, this histoscape has 204 vertices and 104 faces. Only 89 of the vertices are visible in this particular view, and only 49 of the faces are at least partly visible. There are 37 T junctions, 26 V junctions, 41 W junctions, and 22 Y junctions. When half edges are forced at the boundary, the line labeling problem has 16 solutions, because of four independent ambiguities in the interior "canyons"; all but six labels are forced.

▸**Page 163** bottom line ———— 12 Sep 2025

$(2m + 1) \times (2n + 1) \times (2t + 1) \rightsquigarrow (2m + 1) \times (2n + 1) \times (2t - 1)$

▸**Page 164** replacement for lines 2–10 12 Sep 2025

Second, mark all the "visible" faces of cubies, by doing the following three operations for all (i,j,k) with $\bar\imath\bar\jmath\bar k = 111$ and $b_{ijk} = 1$: If $b_{(i\pm2)jk} = 0$, set $b_{(i\pm1)jk} \leftarrow 1$; if $b_{i(j\pm2)k} = 0$, set $b_{i(j\pm1)k} \leftarrow 1$; if $b_{ij(k\pm2)} = 0$, set $b_{ij(k\pm1)} \leftarrow 1$. (We assume that $b_{ijk} = 0$ whenever $i < 0$ or $j < 0$ or $k < 0$ or $i > 2m$ or $j > 2n$ or $k > 2t - 2$.)

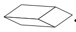
Third, mark all the "visible" edges, by doing the analogous three operations for all (i,j,k) with $\bar\imath + \bar\jmath + \bar k = 2$ and $b_{ijk} = 1$: If $\bar\imath = 1$ and $b_{(i\pm2)jk} = 0$, set $b_{(i\pm1)jk} \leftarrow 1$; if $\bar\jmath = 1$ and $b_{i(j\pm2)k} = 0$, set $b_{i(j\pm1)k} \leftarrow 1$; if $\bar k = 1$ and $b_{ij(k\pm2)} = 0$, set $b_{ij(k\pm1)} \leftarrow 1$.

Fourth, mark all the vertices, by doing those same three operations for all (i,j,k) with $\bar\imath + \bar\jmath + \bar k = 1$.

▸**Page 164** line 5 of answer 31 07 Jun 2025

True $\rightsquigarrow$ False

▸**Page 171** in answer 59 15 Sep 2025

line 2: only 90% squashed: . $\rightsquigarrow$ only 80% squashed: .
lines 5 and 6: It does not ... David" — so HC $\rightsquigarrow$
He aligned the cubes in a more symmetrical way, so that the blank region in the middle forms a "star of David"; thus the HC

▸**Page 174** last line of answer 68 07 Jun 2025

$1 — (m-1) \rightsquigarrow 0 — (m-1)$

Page 184 line 3 of answer 115 28 Dec 2025

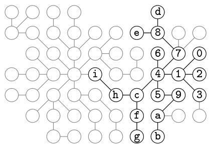
$m_{s+t+1} \rightsquigarrow m = m_{s+t+1}$

Page 202 new line to follow the caption of Fig. A–14 25 Mar 2025

N. Beluhov generalizes (v) to leapers in arXiv:2503.18700 [math.CO] (2025), 12 pages.

▸**Page 205** replacement for answer 213(a) 28 Dec 2025

213. (a) The answer is unique, except for automorphisms of S:



▸**Page 206** in the first two lines of the `sol` matrix 28 Dec 2025

$u \rightsquigarrow$ `UERT`
$v \rightsquigarrow$ `VERT`

▸**Page 235** in step F3 26 Mar 2025

$s \rightsquigarrow s'$ (five changes)

▸**Page 236** in answer 353 26 Mar 2025

$s \rightsquigarrow s'$ (three changes)

▸**Page 258** line 12 12 Dec 2025

022123122, 321210032, 323323302, 331112112, 333021222, 333021222 $\rightsquigarrow$
321210032, 323323302, 331112112, 333021222 $\rightsquigarrow$

Page 264 and following ——— 01 Jan 2025

Miscellaneous changes to the existing index of Volume 4 Fascicle 7 are collected here, including corrections and amendments to the old entries as well as new entries that are occasioned by the new material. Thus, the lines of the full index that have changed serve also as an index to the present document. However, when a correction or amendment has caused an old index entry to be deleted, the deletion is usually not indicated.