☆ THE ART OF COMPUTER PROGRAMMING ☆

☆☆☆☆☆☆☆☆ ERRATA TO VOLUME 4B ☆☆☆☆☆☆☆☆

This document is a transcript of the notes that I have been making in my personal copy of *The Art of Computer Programming*, Volume 4B since it was first printed in 2022.

Three levels of updates — "errors," "amendments," and "plans" — appear, indicated by three different typographic conventions:

▸**Page 666** line 1 ______ 04 Jul 1776

Technical or typographical errors (aka bugs) are the most critical items, so they are flagged with a '▸' preceding the page number. The date on which I first was told about the bug is shown; this is the effective date on which I paid the finder's fee. The necessary corrections are indicated in a straightforward way. If, for example, the book says 'n' where it should have said '$n + 1$', the change is shown thus:

$n \leadsto n + 1$

Page 666 line 2 ______ 14 Jul 1789

Amendments to the text appear in the same format as bugs, but without the '▸'. These are things I wish I had known about or thought of when I wrote the original text, so I added them later. The date is the date I drafted the new text.

Page 666 line 3 . . . . 20 Nov 1917

Plans for the future represent a third kind of item. In such notes I sketched my intentions about things that I wasn't ready to flesh out further when I wrote them down. You can identify these items because they're written in slanted type, and preceded by a bunch of dots '. . . . .' leading to the date on which I recorded the plan in my files.

My personal file of updates also includes a fourth category of items, not shown in this list. They are miscellaneous minor corrections or improvements that most readers don't want to know about, because they are so trivial. If you really want to see all of the gory details, you can download the full list from Internet webpage

`http://www-cs-faculty.stanford.edu/~knuth/taocp.html`

by selecting the "long form" of the errata.

The ultimate, glorious, future editions of The Art of Computer Programming are works in progress. Please let me know of any improvements that you think I ought to make. Send your comments either by snail mail to D. E. Knuth, Computer Science, CoDa Building room W208, 489 Jane Stanford Way, Stanford University, Stanford CA 94305-5008 USA, or by email to `taocp@cs.stanford.edu`*. (Use email for book suggestions only, please — all other correspondence is returned unread to the sender, or discarded, because I have no time to read ordinary email.) Although I'm working full time on*

Volume 4C these days, I will try to reply to all such messages within a year of receipt. Current news is posted on

`http://www-cs-faculty.stanford.edu/~knuth/taocp.html`

and updated regularly.

—Don Knuth, July 2022

Next to the promulgation of new truth,
the best thing, I conceive, that a man can do,
is the recantation of published error.
— JOSEPH LISTER (1878)

☆☆ COMBINATORIAL ALGORITHMS, PART 2 ☆☆

Last updated 16 April 2026

Most of these corrections have already been made in recent printings.

▸**Page ix** lines 10 and 11 ——— 06 Jan 2023

Mar-ijn $\leadsto$ Ma-rijn

▸**Page xiv** in the quote by Digges ——— 27 Apr 2024

aduice $\leadsto$ aduise
thorow; $\leadsto$ thorow,

Page xv replacement for lines 11–13 ——— 26 Nov 2022

Page 2 in (9) ——— 25 Dec 2022

$(\mathrm{E}\,XY) \leadsto \mathrm{E}(XY)$

▸**Page 4** line 11 ——— 13 Mar 2026

707–716 $\leadsto$ 707–714

Page 21 append a new sentence to exercise 102 ——— 04 Apr 2026

(Here 'a.s.' and 'q.s.' refer to an implicit parameter n.)

▸**Page 21** line 17 of exercise 103 ——— 02 Jan 2025

$X \leftarrow$ random bit $\leadsto$ $X_n \leftarrow$ random bit

▸**Page 25** line 9 of exercise 124 ——— 15 Jul 2025

$|E(f) - E_n(f)| \leadsto \mathrm{E}\,|E_n(f) - E(f)|$

Page 27 lines 3 and 4 of exercise 135 ——— 05 Dec 2022

either ($p_k < l$ and $p_{k+1} > l$ and $q_l > k$ and $q_{l+1} < k$) or ($q_k < l$ and $q_{k+1} > l$ and $p_l > k$ and $p_{l+1} < k$). $\leadsto$ either ($q_k < l$ and $p_l > k$ and $p_{l+1} < k$ and $q_{k+1} > l$) or ($q_{k+1} < l$ and $p_l < k$ and $p_{l+1} > k$ and $q_k > l$).

▸**Page 33** in (5) ——— 10 Jan 2025

holds$\}$ $\leadsto$ holds$\bigr\}$

Page 34 line 19 ——— 18 Nov 2024

found in 2016 $\leadsto$ found at T. U. Dresden in 2016

▸**Page 37** line 17 ——— 15 Feb 2023

to run though $\leadsto$ to run through

▸**Page 42** in the caption to Table 1 ——— 27 Nov 2022

`0000` $\leadsto$ `000`
`150f` $\leadsto$ `15f`
`MEM[40d]` $\leadsto$ `MEM[4d]`

▸**Page 43** line 2 after Table 2 ——— 27 Nov 2022

`200` through `204` $\leadsto$ `20` through `24`

Page 49 first line of step E5 ——— 28 Jan 2024

If $d = 0$, terminate. Otherwise set $D \leftarrow D \cdot d$ and $X_l \leftarrow y_I$, $\leadsto$
Set $D \leftarrow D \cdot d$, and terminate if $d = 0$. Otherwise set $X_l \leftarrow y_I$,

Page 49 replacement for the final six lines ——— 28 Jan 2024

know roughly how many there are; Algorithm E will tell us the approximate number. Indeed, the expected final value of D is exactly the total number of solutions, because every solution $X_1 \ldots X_l$ constructed by the algorithm is obtained with probability $1/D$, in a case where $D > 0$. Note that there may be another criterion for successful termination in step E2, even though l might still be $\leq n$.

▸**Page 50** line −2 before the footnote ——— 11 Nov 2023

they defend $\leadsto$ they defined

Page 55 new rating for exercise 15 ——— 14 Jun 2023

[*HM42*] $\leadsto$ [*HM44*]

▸**Page 58** exercises 41 and 42 ——— 27 Nov 2022

`MEM[40d]` $\leadsto$ `MEM[4d]`
`MEM[904]` $\leadsto$ `MEM[94]`
`MEM[a0d]` $\leadsto$ `MEM[ad]`

Page 59 new rating for exercise 58 ——— 17 Feb 2025

58. [*27*] $\leadsto$ ▸**58.** [*30*]

Page 69 line 2 of Algorithm X ——— 17 Nov 2024

described. $\leadsto$ described, together with global variables N, Z, i, j, l, and p.

▸**Page 88** line −10 ——— 06 Dec 2024

the color of any other option $\leadsto$ its color in any other option

▸**Page 91** line −6 ——— 17 Dec 2022

only 5.2 Gμ $\leadsto$ only 4.5 Gμ

Page 95 the line after (66) ——— 10 Jan 2026

to find them $\leadsto$ to find them, and to show that seven distinct letters is impossible,

▸**Page 102** in (86) ——— 19 Jan 2023

60741+ $\leadsto$ 60742−

Page 102 four lines after (86) ——— 03 Aug 2023

average length n ⇝ average length $n/2$

▸**Page 109** in (103) ——— 02 Jan 2025

hide$''(i)$ ⇝ hide$''(p)$

▸**Page 109** in (105) ——— 02 Jan 2025

unhide(i) ⇝ unhide(p)

▸**Page 110** in step P3 ——— 11 Mar 2026

If $i = N$ ⇝ If $i > N$

▸**Page 110** line 1 of step P5 ——— 02 Jan 2023

of its option, go ⇝ of its option, or if $\texttt{COLOR}(x) \neq 0$, go

▸**Page 111** line 11 ——— 14 Dec 2022

30,286,432 solutions ⇝ 30,258,432 solutions

Page 127 line 1 of exercise 33 ——— 12 Oct 2025

construct ⇝ efficiently construct

Page 135 in exercise 104 ——— 15 Sep 2025

line 2: whenever k is not a multiple of p ⇝
line 3: (if x_n is blank) ⇝ (where x_n is considered to be blank)

Page 136 append a sentence to exercise 111 ——— 30 Mar 2023

(Crossword puzzle diagrams are black-and-white grids such as those in exercise 1.3.2–23.)

▸**Page 138** line 8 of exercise 121 ——— 07 Jan 2023

(2, 3, 3, 57) ⇝ (2, 2, 3, 57)

▸**Page 145** line 4 of exercise 165 ——— 02 Jan 2025

Would it better ⇝ Would it be better

Page 145 new rating for exercise 171 ——— 02 Jan 2025

▸**171.** [*35*]

Page 145 and 146 in exercise 172 ——— 03 Jan 2023

[change ‘snake-in-the-box’ to simply ‘snake’, in five places]

Page 146 new rating for exercise 175 ——— 27 Mar 2024

[*M21*] ⇝ [*M22*]

▸**Page 146** lines 2 and 4 of exercise 175 ——— 27 Mar 2024

a_i ⇝ r_i

▸**Page 147** replacement for lines 3 and 4 of exercise 180 ——— 15 Feb 2026

$$o_p = `i_1\ \ldots\ i_p\text{'}, \text{ for } 1 \le p \le t'; \qquad o_{t'+q} = `\ \ldots\ i_{t'}\ \ldots\text{'}, \text{ for } 1 \le q < t.$$

Furthermore, $i_r \in o_{t'+q}$ if and only if $1 \le q < t + r - t'$, for $1 \le r \le t'$.

Page 159 new rating for exercise 300 ——— 27 Oct 2023

[*23*] ⇝ [*24*]

▸**Page 160** lines 2 and 3 of exercise 303(a) ——— 11 Jun 2024

The area of each ... corresponding tree. ⇝

Page 160 new rating for exercise 305 ——— 03 Feb 2023

[*25*] ⇝ [*28*]

Page 161 new rating for exercise 306 ——— 04 Feb 2023

[*30*] ⇝ [*32*]

Page 161 in exercise 306 ——— 03 Jan 2023

[change 'snake-in-the-box' to simply 'snake', in two places]

Page 172 new first paragraph for exercise 372(b) ——— 06 Dec 2022

b) A *twintree* is a data structure whose nodes x have four pointer fields, $\mathtt{L0}(x)$, $\mathtt{R0}(x)$, $\mathtt{L1}(x)$, $\mathtt{R1}(x)$. It defines two binary trees T_0 and T_1 on the nodes, where T_θ is rooted at $\mathtt{ROOT}\theta$ and has child links $(\mathtt{L}\theta, \mathtt{R}\theta)$. These two trees satisfy $\mathrm{inorder}(T_0) = \mathrm{inorder}(T_1) = x_1 \ldots x_n$; $\mathtt{L0}(x_k) = \Lambda \iff \mathtt{L1}(x_k) \neq \Lambda$, for $1 < k \leq n$; $\mathtt{R0}(x_k) = \Lambda \iff \mathtt{R1}(x_k) \neq \Lambda$, for $1 \leq k < n$.

Page 172 last line of exercise 372 ——— 30 Nov 2022

twin tree ⇝ twintree

▸**Page 175** in exercise 398 ——— 07 Apr 2023

entries in each of ⇝ entries in some or all of

simply states its contents, without any operation; hence its solution is a no-brainer. ⇝ without any operation, is a no-brainer, unless no clue has been specified for it.

Page 176 in exercise 399 ——— 07 Apr 2023

[*22*] ⇝ [*23*] and Algorithm C ⇝ Algorithm X

Page 178 in exercise 417 ——— 01 Jan 2024

such square puzzles ⇝ such triples of square puzzles

Page 181 line 3 ——— 05 Jul 2023

Hint: Use bitmaps. ⇝ *Hint:* Work bitwise.

Page 187 line −8 ——— 19 Jun 2023

deals only with clauses whose ⇝ deals only with "k-clauses," whose

Page 192 in (20) ——— 16 Sep 2025

$1 \leq i+j \leq t_k+1$ ⇝ $1 \leq i+j \leq r+1$

▸**Page 193** line −12 ——— 13 Mar 2026

34 (1964) ⇝ **34** (1965)

Page 214 lines 26 and 27 ——— 16 Sep 2025

we can find another nonfalse literal ... a clause of size 1. ⇝ we can try to find another nonfalse literal to swap into its place.

Page 214 bottom line ——— 23 Feb 2025

Moreover, our new ⇝ Variable W_l, if nonzero, will point to the first clause that watches literal l. Our new

Page 215 line −7 ——— 16 Sep 2025

unit clauses ⇝ unit clauses, the clauses of size 1

▸**Page 252** bottom line ——— 17 Mar 2026

$R_{l'} \leftarrow c$ ⇝ $R_{\bar{l}'} \leftarrow c$

Page 281 line 7 ——— 19 Jun 2023

is certifiable ⇝ is certifiable (see (119))

Page 286 line 7 after (172) ——— 19 Jun 2023

12-clause ⇝ twelve-clause

▸**Page 291** in Table 5, entry for $Z(13,22)$ ——— 17 Mar 2026

80 ⇝ 70

▸**Page 297** line 16 ——— 13 Mar 2026

5 (1998) ⇝ **5** (1996)

Page 317 replacement for exercise 5 ——— 18 May 2023

5. [*HM47*] Determine the asymptotic behavior of $W(3,k)$ as $k \to \infty$.

Page 322 rewording of exercise 68 ——— 26 Feb 2026

68. [*39*] Find a maximum-length mobile Life path on the chessboard in which 6 to 10 cells are always alive.

Page 339 new rating for exercise 264 ——— 26 Mar 2026

[*21*] ⇝ [*24*]

Page 344 new rating for exercise 318 ——— 02 Feb 2024

[*HM27*] ⇝ [*HM33*]

Page 350 line 3 of exercise 363(g) ——— 10 Feb 2024

strict Horn clauses ⇝ definite Horn clauses

Page 357 new rating for exercise 415 ——— 16 Sep 2025

[*M22*] ⇝ [*M27*]

▸**Page 364** line 8 of exercise 486 ——— 29 Jul 2023

Ten captures ⇝ Eleven captures

Page 370 new answer for Notes on the Exercises ——— 24 Nov 2023

2. They help teach you how to ask good questions.

Page 370 last line of MPR answer 1 ——— 28 Jun 2023

125 (2018), 387–399 ⇝ **125** (2018), 387–399; M. Purcell, *AMM* **130** (2023), 421–436

▸**Page 372** last line of answer 18 —— 13 Mar 2026

353–356 $\rightsquigarrow$ 353–357

▸**Page 378** line 3 of answer 51 —— 06 Apr 2026

$\bar{x}_1\bar{x}_4 + \bar{x}_2\bar{x}_4 + \bar{x}_3\bar{x}_4 + \bar{x}_2\bar{x}_5 + \bar{x}_3\bar{x}_5 \rightsquigarrow \bar{x}_1\bar{x}_4 \vee \bar{x}_2\bar{x}_4 \vee \bar{x}_3\bar{x}_4 \vee \bar{x}_2\bar{x}_5 \vee \bar{x}_3\bar{x}_5$

▸**Page 385** line 2 of answer 100 —— 19 Jun 2023

0). $\rightsquigarrow$ 0).)

▸**Page 385** in answer 104 —— 03 Mar 2026

line 1: $(q^2, 2pq, p^2) \rightsquigarrow (q^2, pq, p^2)/(1-pq)$
line 2: $(p^2+pq^2, 0, q^2+qp^2) \rightsquigarrow (q^2(1+p), 0, p^2(1+q))/(1-pq)$

▸**Page 394** last line of answer 133 —— 10 Nov 2022

143–145 $\rightsquigarrow$ 145–147

▸**Page 394** in answer 135 —— 10 Nov 2022

line 19: the interesting recurrence $\rightsquigarrow$ an interesting recurrence for nonnegative i, j, k, n:
line 20: $B_1(i,j,k) \rightsquigarrow B_0(i,j,k)$
line 21: the solution $\rightsquigarrow$ the solution for $i+j \le n$

▸**Page 395** line -2 of answer 135 —— 10 Nov 2022

$b_1 = 1 \rightsquigarrow b_0 = 1$

▸**Page 396** line 2 of answer 140(e) —— 20 Jan 2023

$(**)$ $(\epsilon \rightsquigarrow (**)\ \epsilon$

▸**Page 397** line 3 —— 26 Nov 2024

their Theorem 10 $\rightsquigarrow$ their Theorem 11

▸**Page 397** line 2 of answer 144(d) —— 21 Jan 2023

$X_1^+ \cdots + X_n^+ \rightsquigarrow X_1^+ + \cdots + X_n^+$

▸**Page 398** last line of answer 10 —— 21 Jan 2026

$a_l - a_{l-1} \rightsquigarrow a_{l+1} - a_l$

Page 399 third line of answer 15 —— 04 Jul 2025

[arXiv:2107.13460 [math.CO] (2021), 51 pages] $\rightsquigarrow$ [*Advances in Math.* **427** (2023), 109127:1–83]

Page 399 last line of answer 15 —— 09 May 2023

[arXiv:2112.03336 [math.CO] (2021), 14 pages]. $\rightsquigarrow$ [*Optimization Letters* **17** (2023), 1229–1240].

Page 406 append new line to answer 48 —— 18 Feb 2023

See *Journal of Automated Reasoning* **67** (2023), 12:1–12:10.

▸**Page 407** in answer 58 —— 14 Feb 2025

line 3: every path $\rightsquigarrow$ every shortest path
lines 12–13: Otherwise make w ... if $u \ne \Lambda$. $\rightsquigarrow$ Otherwise make w a resource (see below), and set $u \leftarrow$ `LINK(u)`; return to $(*)$ if $u \ne \Lambda$, otherwise append `S` to `R` (see below) and set $d \leftarrow d+1$.

Page 408 replacement for line 10 — 14 Feb 2025

`LINK(`w`)` $\leftarrow$ `S`, and `S` $\leftarrow w$. (It works, although `S` does double duty!)

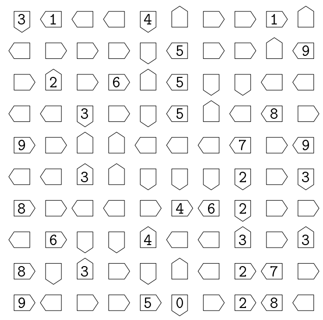
▸**Page 410** line 10 — 18 Feb 2023

pages 28 $\rightsquigarrow$ page 28

Page 411 replacement for answer 69 — 14 Mar 2023

69. (The main point is that the 33rd digit of π is zero.) On 14 March 2023, Germán González-Morris rose to this challenge by promulgating the puzzle shown here, which actually exhibits 35 of the requested digits(!). (The answer can be found in Appendix E.)

Is it possible that there might also be a puzzle of this kind whose empty boxes are symmetrically located about the center, as they are in exercise 68?



▸**Page 419** last line of answer 20 — 05 Jan 2026

$y_{j\&-2^{\lfloor \lg(j-k)\rfloor}} \rightsquigarrow y_{j\&-2^{\lfloor \lg(j\&\sim k)\rfloor}}$

▸**Page 420** line 14 of answer 23 — 16 Nov 2024

(ratio $\approx$ 3.68) $\rightsquigarrow$ (ratio $\approx$ 3.58)

▸**Page 421** in answer 30 — 22 Mar 2023

construction, but change '$C\backslash C_j$' to 'C' if T_j is a solution node. $\rightsquigarrow$ construction; but if T is just a root node marked as a solution, let there be no rows and no columns.

Page 422 replacement for answer 33 — 20 Dec 2025

33. (Solution by N. Posting.) Add a new row at the bottom of A, all 1s. This $(m+1)\times n$ matrix A' has one solution that chooses only the last row. All other solutions choose rows that solve A. (Another construction, which adds a new column, can be used if the all-1s row is already present.)

▸**Page 422** line 4 of answer 34 — 02 Jan 2025

$011\alpha_k$ if $k > t/2$ $\rightsquigarrow$ $001\alpha_k$ if $k > t/2$

Page 431 and 432, replacement for answer 73 — 10 Aug 2023

73. From 06151146/66611014/56132144/55326224/53632022/34300525/33040054 we get 807,752 solutions, which is the current record. This array, found in 2023 by Germán González-Morris based on previous work by Michael Keller, has the surprising property that *every* candidate placement occurs in at least 35,082 solutions!

Page 432 append to answer 75 — 13 Feb 2024

"Gropes" of a completely different kind were introduced into the theory of topological manifolds by J. W. Cannon, *Bull. Amer. Math. Soc.* **84** (1978), 832–866.

▸**Page 438** in answer 98 — 02 Jan 2025

following options ... That problem $\rightsquigarrow$ following $6m$ options: (i) '$i1$ $i2$', '$i2$ $i3$', '$i3$ $i1$'; (ii) 'ij x_k:`1`' if $l_{ij} = x_k$, 'ij x_k:`0`' if $l_{ij} = \bar{x}_k$. That problem

Page 440 line 1 ——— 03 Jul 2025

See Richard Stong in *AMM*, to appear. $\leadsto$ See N. Hodges et al. in *AMM* **132** (2025), 94–96.

Page 442 line 2 ——— 19 Dec 2022

found in 20 Gμ $\leadsto$ found in 23 Gμ

▶**Page 443** last line of answer 111 ——— 04 Nov 2023

591 Gμ $\leadsto$ 550 Gμ

▶**Page 443** first line of answer 112(c) ——— 08 Apr 2023

140 ways $\leadsto$ 416 ways

▶**Page 443** in the rightmost Torto of answer 112(c) ——— 08 Apr 2023

`ETV` $\leadsto$ `YTE`

Page 444 line 4 ——— 07 Jan 2023

225 Gμ $\leadsto$ 202 Gμ

▶**Page 444** line 5 ——— 07 Jan 2023

$\{\mathtt{C},\mathtt{E},\mathtt{T},\mathtt{R},\mathtt{U},\mathtt{Y}\}$ $\leadsto$ $\{\mathtt{C},\mathtt{E},\mathtt{I},\mathtt{R},\mathtt{U},\mathtt{Y}\}$

Page 444 replacement for answer 114 ——— 02 Jan 2023

114. (Solution improved by P. Weigel.) Besides the primary items p_{ij}, r_{ik}, c_{jk}, b_{xk} of (30), introduce u_k and v_l to define a permutation. Also introduce secondary items π_k to record the permutation and ij to record the value at cell (i,j). The permutation is defined by 81 options 'u_k v_l π_k:l' for $1 \le k,l \le 9$. And there are $9^4 = 6561$ other options, one for each cell (i,j) of the board and each pair (k,l) of values before and after α is applied. If $(ij)\alpha = i'j'$, let $x' = 3\lfloor i'/3\rfloor + \lfloor j'/3\rfloor$. Then option (i,j,k,l) is normally '$p_{i'j'}$ $r_{i'l}$ $c_{j'l}$ $b_{x'l}$ ij:k $i'j'$:l π_k:l'. However, if $i' = i$ and $j' = j$, that option is shortened to 'p_{ij} r_{ik} c_{jk} b_{xk} ij:k π_k:k', and omitted when $k \ne l$. The options (i,j,k,l) are also omitted when $i = 0$ and $k \ne j+1$, or $i' = 0$ and $l \ne j'+1$, or $i > 0$ and $k = j+1$, or $i' > 0$ and $l = j'+1$, in order to force '`123456789`' on the top row.

With that top row and with α = transposition, Algorithm C produces 30,258,432 solutions in 171 gigamems. (Those solutions were first enumerated in 2005 by E. Russell; see `www.afjarvis.org.uk/sudoku/sudgroup.html`.)

Page 444 line 1 of answer 115 ——— 02 Jan 2023

b'_{yk} and $B'_{y'l}$ $\leadsto$ b'_{yk}

▶**Page 446** new solution for answer 121(c) ——— 08 Jan 2023

(c) With δRU in the middle, another solution has C_{k-1}, D_{k-1}, A_{k-1}, B_{k-1} at the corners. With δLD, another solution has corners B_{k-1}, A_{k-1}, D_{k-1}, C_{k-1}. Both of those solutions work with δLU and δSU; and δSU also has 54 additional solutions, with C_{k-2} in the upper left corner. They use $\delta\{\{L,P,S,T\}\{J,U\},RU\}$ in the middle of row 2^{k-2}, and independently $\delta\{L,K,P,R,S,T\}U$ in the middle of row $3\cdot 2^{k-2}$.

▶**Page 446** line 4 of answer 123 ——— 26 Nov 2024

7.2.1.5–4 $\leadsto$ 7.2.1.5–(4)

▸**Page 450** lines 4 and 5 of answer 133 ——— 16 Jan 2023

We save a factor ... another factor $\leadsto$ Remove options with non-`a` on the border; also limit `aaaa` to four border positions; and save another factor

Page 451 simplifying the matrices in answer 136 ——— 06 Apr 2025

three reflection matrices and two orthogonal matrices, ⟨ display ⟩ $\leadsto$
a reflection matrix and two orthogonal matrices,

$$D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & +1 \end{pmatrix}, \qquad P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \qquad Q = \frac{1}{2}\begin{pmatrix} 1 & -\phi & 1/\phi \\ \phi & 1/\phi & -1 \\ 1/\phi & 1 & \phi \end{pmatrix}.$$

▸**Page 455** line 11 ——— 20 Dec 2022

$q \equiv x$ (modulo 2) $\leadsto$ $q \equiv x/q$ (modulo 2)

▸**Page 456** line −3 of answer 146 ——— 13 Mar 2026

Scientific American **235**, 3 $\leadsto$ *Scientific American* **239**, 3

▸**Page 457** line 22 ——— 07 Apr 2026

(1893) $\leadsto$ (1892)

Page 457 line 22 ——— 19 Jun 2023

8-cube $\leadsto$ eight-cube

Page 459 line 13 of answer 151 ——— 05 Nov 2022

with Algorithm X $\leadsto$ with Algorithm X and exercise 413

Page 460 line −3 ——— 07 Apr 2026

in 1981, and C. H. Jepsen and S. Ahearn $\leadsto$ in 1981; S. T. Ahearn and C. H. Jepsen

▸**Page 462** line 3 of answer 162 ——— 13 Oct 2023

$r_{j+1}\ c_{k+3}\ a_{j+k+3}\ b_{j-k-3}$ $\leadsto$ $r_{j+1}\ c_{k+3}\ a_{j+k+4}\ b_{j-k-2}$

▸**Page 462** in answer 162 ——— 13 Oct 2023

line 5: multiplicity. $\leadsto$ multiplcity, and we'll use the sharp preference heuristic.
line 8: 1.2 teramems. $\leadsto$ 150 megamems.
line 10: Only 35 Gμ $\leadsto$ Less than 200 Mμ
line 13: 6 teramems $\leadsto$ 180 megamems
and in the largest displayed solution, put shading on the $\mathcal{Q}_5$.

▸**Page 463** last line of answer 166 ——— 26 Nov 2024

`RLINK`(p). $\leadsto$ `RLINK`(p).]

▸**Page 463** replacement for answer 171 ——— 02 Jan 2025

171. Let there be ten primary items v, for $0 \le v < 10$; also fifteen primary items $\#uv$, for each edge u — v, where the edges are 0 — 1 — 2 — 3 — 4 — 0, 0 — 5, 1 — 6, 2 — 7, 3 — 8, 4 — 9, and 5 — 7 — 9 — 6 — 8 — 5. Let there be $26 \cdot 10$ secondary items $\mathtt{a}_v$ through $\mathtt{z}_v$, for $0 \le v < 10$; also $26 \cdot 30$ secondary items $\mathtt{a}_{uv}$ through $\mathtt{z}_{uv}$, for $u \not\!\!—\, v$; also a secondary item w for each word in, say, `WORDS`(1000). There are 26 options, '$\#uv$ $\mathtt{a}_u$:1 $\mathtt{a}_v$:1' through '$\#uv$ $\mathtt{z}_u$:1 $\mathtt{z}_v$:1', for each edge. And there are 10 options for each word; for example, the option for `added` at vertex 0 is '0 $\mathtt{a}_0$:1 $\mathtt{b}_0$:0 $\mathtt{c}_0$:0 $\mathtt{d}_0$:1 $\mathtt{e}_0$:1 $\mathtt{f}_0$:0 ... $\mathtt{z}_0$:0 $\mathtt{a}_{02}$ $\mathtt{a}_{03}$ $\mathtt{a}_{06}$ $\mathtt{a}_{07}$ $\mathtt{a}_{08}$ $\mathtt{a}_{09}$ $\mathtt{d}_{02}$ ... $\mathtt{e}_{09}$ `added`'.

Every solution will be obtained at least 120 times, because the Petersen graph has 120 automorphisms, and the $\#uv$ options might apply more than once. But symmetry can be broken by introducing additional secondary items for pairwise ordering, or by modifying Algorithm C to choose the labels of 0, 1, and 3 first (see exercise 122).

There are two solutions in `WORDS`(834), namely `muddy`, `thumb`, `books`, `knock`, `ended`, `apply`, `fifth`, `grass`, `civil`, (`refer` or `fewer`).

Page 464 lines 5 and 6 of answer 172 ——— 02 Feb 2023

$= 2$. For the path ... $= 1$ $\leadsto$ $= 2$. We can safely omit options where $v_i \text{ — } v_j$ and $a_i = a_j = 1$; they force a triangle. For the path problem, the starting vertex should have d options, with $a_1 + \cdots + a_d = 1$

Page 464 line 4 of answer 172(a) ——— 03 Jan 2023

snake-in-the-box $\leadsto$ snake

Page 464 in answer 172(a) ——— 02 Feb 2023

line 5: 6 Tμ $\leadsto$ 270 Gμ
line 12: 13 Mμ $\leadsto$ 54 Mμ
line 16: (Algorithm M ... 625 Gμ $\leadsto$ (If we force the first step downward, Algorithm M finds the $7!^2 = 25401600$ solutions in 365 Gμ
line 21: in 17 Gμ, despite having 16788 options of total length 454380(!). $\leadsto$ in 9.4 Gμ, despite having 10422 options of total length 281934(!).

▸**Page 464** replacement for most of the last paragraph ——— 02 Feb 2023

Now let's ... dual transposition. $\leadsto$

Now let's consider paths that start in cell $(0, 1)$ and do not end in a corner. (i) No such solutions have 32 kings (proved in 166 Gμ). (ii) Knights, however, yield a big surprise: There's a unique path of length 33, doubly counted! (Found in 43 Gμ.) (iii) Bishop paths can't have length 12 unless they start or end in a corner. (iv) There are $N = (n-1)!^2 - 2(n-2)!^2$ solutions where the rook first moves down, and N where it first moves sideways. Of these, $2(n-2)!^2$ end at $(1, n-1)$ and are equivalent to those ending at $(n-2, 0)$; $(n-2)!^2$ end at $(n-1, n-2)$ and are double-counted by central symmetry; $(n-2)!^2 - (n-2)!$ end at $(1, 0)$ and are double-counted by transposition; $(n-2)!^2 - (n-2)!$ end at $(n-2, n-1)$ and are double-counted by dual transposition.

▸**Page 465** line 1 ——— 02 Feb 2023

So there are ... 47691936 $\leadsto$ So there are $2(n-1)!^2 - 9(n-2)!^2 + 2(n-2)! = 46139040$

Page 465 line 6 ——— 02 Feb 2023

fast, except that the kings need 6.3 Tμ. $\leadsto$ fast; kings need the max time (170 Gμ).

▸**Page 465** line 5 of answer 172(b) ——— 16 Mar 2023

bishop has 36 $\leadsto$ bishop has 72

▸**Page 465** lines 7 and 8 of answer 172(b) ——— 03 Jan 2023

under both horizontal and vertical reflection. Every rook snake-in-the-box $\leadsto$ under reflection about either diagonal. Every rook snake

▸**Page 465** in Kautz reference on line -6 or -7 ——— 13 Mar 2026

177–180 $\leadsto$ 179–189

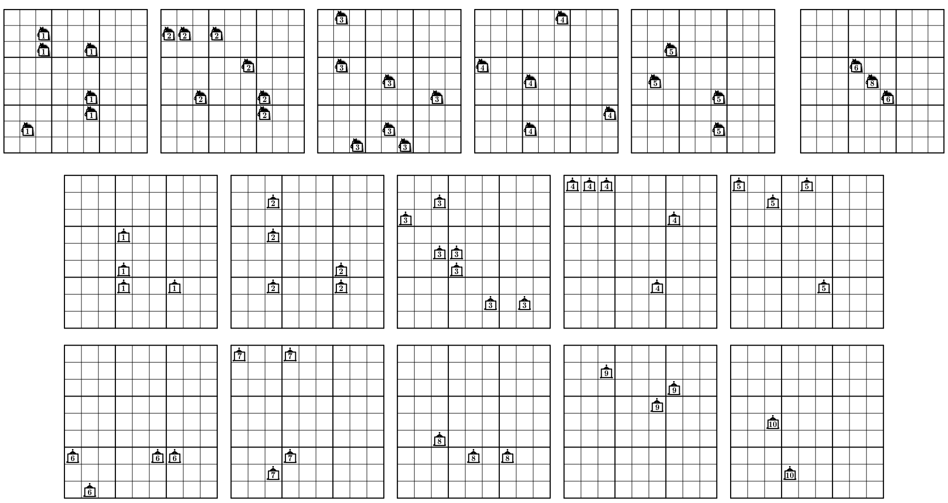
Page 465 and 466, replacement for final paragraphs of answer 172 ——— 08 Sep 2023

Nikolai . . . [To appear.] $\rightsquigarrow$

The maximum length of an $n \times n$ snake king path when $n = 100$ was the topic of a math olympiad problem in 2008, posed by A. and M. Chapovalov. Nikolai Beluhov has shown that such maximum paths can be characterized completely; they're related to spirals when n is even, and to stamp-folding when n is odd(!). When $n \geq 6$ is even, exactly $2n + (n \bmod 4)/2$ such paths are distinct under symmetry, and they all start in a corner. Furthermore there are exactly six distinct snake king *cycles* of length $n^2/2 - 1$, when $n \geq 8$ is a multiple of 4. Beluhov has also proved that the longest snake paths and cycles of a *knight*, on an $m \times n$ board, have length $mn/2 - O(m + n)$. [See *Enumerative Combinatorics and Applications* **3:2** (2023), #S2R16, 1–23.]

Page 466 replacement for Fig. A–3 and the three previous lines ——— 12 Oct 2023

(b, c, d) See Fig. A–3. The knight puzzles with labels ≥ 6, and the bishop puzzles with labels 0, 10, and 12, are due to N. Beluhov; the others are mostly due to F. Stappers (and are minimum if they have < 5 clues). Solutions can be found in Appendix E.



Page 467 in answer 175 ——— 27 Mar 2024

line 5: $a_i = a + 1 \rightsquigarrow r_i = r + 1$
line 5: $2a + 1 \rightsquigarrow 2r + 1$
line 6: '$\#_{ai}\ x_{ai}$', '$\#_{ai}\ \alpha$' $\rightsquigarrow$ '$\#_{ri}\ x_{ri}$', '$\#_{ri}\ \alpha$'

Page 468 append to answer 185 ——— 10 Aug 2023

(See OEIS sequence A113547 for other interpretations of these numbers.)

Page 482 lines -5 and -4 ——— 07 Apr 2026

arXiv:math/9801068 [math.CO] (1995), 46 pages $\rightsquigarrow$
arXiv:9801068v2 [math.CO] (1995, 2026), 42 pages

▸**Page 485** line 12 ——— 01 Nov 2023

essentially unity $\rightsquigarrow$ essential unity

▸**Page 485** line 5 of answer 268 — 02 Jan 2025

upper left corner $\leadsto$ upper left quarter

▸**Page 486** line 2 of answer 270 — 02 Jan 2025

upper left corner $\leadsto$ upper left quarter

▸**Page 490** line −3 of answer 285 — 13 Mar 2026

New Scientist **12** (1962) $\leadsto$ *New Scientist* **12** (1961)

▸**Page 493** line 2 — 14 Mar 2026

1803 "really" distinct $\leadsto$ 1843 "really" distinct

▸**Page 494** line 6 of answer 298 — 02 Jan 2025

upper left corner $\leadsto$ upper left quarter

▸**Page 494** line 3 of answer 300 — 02 Jan 2025

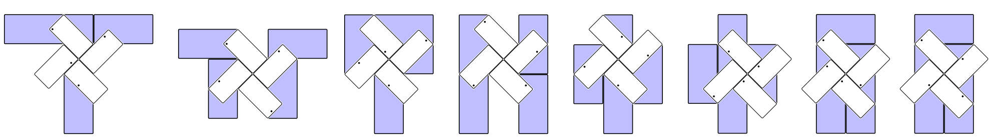
upper left corner $\leadsto$ upper left quarter

Page 497 lines 2 and 3 of answer 303(d) — 12 Oct 2025

$801 \times 4 \leadsto 4 \cdot 801$
$129 \times 4 \leadsto 4 \cdot 129$
$16 \times 4 \leadsto 4 \cdot 16$

▸**Page 497** replacement for answer 305(a) — 04 Feb 2023

(a) Two cases: We can use a 5×5 box, and require that the small squares of each option are either $\{34, 45\}$, $\{47, 56\}$, $\{76, 65\}$, or $\{63, 54\}$; or a 6×6 box, with those small-square coordinates shifted by 11. For example, if we call the leftmost piece '0', its 5×5 options are '0 35 37 34 45', '0 57 77 47 56', '0 53 33 63 54', '0 75 73 76 65'. The 5×5 problem has $4 \cdot 183$ solutions, in groups of four that are related by 90° rotation; the 6×6 problem, similarly, has $4 \cdot 209$ solutions. Reflection gives $4 \cdot 183 + 4 \cdot 209$ more. Here are some of the $8 + 5$ classes of equivalent solutions whose large squares form a symmetric shape; the last two of these look the same, but use different pieces(!):



Page 498 and 499 in answer 306 — 03 Jan 2023

[change 'snake-in-the-box' to simply 'snake', in two places]

▸**Page 498** and 499 in answer 306 — 22 Jan 2023

line 2: both even, and $0 < x < 2n$, $0 < y < 2m$. $\leadsto$ both even. But now we extend the ranges to $0 \le x \le 2n$ and $0 \le y \le 2m$, allowing small squares to cross the boundary.
in 3rd paragraph: Algorithm M finds ... single 20-cycle. $\leadsto$ Algorithm M finds respectively $(0, 0, 4 \cdot 10, 8 \cdot 10)$ solutions in those four cases. However, $(0, 0, 4 \cdot 1, 8 \cdot 8)$ of those solutions are spurious, because their small squares form disjoint cycles instead of a single 20-cycle.

▸**Page 499** last line of answer 308 — 13 Feb 2023

3, 1, and 4 distinct $\leadsto$ 23, 7, and 8 distinct

▸**Page 500** line 6 ——— 21 Dec 2022

168 Gμ to 135 Gμ $\rightsquigarrow$ 16.8 Gμ to 13.5 Gμ

▸**Page 502** line 1 of answer 315 ——— 09 Apr 2023

$(x,y) \mapsto (x+y, x_{\max} - x)$ $\rightsquigarrow$ $(x,y) \mapsto (x+y, -x)$

▸**Page 503** in answer 319 ——— 21 Dec 2022

[The H-grid representation of the L tetrabolo should be

```
· · · · L
L L · · L
· · L L ·
· · L L ·
```

and the rightmost illustration in the two-layer tetrabolo packing displayed on line -4 should be flipped vertically.]

Page 506 lines 3 and 4 ——— 23 Jan 2023

`userpages.monmouth.com/~colonel/polycur.html` $\rightsquigarrow$
`sicherman.net/polycur.html`

Page 506 replacement for four paragraphs in answer 325 ——— 13 Nov 2023

(a) The strong Somap has vertex degrees $7^1\,6^7\,5^{19}\,4^{31}\,3^{59}\,2^{63}\,1^{45}\,0^{15}$; so an "average" solution has $(1\cdot 7 + 7\cdot 6 + \cdots + 15\cdot 0)/240 \approx 2.57$ strong neighbors. (The unique vertex of degree 7 has the level-by-level structure $\substack{333\\534\\552}\ \substack{114\\674\\652}\ \substack{174\\772\\662}$ from bottom to top.) This graph has two edges between $\substack{333\\534\\552}\ \substack{614\\114\\752}\ \substack{664\\762\\772}$ and $\substack{333\\534\\554}\ \substack{614\\114\\752}\ \substack{662\\762\\772}$, so it's actually a multigraph. The two-piece substructure  occurs only in the two solutions $\substack{333\\436\\466}\ \substack{412\\452\\762}\ \substack{115\\755\\772}$ and $\substack{333\\436\\466}\ \substack{412\\455\\765}\ \substack{112\\752\\772}$.

The full Somap has vertex degrees $21^2\,18^1\,16^9\,15^{13}\,14^{10}\,13^{16}\,12^{17}\,11^{12}\,10^{16}\,9^{28}\,8^{26}\,7^{25}\,6^{26}\,5^{16}\,4^{17}\,3^3\,2^1\,1^1\,0^1$, giving an average degree ≈ 9.14. (Its unique isolated vertex is $\substack{333\\432\\462}\ \substack{455\\466\\762}\ \substack{115\\715\\772}$, and its only pendant vertex is $\substack{333\\435\\222}\ \substack{755\\465\\462}\ \substack{771\\711\\466}$. It has 14 instances of repeated edges.)

(b) The Somap has just two components, namely the isolated vertex and the 239 others. The latter has just three bicomponents, namely the pendant vertex, its neighbor, and the 237 others. Its diameter is 8 (or 21, if we use the edge lengths 2 and 3).

The strong Somap has a much sparser and more intricate structure. Besides the 15 isolated vertices, there are 25 components of sizes $\{8\times 2, 6\times 3, 4, 3\times 5, 2\times 6, 7, 8, 11, 16, 118\}$. Using the algorithm of Section 7.4.1.2, the large component breaks down into nine bicomponents (seven of size 1, and another of size 2, not counting articulation points; the other of size 109); the 16-vertex component breaks into seven; and so on, totalling 58 bicomponents altogether for the nonisolated vertices.

Page 509 in answer 335(b) ——— 03 Mar 2026

George $\rightsquigarrow$ [See Rodolfo Marcelo Kurchan, *JRM* **25** (1993), 300; **26** (1994), 311–312.] George

Page 509 and 510, replacement for answer 336 ——— 31 Dec 2022

336. (Solution by P. Weigel.) We can model such a piece by three cubies and one dowel, where the dowel is not allowed to have the same coordinates as the solid cubie: Let there be 27 primary items $C(x,y,z)$ and 27 secondary items $D(x,y,z)$, for $0 \le x,y,z < 3$, representing cubies and dowels. There are $2\cdot 2\cdot 2\cdot 24 = 192$ options, obtained by translation and rotation of the prototype piece '$C(0,1,0)$ $C(1,1,0)$ $C(0,0,0)$ $D(0,0,0)$ $D(0,0,1)$'. (In other words, we consider the solid cubie to be essentially the same as a drilled cubie that's occupied by a dowel.)

Algorithm X needs only 500 kilomems to discover 24 solutions, all of which are equivalent under rotation. One solution has the cubies in positions $\substack{112\\917\\977}\ \substack{322\\388\\986}\ \substack{344\\546\\556}$ and their dowels in positions $\substack{100\\997\\000}\ \substack{124\\527\\080}\ \substack{334\\566\\080}$.

Our model has a surprising corollary that was noted by Ronald Kint-Bruynseels, the designer of this remarkable puzzle: It's possible to drill two holes in each solid cubie, perpendicular to the dowel, without destroying the uniqueness of the solution. [*Cubism For Fun* **75** (2008), 16–19; **77** (2008), 13–18.]

Page 511 line 6 of answer 340(b) ——— 12 Oct 2025

$3940 \times 8 \rightsquigarrow 8 \cdot 3940$

▸**Page 512** in answer 342 ——— 15 Dec 2022

line 6: 2.5 Tμ $\rightsquigarrow$ 240 Gμ
line 8: 2075 sets $\rightsquigarrow$ 2074 sets

▸**Page 512** line 8 of answer 343 ——— 12 Sep 2022

'y 212 311 312 412 512' $\rightsquigarrow$ 'y 122 113 123 124 125'

▸**Page 513** in answer 345's solution ——— 17 Dec 2022

`nnnn` $\rightsquigarrow$ `bbbb`

Page 514 beginning of answer 347 ——— 04 Jun 2025

Fourteen proofs ... [For $\rightsquigarrow$ Many proofs have been given by S. Wagon, *AMM* **94** (1987), 601–617. [No simple necessary and sufficient conditions for $a \times b \times c$ packing $l \times m \times n$ are known. For

Page 522 and 523 in answer 372(b) ——— 30 Nov 2022

twin tree $\rightsquigarrow$ twintree (6 times)

▸**Page 529** line 3 of answer 390 ——— 09 Jun 2023

exercise 389. $\rightsquigarrow$ exercise 389, except when k is ruled out by a strong clue.

▸**Page 529** replacement for third paragraph of answer 390 ——— 09 Jun 2023

For example, the options for cells $(0,0)$ and $(0,1)$ in puzzle 388(b) are '$p_{00}\ r_{03}\ c_{03}\ g_{13}$', '$p_{00}\ r_{04}\ c_{04}$', '$p_{00}\ r_{05}\ c_{05}$'; '$p_{01}\ r_{01}\ c_{11}$', '$p_{01}\ r_{02}\ c_{12}$', '$p_{01}\ r_{03}\ c_{13}\ g_{12}$'. (There's no option for $(i,j,k) = (0,0,2)$ or $(0,1,4)$.) Another option is '$p_{22}\ r_{23}\ c_{23}\ g_{23}\ g_{33}\ g_{43}\ g_{53}$'.

▸**Page 532** in answer 398(b) ——— 16 Mar 2023

in the leftmost column, second row from bottom, 3 $\rightsquigarrow$ 2

Page 532 replacement for the first paragraph of answer 399 ——— 07 Apr 2023

399. (Solution by P. Weigel.) Let r_{ik} and c_{jk} be primary items for $0 \le i,j < n$ and $1 \le k \le n$. Also, if there are w cages, introduce primary items g_t for $1 \le t \le w$. Furthermore, introduce primary items p_{ij} for all cells ij that belong to clueless cages. Every such p_{ij} has n options '$p_{ij}\ r_{ik}\ c_{jk}$', for $1 \le k \le n$. Finally, let C_t be the cells of the tth cage, and let there be an option

$$\text{`}g_t \bigcup\{r_{il(i_t,j_t)},\ c_{jl(i_t,j_t)}\}\text{'}$$

for every feasible way to assign labels $l(i_t,j_t)$ to the cells of C_t. For example, there are two labelings that satisfy the clue '15×' in the third cage in puzzle 398(a), namely either $l(0,3) = 3$ and $l(0,4) = 5$ or $l(0,3) = 5$ and $l(0,4) = 3$; the two options for g_3 are therefore '$g_3\ r_{03}\ c_{33}\ r_{05}\ c_{45}$' and '$g_3\ r_{05}\ c_{35}\ r_{03}\ c_{43}$'. The cage of that puzzle whose clue is '9×' has just one option: '$g_4\ r_{13}\ c_{03}\ r_{11}\ c_{11}\ r_{23}\ c_{13}$'.

Page 533 at the beginning of answer 400 ——— 07 Apr 2023

The formulation ... problems, and $\rightsquigarrow$ Algorithm X breezes through those 2048 problems almost instantaneously, and

Page 533 in answer 401 ——— 07 Apr 2023

within the leftmost puzzle on line -3: $2\div \rightsquigarrow 2-$ and $2- \rightsquigarrow 3\div$
line -2: the most $\rightsquigarrow$ among the most
line -1: $(134) \rightsquigarrow (12)$

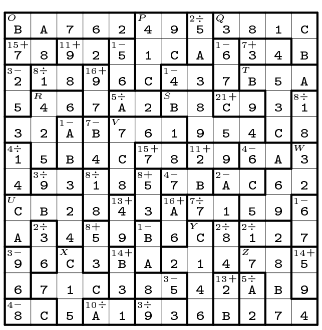
Page 533 replacement for answer 402 ——— 07 Apr 2023

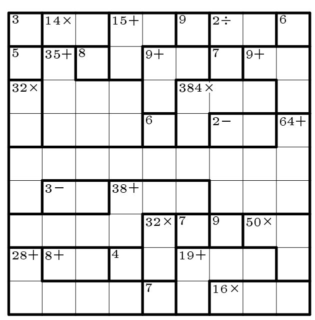
402. (The solution appears below, following answer 403.) The author constructed this puzzle by first designing the cages, then generating a dozen or so random latin squares via exercise 86 until finding one that had a unique solution. Then the domino clues were permuted at random, ten times; the most difficult of those ten puzzles was selected.

The construction of answer 399 gives an XC problem with 9186 options, 342 items, and 107388 total entries. There are respectively (720, 684, 744, 1310, 990, 360, 792, 708, 568, 1200, 606, 30) options for the pentominoes (O, P, ..., Z); preprocessing with Algorithm P reduces those counts to (486, 231, 60, 957, 852, 186, 360, 572, 175, 765, 477, 22). Overall, the reduced problem has 5528 options, 324 items, and 62914 total entries. The total time to find the solution and prove its uniqueness was 2.5 Gμ for Algorithm P and 11 Gμ for Algorithm X, with a search tree of 829 kilonodes. (Without preprocessing, Algorithm X would have taken 27.4 Gμ, and its search tree would have had 1.8 meganodes. Could a human being solve this puzzle by hand?)

Page 533 replacement for answer 403 ——— 07 Apr 2023

403. Filip Stappers has devised the stunning puzzle shown below, which matches π's first 42 digits perfectly(!). The construction of answer 399 fails spectacularly on this particular instance, because the monster cage for '64+' has 354,896,640 options. We can, however, work around that problem by simply treating row 4 as an additional cage without a clue, and subtracting $1+2+\cdots+9=45$ from the original cage total. (A latin square is determined by any $n-1$ of its rows.) Then Algorithm X solves the problem handily, with a cost of only 292 Mμ (from 173875 options, 200 items, 2766956 total entries), and with a search tree of only 194 nodes. (See Appendix E.)





▸Page 534 in answer 404 ——— 09 Jun 2023

pairs of unlabeled vertices ... For example, $\rightsquigarrow$ pairs of vertices u — v such that $u=v_k$ or might be labeled k and $v=v_{k+1}$ or might be labeled $k+1$. But we omit $-u\ p_u{:}k\ q_k{:}u$ if $u=v_k$; we omit $+v\ p_v{:}k{+}1\ q_{k+1}{:}v$ if $v=v_{k+1}$. For example,

▸**Page 534** in answer 404 ——— 13 Feb 2023

line 16: 's_3 +10 p_{10}:4 q_4:10' $\rightsquigarrow$ 's_3 +11 p_{11}:4 q_4:11'
line 17: 's_4 −11 p_{11}:4 q_4:11' $\rightsquigarrow$ 's_4 −10 p_{10}:4 q_4:10'

Page 535 replacement for answer 408 ——— 04 Feb 2023

408. (a)

					6
28					
11			24		33

; (b)

	2		6		10
1		5		9	
23	21			13	
24		19	17		14
	25		30		36
26		29		35	

. This 19-clue puzzle is due to Germán González-Morris. (See Appendix E for solutions. Are the numbers 5 and 19 best possible for 6 × 6 hidato?)

Page 535 in answer 411 ——— 28 Aug 2023

Are *three* loops possible? ... solution.] $\rightsquigarrow$ If $m+1$ and $n+1$ are relatively prime, N. Beluhov [arXiv:2308.08798 [math.CO] (2023), 28 pages] has proved that an $m \times n$ slitherlink diagram with all clues given cannot have more than one solution. He has also found $(4k+3) \times (4k+3)$ fully clued examples that are solvable with exactly *three* loops, for all $k > 0$. It's currently unknown whether or not exactly *five* loops are possible.]

▸**Page 539** in answer 422 ——— 31 Dec 2022

line 1: $(2m-1)(2n-1)\ldots y \le 2n-2 \rightsquigarrow 4mn$ pairs xy for $0 \le x < 2m$ and $0 \le y < 2n$
lines 9 and 10: $N(i,j) = C(1,j) - 10, \ldots$ Edges $\rightsquigarrow$
$N(i,j) = C(i,j) - 21$, $NN(i,j) = C(i,j) - 41$, $W(i,j) = C(i,j) - 12$, $WW(i,j) = C(i,j) - 14$; $N(i,j)$ is the edge between cells (i,j) and $(i-1,j)$. Edges

Page 540 last lines of answer 425 ——— 10 Nov 2022

solutions for $k = 5 \ldots 9, 10$. $\rightsquigarrow$ solutions for $5 \le k \le 10$, due to G. J. H. Goh, are also known. (See (xviii)–(xxiv) in Fig. A–5; `https://boonsuan.github.io/masyu.pdf`.)

Page 547 replacements for two illustrations in the bottom line ——— 12 Oct 2025

(α)

a	A	e	E	B	b
A	W	Y	w	X	B
S	z	Q	R	Z	q
s	X	E	S	x	Q
D	Z	y	W	Y	C
d	D	R	r	C	c

(γ)

a	E	e	A	B	b
S	W	E	w	X	Q
A	z	Z	R	Q	q
s	X	Y	S	x	B
D	Z	y	W	Y	C
d	C	D	r	R	c

Page 548 replacement for copy after the display in answer 449 ——— 22 Oct 2023

Literary hitori nuggets longer than 80 characters were first reported by Gary McDonald in 2023: $(7 \times 12 = 84)$ "he stood and carefully examined the sky, to ascertain the time of night from the altitudes of the stars." [Thomas Hardy, *Far from the Madding Crowd* (1874)]. $(11 \times 8 = 88)$ "'...But talk to me of poverty and wealth, and there indeed we touch upon realities.' 'My De-ar, this is becoming Awful—'" [Charles Dickens, *Our Mutual Friend* (1864)]. $(10 \times 10 = 100)$ "shall never forget his flying Henry's kite for him that very windy day last Easter—and ever since his particular kindness" [Jane Austen, *Emma* (1816)].

Page 549 new answer ——— 18 Jun 2023

5. The upper bound $W(3,k) = e^{O(\log k)^{11}}$ follows from (surprising) results of Z. Kelley and R. Meka, arXiv:2302.05537 [math.NT], 79 pages. The (surprising) superpolynomial lower bound $W(3,k) = k^{\Omega(\log k/\log\log k)}$ was proved by B. Green and Z. Hunter in *Forum of Mathematics, Pi* **10** (2022), e18:1–51; *Combinatorica* **42** (2022), 1231–1252.

▶**Page 549** line 7 of answer 10 ——— 11 May 2024

Such a problem, "not-all-equal SAT," is $\leadsto$ Such a problem is

▶**Page 551** replacement for the opening lines of answer 17 ——— 02 Jul 2025

17. Let *mcgregor* (n) be the clauses (15) and (16) for the graph. For example, let the Boolean variable v_{jkc} mean that vertex (j,k) has color c; then (15) is the set of clauses $(v_{jk1} \vee v_{jk2} \vee v_{jk3} \vee v_{jk4})$, for $0 \le j \le n$ and $0 \le k < n$. Add clauses to enforce the symmetric threshold function $S_{\le r}(v_{001},\ldots,v_{n(n-1)1})$, using either (18) and (19) or (20) and (21). The resulting clauses are satisfiable if and only if $f(n) \le r$. Similarly, if $S_{\ge r}$ is substituted for $S_{\le r}$, the clauses are satisfiable if and only if $g(n) \ge r$.

Additional unit clauses that specify the colors of the four clique vertices will speed up the computation: Four cases should be run, one with each clique vertex receiving color 1. If all four cases are unsatisfiable, we've proved that $f(n) > r$ or $g(n) < r$, respectively. Binary search with different values of r will identify the optimum.

▶**Page 552** line 3 of answer 19 ——— 02 Jul 2025

vertices v_x $\leadsto$ variables v_x

▶**Page 552** line 8 of answer 19 ——— 24 May 2024

(1, 3, 4, 8, 9, 13) $\leadsto$ (1, 3, 4, 7, 9, 13)

▶**Page 554** line −2 of answer 31 ——— 13 Mar 2026

(2) **11** (1936) $\leadsto$ **11** (1936)

▶**Page 555** line 1 of answer 35 ——— 02 Jan 2025

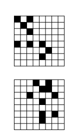
$\chi^*(G) \ge 7/2$ $\leadsto$ $\chi^*(G) \le 7/2$

▶**Page 557** in answer 49 ——— 17 Mar 2026

line 2: $\bar{a}_2^2$, $\bar{a}_3^2$ $\leadsto$ $\bar{a}_3^2$
line 9: at least three of these five $\leadsto$ at least two of those five

Page 561 replacement for answer 68 ——— 26 Feb 2026

68. (Solution by Ben Chaffin.) Exhaustive enumeration is feasible with bitwise computations (and without SAT). The longest paths achievable with (6, 7, 8, 9, 10) live starting cells have (8, 9, 10, 11, 11) generations; starting patterns that achieve length 11 are shown. Roughly (69%, 8%, 10%, 4%, 13%) of all starts end up (too dead, too live, immobile, still, out-of-bounds), respectively.



Page 565 at end of answer 84 ——— 10 Jan 2024

There's no known way ... for all $j \ge 0$.] $\leadsto$ There's no known way to prove even the special cases that, say, $f^*(j,2j) = 6j$ for all $j \ge 0$. There has, however, been a recent spectacular breakthrough: As of February 2024, Tomas Rokicki has verified equality for all cells (i,j) with $i \le 6$ or $j \le 6$.]

Page 588 in final paragraph of answer 205 ——— 22 Jul 2024

delete 'Is there a shorter solution?' at the beginning; add this at the end:
See T. Zhang, T. Peitl, and S. Szeider, arXiv:2405.16149 [cs.DM] (2024), 25 pages.

▸**Page 598** replacement for second paragraph of answer 263 ——— 26 Mar 2026

When step C7 is entered from C4, assuming that $d > 0$, set $d' \leftarrow r \leftarrow 0$, $q \leftarrow -1$, $s \leftarrow s{+}3$, and do $blit(l_j)$ for $0 \le j < k$. Also set $t \leftarrow \max\bigl(\texttt{TLOC}(|l_0|), \ldots, \texttt{TLOC}(|l_{k-1}|)\bigr)$. Then, while $q > 0$, set $l \leftarrow L_t$, $t \leftarrow t-1$; if $\texttt{S}(|l|) = s$ then set $q \leftarrow q-1$, and if $R_l \ne \Lambda$ let clause R_l be $l_0 l_1 \ldots l_{k-1}$ and do $blit(l_j)$ for $1 \le j < k$. Finally set $l' \leftarrow L_t$, and while $\texttt{S}(|l'|) \ne s$ set $t \leftarrow t-1$ and $l' \leftarrow L_t$.

Page 602 append new sentence to answer 283 ——— 26 Oct 2024

Note: Exercise 7.2.2.3–232 has further information.

▸**Page 610** replacement for the last line of answer 313 ——— 04 Jan 2025

From 0110 the algorithm goes "uphill" to either 1110 or 0111.

▸**Page 611** line 6 of answer 318 ——— 02 Feb 2024

$g_n - g_{n+1} = p/g_n^t - p/g_{n+1}^t > 0$ when $g_{n+1} < g_n \rightsquigarrow g_n - g_{n+1} = p/g_n^t - p/g_{n-1}^t > 0$ when $g_n < g_{n-1}$

▸**Page 621** top line ——— 15 Nov 2025

$C_1, \ldots, C_a, C'_1, \ldots, C'_b \rightsquigarrow C_1, \ldots, C_p, C'_1, \ldots, C'_q$

▸**Page 624** last line of answer 379 ——— 13 Sep 2022

property. The erp rule . . . sufficient. $\rightsquigarrow$ property.

▸**Page 627** line 5 of answer 392 ——— 20 Nov 2024

Serhiy A. Grabarchuk $\rightsquigarrow$ Serhiy O. Grabarchuk

▸**Page 632** in answer 410 ——— 08 Oct 2022

line 4: $(c_0 z_0)_2 \ge x_0 + x_1 \rightsquigarrow (c_0 z_0)_2 \ge x_0 + x_2$
line 8: $z_i \ge v \oplus s$, $q_i \ge v \wedge s \rightsquigarrow z_i \ge v \oplus s_i$, $q_i \ge v \wedge s_i$
lines 11–13: ternary clauses; . . . But the order encoding $\rightsquigarrow$ ternary clauses, together with $(\bar c_7) \wedge (\bar z_7)$. Unit propagation, and the removal of clauses with the pure literals z_0, z_2, z_4, z_6, now reduce the result. But the order encoding

▸**Page 633** replacement for first two lines of answer 415 ——— 16 Sep 2025

415. Via (166), construct clauses for $1 \le k \le n$ to encode 'a_{k-1} implies $x_k < y_k + a_k$':

$$\Bigl(\bigwedge_{j=1}^{d-1} (\bar a_{k-1} \vee \bar x_k^j \vee y_k^j)\Bigr) \wedge \Bigl(\bigwedge_{j=0}^{d-1} (\bar a_{k-1} \vee a_k \vee \bar x_k^j \vee y_k^{j+1})\Bigr), \quad \text{omitting } \bar x_k^0 \text{ and } y_k^d;$$

Page 635 replacement for the first paragraph of answer 436 ——— 02 Apr 2026

436. Let the variables q_k for $0 \le k \le n$ and $q \in Q$ represent the sequence of states, and let the variables t_{kaq} represent a transition when $1 \le k \le n$ and when T contains a triple of the form (q', a, q). The clauses, F, are the following, for $1 \le k \le n$: (i) $(\bar t_{kaq} \vee x_k^a) \wedge (\bar t_{kaq} \vee q_k)$, when t_{kaq} exists, where x_k^0 denotes $\bar x_k$ and x_k^1 denotes x_k; (ii) $(\bar q_{k-1} \vee \bigvee\{t_{kaq'} \mid (q,a,q') \in T\})$, for $q \in Q$; (iii) $(\bar q_k \vee \bigvee\{t_{kaq} \mid (q',a,q) \in T\})$, for $q \in Q$; (iv) $(\bar x_k^a \vee \bigvee\{t_{kaq} \mid (q',a,q) \in T\})$, for $a \in A$; (v) $(\bar t_{kaq'} \vee \bigvee\{q_{k-1} \mid (q,a,q') \in T\})$, when $t_{kaq'}$ exists. And (vi) $(\bar q_0)$ for $q \in Q\setminus I$, $(\bar q_n)$ for $q \in Q\setminus O$.

▸**Page 635** line −7 of answer 436 ——— 02 Apr 2026

$8n + 3 \rightsquigarrow 7n + 3$

▶**Page 639** in answer 456 —— 20 Nov 2024

0.06% $\rightsquigarrow$ 0.62%

▶**Page 644** line 2 of answer 480(c) —— 02 Jan 2025

$c_{n-3} \le c_n + 9 \rightsquigarrow c_{n+3} \le c_n + 9$

▶**Page 645** line 8 of answer 483 —— 13 Mar 2026

191-vertex graph $\rightsquigarrow$ 3071-vertex graph

▶**Page 647** line −3 of answer 488 —— 13 Mar 2026

2–16, for generalizations $\rightsquigarrow$ 12–16, for generalizations

Page 648 replacement for last two lines of answer 495 —— 16 Feb 2024

row increase lexicographically; and the same holds for columns. [See Anna Lubiw, *SICOMP* **16** (1987), 854–879. The matrix entries needn't be only 0s and 1s.]

Page 653 line 2 of answer 521 —— 19 Jan 2023

arXiv:1402.1956 [cs.AI] (2014), 13 pages. $\rightsquigarrow$ *Int. J. Artificial Intelligence Tools* **27**,8 (2018), 1850033:1–19.

▶**Page 662** line 8 —— 05 Aug 2025

$\Gamma(x+k)/\Gamma(k) \rightsquigarrow \Gamma(x+k)/\Gamma(x)$

Page 662 line 16 —— 26 Feb 2024

Stirling subset number $\rightsquigarrow$ Stirling partition number

▶**Page 662** line −4 —— 15 Aug 2022

saturated subtraction $\rightsquigarrow$ saturating subtraction

▶**Page 665** lines −4 and −3 —— 15 Sep 2025

In the end, however, I did put in one equation,
Einstein's famous equation, $E = mc^2$.

Page 668 in the Independent Set entry —— 29 Dec 2024

no-three-on-a-line problem $\rightsquigarrow$ no-three-in-line problem

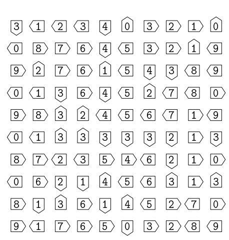
▶**Page 668** in the Clique entry —— 19 Jan 2023

$\sim G \rightsquigarrow \overline{G}$

Page 671 line 3 —— 03 Jan 2023

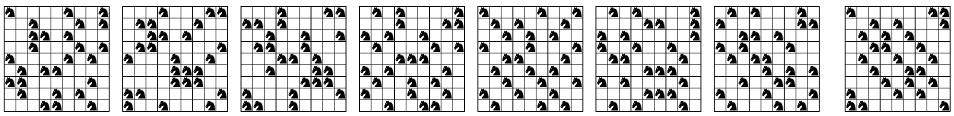
in Section 7.2.2.1. $\rightsquigarrow$ in Section 7.2.2.1, unless stated otherwise.

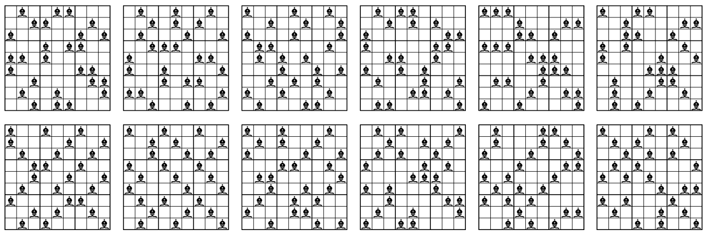
Page 671 new answer inserted at left of line 4 —— 14 Mar 2023



(see answer 7.2.2–69)

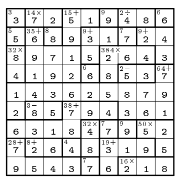
Page 671 replacement for the answers re answer 173 ——— 12 Oct 2023





(see answer 173)

Page 672 solution to the puzzle in the new answer 403 ——— 18 Feb 2023



(see answer 403)

Page 672 rightmost solution for answer 408 ——— 04 Feb 2023

3	2	7	6	11	10
1	4	5	8	9	12
23	21	20	18	13	15
24	22	19	17	16	14
27	25	31	30	33	36
26	28	29	32	35	34

Page 673 sneak in a new answer near the bottom ——— 22 Oct 2023

S	H	A	L	L	N	E	V	E	R
F	O	R	G	E	T	H	I	S	F
L	Y	I	N	G	H	E	N	R	Y
S	K	I	T	E	F	O	R	H	I
M	T	H	A	T	V	E	R	Y	W
I	N	D	Y	D	A	Y	L	A	S
T	E	A	S	T	E	R	A	N	D
E	V	E	R	S	I	N	C	E	H
I	S	P	A	R	T	I	C	U	L
A	R	K	I	N	D	N	E	S	S

(see answer 449)

Page 674 and following ——— 23 Jul 2022

Miscellaneous changes to the existing index of Volume 4B are collected here, including corrections and amendments to the old entries as well as new entries that are occasioned by the new material. Thus, the lines of the full index that have changed serve also as an index to the present document. However, when a correction or amendment has caused an old index entry to be deleted, the deletion is usually not indicated.

▸**Page 714** line −6 ——— 14 Nov 2022

on an Dell Precision $\leadsto$ on a Dell Precision