☆ THE ART OF COMPUTER PROGRAMMING ☆

☆☆☆ ERRATA TO VOLUME 4A (after 2021) ☆☆☆

This document is a transcript of the notes that I have been making in my personal copy of *The Art of Computer Programming*, Volume 4A (20th printing), since it was first printed in 2022. Previous errata are recorded in another file '`all4a-pre.ps`'.

Four levels of updates — "errors," "amendments," "plans," and "improvements" — appear, indicated by four different typographic conventions:

▸**Page 666** line 1 ——— 04 Jul 1776

Technical or typographical errors (aka bugs) are the most critical items, so they are flagged with a '▸' preceding the page number. The date on which I first was told about the bug is shown; this is the effective date on which I paid the finder's fee. The necessary corrections are indicated in a straightforward way. If, for example, the book says 'n' where it should have said '$n+1$', the change is shown thus:

$n \rightsquigarrow n+1$

Page 666 line 2 ——— 14 Jul 1789

Amendments to the text appear in the same format as bugs, but without the '▸'. These are things I wish I had known about or thought of when I wrote the original text, so I added them later. The date is the date I drafted the new text.

Page 666 line 3 . . . . 20 Nov 1917

Plans for the future represent a third kind of item. In such notes I sketched my intentions about things that I wasn't ready to flesh out further when I wrote them down. You can identify these items because they're written in slanted type, and preceded by a bunch of dots '. . . . .' leading to the date on which I recorded the plan in my files.

Page 666 line 4 ——— 10 Jan 1938

The fourth and final category — indicated by page and line number in smaller, slanted type — consists of minor corrections or improvements that most readers don't want to know about, because they are so trivial. You wouldn't even be seeing these items if you hadn't specifically chosen to print the complete errata list in all its gory details. Are you sure you wanted to do that?

The ultimate, glorious, future editions of The Art of Computer Programming are works in progress. Please let me know of any improvements that you think I ought to make. Send your comments either by snail mail to D. E. Knuth, Computer Science, Gates Building 1B, Stanford University, Stanford CA 94305-9015, or by email to `taocp@cs.stanford.edu`. *(Use email for book suggestions only, please — all other correspondence is returned unread to the sender, or discarded, because I have no time to read ordinary email.) Although*

I'm working full time on Volume 4 these days, I will try to reply to all such messages within a year of receipt. Current news is posted on

`http://www-cs-faculty.stanford.edu/~knuth/taocp.html`

and updated regularly.

— Don Knuth, January 2011

If there happen to bee any iarre or dissonance, blame not the Printer,
who (I doe assure thee) through his great paines and diligence,
doth heere deliuer to thee a perfect and true Coppie.
If . . . there bee any fault by mee committed,
I desire the skilfull, eyther with courtesie to let the same bee concealed,
or in friendly sort to bee thereof admonished:
and at the next Impression he shall find error reformed:
remembring alwaies, that it is more easie to finde a fault then to amend it.

— WILLIAM BYRD, *Psalmes, Sonets, & songs of sadnes and pietie* (1588)

He did a lot of editing. That's how he liked to work.
Sometimes he even made alterations between printings.

— P. D. JAMES, *The Lighthouse* (2005)

☆☆ COMBINATORIAL ALGORITHMS, PART 1 ☆☆

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Last updated 9 September 2025

Most of these corrections have already been made in recent printings.

Page v lines −6 and −5 ______ 15 Jun 2022

because, in fact, ... by 1. $\leadsto$ because the size of a combinatorial problem can increase more than 100,000-fold when a parameter n increases by just 1.

Page ix new paragraph to follow line 4 ______ 09 Oct 2023

Many of the computer programs that I wrote while preparing this material are listed Internet on the following webpage:

`http://www-cs-faculty.stanford.edu/~knuth/programs.html`

In particular, you can download BDD14 and BDD15, which are the experimental toolkits that I played with when studying BDDs and ZDDs, respectively, in Section 7.1.4. Another example is a program called SIMPATH, for exercise 7.1.4–225.

Page x line −3 ______ 24 Nov 2023

For example, $\nu 10 = 2$, $\lambda 10 = 3$, $\rho 10 = 1$. $\leadsto$
For example, $\nu 18 = 2$, $\lambda 18 = 4$, $\rho 18 = 1$.

Page xiv line 4 of Boswell's quote ______ 27 Apr 2024

shown $\leadsto$ *shewn*

Page 31 line 14 ______ 28 Nov 2024

pixel density $\leadsto$ level of gray

Page 33 line 4 ______ 10 Feb 2024

incident with e. The hypergraph $\leadsto$ incident with e. (We assume that $V \cap E = \emptyset$.) The hypergraph

Page 34 line 2 before (61) ______ 28 Nov 2024

all $w \notin W$ $\leadsto$ all $w \in V \setminus W$

Page 42 line 1 of exercise 80 ______ 28 Nov 2024

graph must $\leadsto$ graph on two or more vertices must

▸**Page 48** line 8 ______ 15 Nov 2023

Looking Glass $\leadsto$ *Looking-Glass*

▸**Page 60** line 2 ______ 28 Feb 2022

$l \leftarrow l + k$ $\leadsto$ $\mathtt{START}(c) \leftarrow l$, $l \leftarrow l + k$

Page 60 new steps C3 and C5 ———— 30 May 2023

C3. [Done with loop?] If $c = \Lambda$, return to C2. Otherwise set $c' \leftarrow$ `PREV`(c), $q \leftarrow$ `CONCLUSION`(c), and go to C6 if `TRUTH`$(q) = 1$. Otherwise set $l \leftarrow$ `START`(c) and $k \leftarrow$ `COUNT`$(c) - 1$.

C5. [Deduce `CONCLUSION`(c).] Set `TRUTH`$(q) \leftarrow 1$, $S_s \leftarrow q$, $s \leftarrow s + 1$.

Page 98 line 8 ———— 15 Jun 2024

its complexity $\rightsquigarrow$ its cost

Page 112 new copy to follow line 3 ———— 15 Jul 2023

[An even better upper bound, replacing $3 \lg n$ by $\lg n + \lg\lg n$, has been proved by S. A. Lozhkin, *Moscow University Mathematics Bulletin* **77** (2022), 144–153.]

Page 113 replacement for (43) and preceding lines ———— 21 Feb 2025

a chain of 22 steps does emerge; and David Stevenson . . . [end of display] $\rightsquigarrow$
a chain of 22 steps does emerge. And Oliver Runge has shown that only 19 steps are actually needed(!), if we do a "hungry search" that uses footprints more cleverly [`https://orunge.org/boolean-chains/`]:

$$\begin{array}{rrr}
x_5 = \bar{x}_3 \wedge x_4, & x_{12} = x_1 \vee x_{11}, & x_{18} = x_{11} \oplus x_{16}, \\
x_6 = x_3 \oplus x_4, & g = x_{13} = x_7 \vee x_{12}, & \bar{a} = x_{19} = x_8 \wedge \bar{x}_{18}, \\
x_7 = x_2 \oplus x_3, & x_{14} = x_7 \wedge \bar{x}_{11}, & x_{20} = x_4 \oplus x_{14}, \\
x_8 = x_5 \vee x_7, & x_{15} = x_2 \oplus x_{14}, & \bar{e} = x_{21} = \bar{x}_{16} \wedge x_{20}, \\
x_9 = x_1 \oplus x_2, & \bar{c} = x_{16} = \bar{x}_9 \wedge x_{15}, & x_{22} = x_{19} \oplus x_{20}, \\
\bar{f} = x_{10} = x_8 \wedge \bar{x}_9, & \bar{b} = x_{17} = x_{12} \wedge x_{15}, & \bar{d} = x_{23} = x_8 \oplus x_{22}. \\
x_{11} = x_6 \oplus x_{10}, & &
\end{array} \qquad (43)$$

Page 115 line 6 after (46) ———— 18 Feb 2025

telephone dial $\rightsquigarrow$ telephone keypad

Page 125 new rating for exercise 11 ———— 22 Mar 2025

[*22*] $\rightsquigarrow$ [*23*]

Page 127 line 7 of exercise 38 ———— 16 Mar 2025

functions $\{f_1, f_2, \ldots, f_n\}$ can be computed in $O(n)$ steps $\rightsquigarrow$ functions can be computed in $O(n)$ steps,

Page 133 replacement for the Colman quote ———— 27 Apr 2024

Lady C. — *Pshaw, that's such a hack.*
Sir Sim. — *A hack, Lady Caroline, that the knowing one's have warranted sound.*

▸**Page 142** line 6 ———— 24 May 2022

64 substrings $\rightsquigarrow$ 64 truncated shifts

Page 186 in exercise 23 ———— 14 Jun 2024

line 2: right parenthesis $\rightsquigarrow$ left parenthesis
line 3: $(001101)_2$, the number 13. $\rightsquigarrow$ $(110010)_2$, the number 50.
line 8: assuming that $\nu x \leq 32$. $\rightsquigarrow$ assuming that $m < 32$.

Page 197 line 4 of exercise 167 19 May 2025

life histories $\rightsquigarrow$ Life histories

▸**Page 200** line 2 19 May 2025

$\bmod (x^N+1)$ and $\bmod 2$ $\rightsquigarrow$ $\bmod (x^N+1)\ \bmod 2$

▸**Page 215** the line before (34) 19 May 2025

Then M_k computes $\rightsquigarrow$ We can make M_k compute

Page 262 replacement for exercise 45 19 May 2025

45. [*25*] Design precise specifications for Boolean modules that will compute the three-in-a-row function as in (33) and (34), and show that your network is well defined.

Page 262 new rating for exercise 47 19 Sep 2025

[*M21*] $\rightsquigarrow$ [*M26*]

Page 274 new rating for exercise 206 26 Sep 2024

[*M46*] $\rightsquigarrow$ [*M40*]

Page 299 line 1 24 May 2022

the sequence $\rightsquigarrow$ the digits change by ±1 and the sequence

▸**Page 334** line 21 20 Nov 2022

[*Inf. Proc. Letters* **17** (1983), 231–234] $\rightsquigarrow$ [*Inf. Proc. Letters* **17** (1983), 231–233]

Page 363 first line of step R4 24 May 2022

$c_j = c_{j-1}+1$ $\rightsquigarrow$ $c_{j-1} = c_j - 1$

Page 374 a new sentence to be inserted after (67) 02 Aug 2025

We assume that each bound m_j is 2 or more, so that no component is identically 0.

Page 376 in Theorem W 02 Aug 2025

where $m_1 \le \cdots \le m_n$ $\rightsquigarrow$ where $2 \le m_1 \le \cdots \le m_n$

Page 379 second line of exercise 1 08 Oct 2023

$\infty \cdot n - t$ $\rightsquigarrow$ $\infty \cdot (n{-}t)$

Page 389 in exercise 110 24 May 2022

line 4: where one card ... choice of k: $\rightsquigarrow$ where a player holds $\{c_1, c_2, c_3, c_4\}$ and card c_5 is called the *starter*. The score is the sum of points computed as follows, for each subset S of C:
lines 11 and 12: If $s = 4$ ... same suit as $\rightsquigarrow$ If $S = \{c_1, c_2, c_3, c_4\}$ and all cards of S have the same suit, score $4 + [c_5$ has the same suit as
line 13: $c_k \rightsquigarrow c_5$ (twice)
line -2: combinations and starter choices $\rightsquigarrow$ combinations

▸**Page 425** line 14 20 Nov 2022

Duke Math. J. **25** (1957), 29–43 $\rightsquigarrow$ *Duke Math. J.* **25** (1958), 29–43

Page 428 insert new algorithm in the middle of the page ——— 05 May 2023

line 3 before (53): A nice way $\leadsto$ An interesting way
line 5 after (53): $1/\varpi_n$. $\leadsto$ $1/\varpi_n$. We can generate M by using 3.4.1–(3); it tends to be approximately $n/\xi = e^{\xi}$ (see exercise 67).

now replace the following paragraph by new copy:

There's also an all-integer method, based on Peirce's triangle:

Algorithm P (*Uniformly random set partitions*). Given $N \geq 1$, this algorithm generates the blocks of a random partition of $\{1, \ldots, N\}$, using ideas explained in exercise 26. Three auxiliary stacks, S, T, and B, hold auxiliary data.

P1. [Initialize.] Set $n \leftarrow N$, $k \leftarrow 1$, $S \leftarrow \{1, \ldots, N\}$, and $T \leftarrow B \leftarrow \emptyset$.

P2. [Begin a block.] Pop $x \Leftarrow S$ and push $B \Leftarrow x$. (See 2.2.1–(2) and 2.2.1–(1).)

P3. [Done with block?] If $n = k$, output B as a block of the partition, and set $n \leftarrow n-1$, $k \leftarrow 1$, $S \leftarrow T$, $T \leftarrow B \leftarrow \emptyset$; terminate if $n = 0$, else go to P2.

P4. [Add to block?] Pop $x \Leftarrow S$ and let U be a random integer, $0 \leq U < \varpi_{nk}$. If $U < \varpi_{(n-1)k}$, push $B \Leftarrow x$ and set $n \leftarrow n-1$; otherwise push $T \Leftarrow x$ and set $k \leftarrow k+1$. Return to P3. ▮

Page 453 line 6 above Figure 57 ——— 05 Aug 2025

forests that lie above it $\leadsto$ forest whose worm walk lies below it

Page 462 new paragraph before the Spanning trees subsection ——— 01 Nov 2022

E. F. Harding [*Advances in Appl. Prob.* **3** (1971), 44–77] has shown how to generate the kth oriented *binary* tree with n vertices (see exercise 2.3.4.4–6).

Page 464 line 2 of Algorithm S becomes two lines ——— 15 Dec 2022

trees. $\leadsto$ trees in a revolving-door Gray code order.

▸**Page 466** line 7 ——— 05 Aug 2025

nodes of degree 1 $\leadsto$ inner nodes of degree 1

Page 484 replacement for last two lines of exercise 122 ——— 08 Jan 2024

$\langle$number$\rangle \rightarrow \langle$digit string$\rangle$ | .$\langle$digit string$\rangle$ | $\langle$digit string$\rangle$.$\langle$digit string$\rangle$

For example, $100 = ((.1 \times .2 + 3)/.4)/.5 + 6 + 78.9$, amazingly enough.

Page 499 line −4 ——— 05 Jan 2025

"Lotus Delight of Calculation" $\leadsto$ "Mathematics Illuminated"

Page 505 line 15 ——— 26 Feb 2024

Stirling subset numbers $\leadsto$ Stirling partition numbers

Page 505 new intro to the subsection on integer partitions ——— 12 Apr 2024

Integer partitions. Partitions of integers ... Bishop Wibold $\leadsto$
Integer partitions. Partitions of integers can be found in Chapter 12, Part 4 of the *Bhagavatī Sūtra*, an encyclopedic work in the canon of Jainism that was completed before A.D. 500. Every partition of every number up to 10 is explicitly

mentioned, in words (not symbols), except that $2+2+5$ and $1+2+2+5$ were omitted. [See D. Jadhav, *Mathematics in Ancient Jaina Literature* (World Scientific, 2023), 87–105.]

The earliest known European references came much later. We saw above that Bishop Wibold

▸**Page 507** line −5 ———— 13 Apr 2023

Zahlen durch ⇝ *Zahlen, durch*

Page 514 replacement for the first answer 3 ———— 26 Jul 2022

3. See H. Poincaré, *Rendiconti del Circolo Matematico di Palermo* **18** (1904), 45–110; R. H. Bing, *Annals of Math.* (2) **68** (1958), 17–37; G. Perelman, arXiv:math/0211159 [math.DG] (2002), 39 pages; 0303109 and 0307245 [math.DG] (2003), 22+7 pages.

Page 516 last line of answer 9 ———— 09 Dec 2022

132 rows ⇝ 132 options

▸**Page 516** last line of answer 13 ———— 13 Feb 2024

(2) **50** ⇝ **50**

Page 519 end of answer 23 ———— 28 Nov 2024

[See S. W. Golomb ... code.]⇝ (The literature of coding theory often denotes a code $C(b,n,r)$ of distance d by the symbol $(n+r,b^n,d)_b$. Thus, a b-ary orthogonal array of depth m is essentially an $(m,b^2,m-1)_b$ code.)

[See R. W. Hamming, *The Bell System Technical Journal* **29** (1950), 147–160; S. W. Golomb and E. C. Posner, *IEEE Transactions* **IT-10** (1964), 196–208.]

Page 519 replacement for answer 26 ———— 13 Feb 2025

26. "`local press print scoop: daily, young sharp geeks check dodgy input, track stray flaws, bring quick fixes, about fifty years after grand elder minds first wrote crisp codes using bytes every night`." [Filip Stappers, 2025.]

▸**Page 521** in answer 47 ———— 10 Nov 2022

Geometrie der Gewebe (1937) ⇝ *Geometrie der Gewebe* (1938)

▸**Page 522** in answer 54 ———— 10 Nov 2022

Darryl Francis, Philip Cohen, and A. Ross Eckler in *Word Ways* **19** (1976), 241; **20** (1977), 8. ⇝ Darryl Francis and Philip Cohen in *Word Ways* **9** (1976), 241; **10** (1977), 46–47.

Page 522 beginning of answer 59(c) ———— 28 Nov 2024

Yes. ⇝ Yes, up to isomorphism.

Page 562 replacement for the first part of answer 11 ———— 22 Mar 2025

11. The only subtle point is that j should *decrease* in step U3; then we'll never have $\phi(g)\,\&\,\phi(h)\neq 0$ when $j=0$, so all cases of cost $r-1$ will be discovered before we begin to look at list $r-1$. Some normal functions x_{n+1}, ..., x_m may also be given (cost 0).

U1. [Initialize.] Set $U(0)\leftarrow\phi(0)\leftarrow 0$ and $U(f)\leftarrow\infty$ for $1\le f<2^{2^n-1}$. Then set $U(x_k)\leftarrow\phi(x_k)\leftarrow 0$ and put x_k into list 0, for $1\le k\le m$. Let there be t distinct functions of the form $f=x_j\circ x_k$, where $1\le j<k\le m$, $\circ$ is

a normal operator, and $U(f) = \infty$. If f is the pth such function, where $1 \le p \le t$, put f in list 1 and set $U(f) \leftarrow 1$, $\phi(f) \leftarrow 0^{p-1}10^{t-p}$. Finally set $c \leftarrow 2^{2^n-1} - 1 - m - t$ (the number of f with $U(f) = \infty$).

Page 571 revise answer 51 by making its notation systematic ______ 16 Mar 2025

51. $f = \bigl([(x_5x_6)_2 \in \{1\}] \wedge [(x_1x_2x_3x_4)_2 \in \{1,3,4,7,9,10,13,15\}]\bigr) \vee \bigl([(x_5x_6)_2 \in \{2,3\}] \wedge [(x_1x_2x_3x_4)_2 \in \{0\}]\bigr) \vee \bigl([(x_5x_6)_2 \in \{3\}] \wedge [(x_1x_2x_3x_4)_2 \in \{1,2,4,5,7,10,11,14\}]\bigr)$.

Page 571 replacement for opening lines of answer 54 ______ 24 Mar 2025

The greedy-footprint … minimum-first method $\rightsquigarrow$
Oliver Runge's hungry-search heuristic gives a chain of length 13,

$$\begin{array}{rlrlrl}
x_5 &= x_2 \oplus x_3, & x_{10} &= \bar x_7 \wedge x_8, & f_6 = x_{14} &= x_4 \wedge x_{13},\\
x_6 &= x_4 \wedge \bar x_5, & f_3 = x_{11} &= x_4 \wedge x_{10}, & x_{15} &= x_8 \wedge x_{13},\\
f_4 = x_7 &= x_1 \wedge x_6, & f_5 = x_{12} &= x_1 \wedge x_{11}, & x_{16} &= x_3 \wedge x_{15},\\
x_8 &= x_1 \oplus x_3, & x_{13} &= x_5 \wedge \bar x_{11}, & f_1 = x_{17} &= x_{14} \vee x_{16},\\
f_2 = x_9 &= x_6 \wedge \bar x_8, & & & &
\end{array}$$

which beats the greedy-footprint heuristic by one step; and via exhaustive enumeration he has shown that there is no 12-step chain. The minterm-first method

Page 571 line 2 of answer 55 ______ 22 Mar 2025

costs 29 $\rightsquigarrow$ costs 28

Page 572 replacement for first lines of answer 59 ______ 24 Mar 2025

59. The greedy-footprint heuristic produces a 17-step chain, and Runge's hungry search achieves 15:

$$\begin{array}{rlrlrl}
x_5 &= x_2 \oplus x_4, & x_{10} &= \bar x_6 \wedge x_9, & x_{15} &= x_1 \wedge \bar x_{14},\\
x_6 &= x_2 \oplus x_3, & f_3 = x_{11} &= x_7 \oplus x_{10}, & x_{16} &= x_6 \wedge \bar x_{15},\\
x_7 &= x_1 \oplus x_5, & x_{12} &= x_3 \wedge x_7, & f_4 = x_{17} &= x_{12} \vee x_{16},\\
x_8 &= x_7 \wedge \bar x_4, & x_{13} &= x_5 \vee x_{12}, & x_{18} &= x_{12} \oplus x_{16},\\
x_9 &= x_3 \oplus x_8, & f_2 = x_{14} &= x_9 \oplus x_{13}, & f_1 = x_{19} &= x_9 \oplus x_{18}.
\end{array}$$

Page 586 line 2 of answer 22 ______ 13 Jun 2024

`ADDU k,k,2; SRU t,t,k;` $\rightsquigarrow$ `SRU t,t,k; SRU t,t,2;`

Page 586 and page 587, new answer 23 ______ 14 Jun 2024

23. (a) $(1010\ldots10)_2 = (2^{2m+1} - 2)/3$ and $(11\ldots100\ldots0)_2 = 2^{2m} - 2^m$.

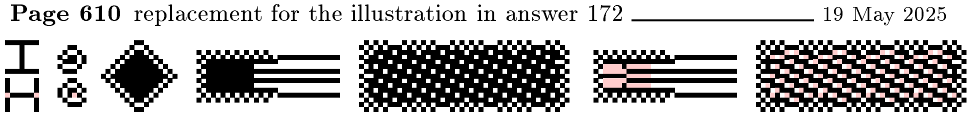
(b) This solution uses the 2-adic constant $\bar\mu_0 = (\ldots101010)_2 = -2/3$:

$$u \leftarrow x \mathbin{\&} -x; \quad v \leftarrow x + u; \quad w \leftarrow 1 + (((v \oplus x)/u) \gg 2); \quad y \leftarrow v + ((w \cdot w - 1) \mathbin{\&} \bar\mu_0).$$

If $x = 2^{2m} - 2^m$, $y > 2^{2m}$. [See C. Neri, arXiv:1602.06426 [cs.DS] (2016), 9 pages.]

(c) `SUBU t,x,1; ANDN u,x,t; SADD k,t,x; ADDU v,x,u; SUBU t,x,1; ANDN u,x,t; SADD k,t,x; ADDU v,x,u; SADD k,x,v; SUB k,xxxiii,k; SRU t,mu0bar,k; SRU t,t,k; ADDU y,v,t`. (Here `xxxiii` is the constant 33.)

Page 610 replacement for the illustration in answer 172 ______ 19 May 2025



Page 610 line 3 of answer 172 — 19 May 2025

The 'I' and 'H' $\leadsto$ (Look closely: The light-colored pixels were originally black.) The 'I' and 'H'

Page 616 line 4 of answer 191 — 19 May 2025

$F_j(y)$, mod $\leadsto$ $F_j(y)$, modulo

Page 627 first lines of answer 45 — 05 Sep 2025

Module M_1 ... and one output $\leadsto$ Give module M_1 only three inputs (x_1, y_1, z_1), and only three outputs $u_2 = x_1$, $v_2 = y_1x_1$, $w_2 = z_1x_1$. Make module M_{n-1} almost normal, but with no input port for z_{n-1} and no output port for u_n; it sets $z_{n-2} = x_{n-1}y_{n-1}$. Give module M_n only three inputs (v_n, w_n, x_n), and one output

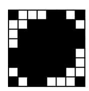
Page 628 lines 2–3 of answer 55 — 19 May 2025

no vertex except 1 appears before all of its neighbors $\leadsto$ every vertex except 1 has a smaller neighbor.

Page 650 line 9 of answer 160 — 01 Jan 2024

the most symmetric solution $\leadsto$ one of the most symmetric

▸**Page 651** replacement for $(A0)$ on line 2 — 01 Jan 2024



▸**Page 658** line 5 of answer 185 — 19 May 2025

the function S_n $\leadsto$ the function Σ_n

▸**Page 659** lines 5 and 14 of answer 198 — 04 Nov 2023

answer 200 $\leadsto$ answer 201

Page 662 new answer — 29 Jan 2025

206. Although all five routines seem to be reasonably fast in practice, they can be exponentially slow in the worst case. For example, the meet and join operations can simulate the hidden weighted bit function. [See K. Nakamura, M. Nishino, and S. Denzumi, *LIPIcs* **322** (2024), 52:1–52:17.]

Page 670 line −12 of answer 237 — 26 Sep 2024

are unknown $\leadsto$ are exponential

Page 672 line 3 of answer 241(d) — 01 Jun 2023

Once again, $\leadsto$ See (C6) and (C7). Once again,

▸**Page 679** in answer 10 — 10 Nov 2022

De Viribus Quantititatis $\leadsto$ *De Viribus Quantitatis*

Page 707 append a new sentence at end of answer 27 — 05 Mar 2023

F. Stappers has found `WRITE` + `WRITE` + `WRITE` + `WRITE` + `WRITE` + `WRITE` = `BOOKS`; `NOTES` + `SOUND` + `TONES` = `MUSIC`.

Page 717 new paragraph before answer 89(e) — 17 Sep 2022

[But when applied to (48), this procedure actually *increases* the value of M in (b).]

Page 724 lines 1 and 2 of answer 1 ——— 08 Oct 2023

by listing the distinct elements first $\leadsto$ first listing its distinct elements (in increasing order)

▸**Page 735** line 1 of answer 55(b) ——— 24 May 2022

$t > 0 \leadsto 0 < t \le 16$

Page 735 replacement for the final reference in answer 56 ——— 21 May 2024

A. Merino, O. Mička, and T. Mütze, *SICOMP* **51** (2022), 379–423; T. Mütze, *Notices Amer. Math. Soc.* **71** (2024), 583–592.

Page 738 line 3 of answer 71 ——— 05 Aug 2025

graph defined by $P_{(1000000)2}$ $\leadsto$ graph $P_{(1000000)2}$ defined in Theorem K

Page 738 line 2 of answer 74 ——— 07 Sep 2025

$10P_{N5} \leadsto (10)P_{N5}$

▸**Page 738** line 5 of answer 77 ——— 05 Aug 2025

find $M + N$ $\leadsto$ find M and N

Page 746 first line of answer 110 ——— 24 May 2022

A nice $\leadsto$ There are $5\binom{52}{5} = 52\binom{51}{4}$ cases, because each card of a 5-combination C can play the role of starter. A nice

Page 746 line −5 of answer 110 ——— 24 May 2022

A four-card flush is not allowed in the “crib.” $\leadsto$ A four-point flush is not counted in the so-called “crib hand.” Then the

▸**Page 751,** line 9 of answer 24 ——— 10 Nov 2022

4 (1953) $\leadsto$ **4** (1954)

▸**Page 762,** last line of answer 8 ——— 27 Feb 2024

k cycles $\leadsto$ m cycles

Page 770, line −5 of answer 30 ——— 05 Aug 2025

when $z = 0$. $\leadsto$ when $z = 0$, and answer 1.2.6–62 defines $n!_q$.

Page 788, in answer 30(d) and answer 31(d) ——— 05 Aug 2025

$f_1 \ldots f_{n-1} \wedge f'_1 \ldots f'_{n-1} \leadsto f_1 \ldots f_{n-1} \mathbin{\&} f'_1 \ldots f'_{n-1}$
$f_1 \ldots f_{n-1} \vee f'_1 \ldots f'_{n-1} \leadsto f_1 \ldots f_{n-1} \mid f'_1 \ldots f'_{n-1}$

Page 810 beginning of answer 122(c) ——— 08 Jan 2024

rises dramatically ... solutions $\leadsto$ rises dramatically to 9,875,514,240, and there now are 137,375 solutions.

Page 810 line −4 of answer 122 ——— 08 Jan 2024

739,361,319 $\leadsto$ 586,220,150

Page 810 and 811, new counts in answer 123 ——— 08 Jan 2024

85597 $\leadsto$ 116222; 17722 $\leadsto$ 19768; 200,765 $\leadsto$ 218,439

Page 816, append to answer 32 ——— 21 Feb 2024

(Partitions into parts that belong to a given set of integers are called *denumerants*.)

▸**Page 821** replacement for lines 4–6 in the quote from Weiner ——— 28 Apr 2024

to so and so many places of accuracy. This means so and
so many significant figures, whether the significance of the digits
begins six places before or six places after the decimal point.
Your accountant works to cents, and he will work to cents

▸**Page 824** line 8 ——— 05 Aug 2025

$\Gamma(x+k)/\Gamma(k) \rightsquigarrow \Gamma(x+k)/\Gamma(x)$

Page 824 line −4 ——— 24 May 2022

saturated subtraction ⇝ saturating subtraction

Page 824 line 16 ——— 26 Feb 2024

Stirling subset number ⇝ Stirling partition number

Page 829 new entry in the rightmost column ——— 05 May 2023

Algorithm 7.2.1.5P, 428.

Page 831 in the Independent Set entry ——— 29 Dec 2024

no-three-on-a-line problem ⇝ no-three-in-line problem

▸**Page 831** in the Clique entry ——— 19 Jan 2023

$\sim G \rightsquigarrow \overline{G}$

Page 834 and following ——— 01 Jan 2011

Miscellaneous changes to the existing index of Volume 4A are collected here, including corrections and amendments to the old entries as well as new entries that are occasioned by the new material. Thus, the lines of the full index that have changed serve also as an index to the present document. However, when a correction or amendment has caused an old index entry to be deleted, the deletion is usually not indicated.