☆ THE ART OF COMPUTER PROGRAMMING ☆

☆☆☆☆ ERRATA TO VOLUME 2 (after 2021) ☆☆☆☆

This document is a transcript of the notes that I have been making in my personal copy of *The Art of Computer Programming*, Volume 2 (third edition, 45th printing) since it was first printed in 2022. Previous errata are recorded in another file '`all2-pre.ps`'.

Four levels of updates — "errors," "amendments," "plans," and "improvements" — appear, indicated by four different typographic conventions:

▸**Page 666** line 1 ________________ 04 Jul 1776

Technical or typographical errors (aka bugs) are the most critical items, so they are flagged with a '▸' preceding the page number. The date on which I first was told about the bug is shown; this is the effective date on which I paid the finder's fee. The necessary corrections are indicated in a straightforward way. If, for example, the book says 'n' where it should have said '$n+1$', the change is shown thus:

$n \leadsto n+1$

Page 666 line 2 ________________ 14 Jul 1789

Amendments to the text appear in the same format as bugs, but without the '▸'. These are things I wish I had known about or thought of when I wrote the original text, so I added them later. The date is the date I drafted the new text.

Page 666 line 3 . . . . 20 Nov 1917

Plans for the future represent a third kind of item. In such notes I sketched my intentions about things that I wasn't ready to flesh out further when I wrote them down. You can identify these items because they're written in slanted type, and preceded by a bunch of dots '. . . . .' leading to the date on which I recorded the plan in my files.

Page 666 line 4 ________________ 10 Jan 1938

The fourth and final category — indicated by page and line number in smaller, slanted type — consists of minor corrections or improvements that most readers don't want to know about, because they are so trivial. You wouldn't even be seeing these items if you hadn't specifically chosen to print the complete errata list in all its gory details. Are you sure you wanted to do that?

My shelves at home are bursting with preprints and reprints of significant research results that I want to digest and summarize, where appropriate, in the ultimate edition of Volume 2. I didn't do that in the third edition because I would surely have to do it over again later: New results continue to pour forth at a great rate, and I will have time to rewrite that volume only once. Volumes 4 and 5 need to be finished first. So I've put most of my effort so far into writing up those parts of the total picture that seem to have converged to their near-final form. It follows, somewhat paradoxically, that the updates in this document are most current in the areas where there has been least activity.

On the other hand I do believe that the changes listed here bring Volume 2 completely up to date in two respects: (1) All of the research problems in the previous edition — i.e., all exercises that were rated 46 and above — have received new ratings of 45 or less whenever I learned of a solution; and in such cases, the answer now refers to that solution. (2) All of the historical information about pioneering developments has been amended whenever new details have come to my attention.

The ultimate, glorious, 100% perfect editions of Volumes 1–4 are works in progress. Please let me know of any improvements that you think I ought to make. Send your comments either by snail mail to D. E. Knuth, Computer Science, Gates Building 1B, Stanford University, Stanford CA 94305-9015, or by email to `taocp@cs.stanford.edu`*. (Use email for book suggestions only, please — all other correspondence is returned unread to the sender, or discarded, because I have no time to read ordinary email.) Although I'm working full time on Volume 4 these days, I will try to reply to all such messages within a year of receipt. Current news about The Art of Computer Programming is posted on*

http://www-cs-faculty.stanford.edu/~knuth/taocp.html

and updated regularly.

— Don Knuth, January 2011

Oh! If only someone would give me time, time,
time to do everything properly,
to read everything at my own *tempo,*
to take it apart and put it together again.
— KARL BARTH (1922)

☆☆☆☆☆☆ SEMINUMERICAL ALGORITHMS ☆☆☆☆☆☆

Last updated 18 July 2025

Most of these corrections have already been made in recent printings.

▸**Page 3** line 12 ——— 23 Sep 2024

Games, Gods, and Gambling ⇝ *Games, Gods and Gambling*

Page 3 line −8 ——— 23 Sep 2024

square it to get ⇝ square it to get a 20-digit product,

▸**Page 27** line 20 ——— 23 Dec 2024

middle square method ⇝ middle-square method

Page 34 line 18 ——— 18 Jun 2024

obtaining ⇝ and m equal to the word size, obtaining

▸**Page 35** lines 11 and 12 ——— 23 Dec 2024

J. Todd and O. Taussky Todd ⇝ O. Taussky and J. Todd

Page 39 line −4 ——— 18 Jun 2024

$\lambda \gg k$ ⇝ λ greatly exceeds k

Page 57 line 3 ——— 18 Jun 2024

$\max\limits_{-\infty<x<+\infty}$ ⇝ $\sup\limits_{-\infty<x<+\infty}$

▸**Page 57** Smirnov reference near the bottom ——— 10 Nov 2022

176–206 ⇝ 179–206

Page 61 line 7 ——— 14 Dec 2022

eleven kinds ⇝ twelve kinds

Page 61 lines 19 and 26 ——— 18 Jun 2024

between zero and one ⇝ in the half-open interval $[0\,..\,1)$
between 0 and $d-1$ ⇝ in the half-open interval $[0\,..\,d)$

Page 61 lines −11 and −10 ——— 24 Sep 2024

the value of d will probably be different in different tests. ⇝ d will generally have different values, depending on the test that is being made.

Page 140 line 2 of exercise 25 ——— 17 Aug 2024

$\frac{1}{2}$. ⇝ $\frac{1}{2}$, and consider the bitwise operations | and & (“or” and “and”).

Page 149 lines 20 and 21 ——— 30 Jun 2024

Probability, Statistics, … 1957), ⇝ *Wahrscheinlichkeit Statistik und Wahrheit* (Vienna: Springer, 1928),

Page 155 lines 8 and 9 — 30 Jun 2024

of points (U_{2n-1}, U_{2n}) is deficient ... other points (U_{2n}, U_{2n+1}) $\leadsto$ of points (U_{2n}, U_{2n+1}) is deficient in some area, the other points (U_{2n+1}, U_{2n+2})

Page 172 addendum to the statement of Lemma P4 — 30 Jun 2024

. $\leadsto$ *, where $T(f)$ and $T(g)$ are the running times needed to compute f and g.*

▸**Page 182** replacement for lines 4 and 5 of exercise 26 — 23 Sep 2024

$$\begin{aligned}(V_{2n}, V_{2n+1}) &= (U_{2n}, U_{2n+1}) \qquad \text{if } (U_{2n}, U_{2n+1}) \in G,\\ (V_{2n}, V_{2n+1}) &= (U_{2n+1}, U_{2n}) \qquad \text{if } (U_{2n}, U_{2n+1}) \notin G,\end{aligned}$$

Page 185 line 2 of point (v) — 30 Jun 2024

in common with m when m is the computer's word size. $\leadsto$ in common with m.

Page 190 new rating for exercise 5 — 15 Nov 2023

[*46*] $\leadsto$ [*45*]

Page 198 line 22 — 08 Nov 2023

1586–1610. $\leadsto$ 1586–1610. (In India, al-Kāshī's contemporary Mādhava had computed $9 \cdot 10^{11}\pi \approx 2827433388233$, according to Nīlakantha's commentary on Gaṇita §10 of the *Āryabhaṭīya*.)

Page 199 line 9 — 08 Nov 2023

man's custom $\leadsto$ human custom

Page 200 lines 13–14 — 10 Aug 2022

as early as 1679 ... "decimal"). $\leadsto$ already in 1679 or earlier, and that he often experimented with radix-16 numbers.

Page 200 line 17 — 10 Aug 2022

1973). Zacher points out $\leadsto$ 1973), and extended by L. Strickland and H. Lewis (Cambridge, Mass.: MIT Press, 2022), who have provided English translations. Zacher pointed out

▸**Page 200** line −18 — 10 Aug 2022

35-digit $\leadsto$ 35-place

Page 200 lines −13 and −12 — 10 Aug 2022

The motive ... could be observed in $\leadsto$ Bernoulli and Leibniz were hoping to discover a simple pattern in

Page 200 new paragraph just before "Charles XII" — 12 Apr 2024

A book about binary arithmetic, *Arithmeticus Perfectus* by W. J. Pelicanus (Prague: 1712), xii + 84 pages, included examples of how one might extract square roots and cube roots in radix 2. This work — "ahead of its time" — was almost totally forgotten for more than 300 years. [See D. Zlatopolski, *Historia Mathematica* **62** (2023), 40–50.]

Page 201 line 1 ———— 10 Aug 2022

More than a century later, a prominent ⇝

The eccentric British schoolmaster T. W. Hill spent more than 50 years of his life experimenting with radix 16. His public lecture in 1845, published posthumously, described ingenious ways to say and to write hexadecimal numbers and fractions [*Selections from the Papers of the Late Thomas Wright Hill, Esq. F.R.A.S.* (London: 1860), 63–85]. But he gave no examples of arithmetic.

Independently, a prominent

Page 201 delete an obsolete paragraph ———— 10 Aug 2022

The book *History of Binary* ... 243]. ⇝

Page 202 line 5 ———— 08 Aug 2022

sexadecimal ⇝ sexdecimal

Page 209 lines 25 and 26 ———— 02 Nov 2022

Several questions ... by W. Parry, ⇝ Intriguing questions concerning *irrational* radices have been investigated by A. Rényi, *Acta Math. Acad. Sci. Hung.* **8** (1957), 477–493; W. Parry,

Page 209 just before the exercises ———— 02 Nov 2022

Gray code ⇝ regular continued fractions (Section 4.5.3); Gray codes

Page 213 line 2 ———— 18 Dec 2023

Crelle **127** (1904), 51–84. ⇝
Jahresbericht der Deutschen Mathematiker-Vereinigung **6** (1897), 83–88.

▸**Page 226** line 17 ———— 15 Nov 2023

Reading, Mass. ⇝ Cambridge, Mass.

Page 232 replacement for (15), (16) and the following reference ———— 23 Sep 2024

$$M_0 = 0, \qquad M_k = M_{k-1} \oplus (x_k \ominus M_{k-1}) \oslash k, \tag{15}$$

$$S_0 = 0, \qquad S_k = S_{k-1} \oplus (x_k \ominus M_{k-1}) \otimes (x_k \ominus M_k), \tag{16}$$

for $1 \le k \le n$, where $\sigma = \sqrt{S_n/(n-1)}$. [See B. P. Welford, *Technometrics* **4** (1962), 419–420; A. J. van Reeken, *CACM* **11** (1968), 149–150.]

Page 245 line −6 of exercise 28 ———— 12 Feb 2025

precision b^p ⇝ fraction size b^p

Page 263 line −4 of exercise 12 ———— 18 Jun 2024

max ⇝ sup

▸**Page 311** in the Harvey / van der Hoeven reference ———— 10 Nov 2022

191 ⇝ (2) **193**

Page 324 line 10 ———— 25 Sep 2024

correct. ⇝ correct. (For example, after three steps we could check that $6+6+8 \equiv 4-3+2-1$ (modulo 9).)

▸**Page 328** last line of exercise 9 — 29 Jul 2024

$\lfloor v_k[u]/16 \rfloor \leadsto \lfloor v_k(u)/16 \rfloor$

Page 330 the quote by Hofstadter — 09 Oct 2024

(1983) $\leadsto$ (1985)

Page 354 replacement for exercise 11 — 29 Jul 2024

11. [*M22*] Let u and v be random positive integers. (a) What is the probability that $\gcd(u,v) \le 3$? (See Theorem D.) (b) What is the *average* value of $\gcd(u,v)$?

Page 359 line 5 — 29 Jul 2024

pattern that appears $\leadsto$ patterns that appear

Page 409 changes to accommodate the new world record prime — 10 Dec 2024

line 7: add '136279841' at the end of list (26)
line 11: But the remaining exponents $\leadsto$ The next 17
lines 15–16: personal computers ... They used a program $\leadsto$ personal computers. And the biggest one was discovered in 2024 by Luke Durant — who assembled a network of thousands of computers in 17 countries(!). They used programs
line −18: unknown Mersenne ... might $\leadsto$ unknown Mersenne primes less than $2^{136279841} - 1$ might
lines −17 and −16: all exponents ... volunteers $\leadsto$ all exponents $<$ 70,832,851 were checked as of December 10, 2024; his volunteers
line −15: Since ... digits $\leadsto$ Since $2^{136279841}-1$ has more than 41 million decimal digits

Page 409 lines −17 and −16 — 01 Apr 2025

all exponents ...; his $\leadsto$ all exponents $<$ 72,072,997 have been checked as of March 31, 2025; his

▸**Page 423** line 3 of Lemma H — 17 Aug 2024

$u = c_1 \cdot v_1(x) \leadsto u(x) = c_1 \cdot v_1(x)$

▸**Page 431** in Table 1 — 21 Nov 2022

rows C_1 and C_0: $d_2^2 b_6^4/c_4^5 \leadsto d_2^3 b_6^4/c_4^5$
row D_0: $e_2^2 c_4^2/d_2^3 b_6^2 \leadsto e_1^2 c_4^2/d_2^3 b_6^2$

▸**Page 432** three lines before (23) — 08 Dec 2022

$C_{n_2+n_j-1}$ becomes $C_{n_2-n_j-1}$

Page 465 in (2) — 17 Aug 2024

$k \le j < i \leadsto 0 \le k \le j < i$

Page 477 lines 20 and 21 — 12 Aug 2022

using a backtrack ... cases. $\leadsto$ using the backtrack method in a program called `ACHAINO` now on the Internet, involved a detailed examination of more than 100 cases.

Page 477 just before the table of $c(r)$ ——— 24 Sep 2024

The computation of $l(n)$ $\leadsto$ The proof that $l(n) > r - 1$

Page 477 replacement for the table at the bottom ——— 13 Aug 2022

r	$c(r)$	r	$c(r)$	r	$c(r)$	r	$c(r)$
0	1	12	379	24	357887	36	550040063
1	2	13	607	25	685951	37	994660991
2	3	14	1087	26	1176431	38	1886023151
3	5	15	1903	27	2211837	39	3502562143
4	7	16	3583	28	4169527	40	6490123999
5	11	17	6271	29	7624319	41	11889505663
6	19	18	11231	30	14143037	42	22899028607
7	29	19	18287	31	25450463	43	41866170239
8	47	20	34303	32	46444543	44	76086635263
9	71	21	65131	33	89209343	45	142771387391
10	127	22	110591	34	155691199	46	257661019487
11	191	23	196591	35	298695487	47	498691112447

Page 478 lines 4–8 ——— 13 Aug 2022

The values listed . . . this agrees $\leadsto$ The values of $c(r)$ listed here for $18 < r \le 28$ were discovered by Achim Flammenkamp before 1998, except that $c(24)$ was first computed by Daniel Bleichenbacher; $c(29)$ through $c(47)$ are due to Neill Clift, between 2004 and 2021. Notice that the formula $1 + r/(1 + 1/\lg r)$ gives a fairly good approximation to $\lg c(r)$ for $10 < r \le 47$; this agrees

Page 478 three lines before (49) ——— 13 Aug 2022

large r. $\leadsto$ large r. (Empirically, $d(r) \approx 1.7c(r)$ for $35 \le r \le 44$.)

Page 478 lines −11 through −5 ——— 24 Sep 2024

Computer calculations . . . Arnold Scholz $\leadsto$
Notice that $2^n - 1$ is the worst case for the binary method, because $\nu(2^n - 1) = n$. E. G. Thurber [*Discrete Math.* **16** (1976), 279–289] has shown that several of these values, including the case $n = 32$, can actually be calculated by hand. Extensive computer calculations by Neill Clift [*Computing* **91** (2011), 265–284; also Thurber and Clift, *Discrete Math.* **344** (2021), #112200, 15 pp.] show that $l(2^n - 1)$ is in fact exactly *equal* to $n - 1 + l(n)$ whenever $l(n) \le 9$. But Clift discovered in 2024 that equality *fails* when $n = 1541527$!

Arnold Scholz

Page 479 bottom line ——— 24 Sep 2024

(see exercise 42). . . . in doubt. $\leadsto$ and for many larger values of n. (See exercise 42 and its answer.)

Page 485 rewording exercise 37 ——— 14 Aug 2022

contains m given . . . (25). $\leadsto$ contains each of the numbers $n_1, \ldots, n_m$. Generalizing (25), prove that $l(n_1, \ldots, n_m) \le \lambda(n) + m\lambda(n)/\lambda\lambda(n) + O(\lambda(n)\lambda\lambda\lambda(n)/\lambda\lambda(n)^2)$, $n = \max\{n_1, \ldots, n_m\}$.

Page 485 in exercise 43 ——— 24 Sep 2024

Does equality always hold? $\rightsquigarrow$

▸**Page 505** line −2 ——— 17 Jul 2025

MacMillan $\rightsquigarrow$ Macmillan

Page 537 new exercise ——— 01 Apr 2024

▸**29.** [*HM21*] (Ira M. Gessel.) Every bivariate generating function corresponds naturally to a sequence of $n \times n$ matrices, when we let

$$A(w,z)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}a_{m,n}\,w^m z^n \quad\longleftrightarrow\quad A_n=\begin{pmatrix} a_{0,0} & a_{0,1} & \ldots & a_{0,n-1}\\ a_{1,0} & a_{1,1} & \ldots & a_{1,n-1}\\ \vdots & \vdots & \ddots & \vdots\\ a_{n-1,0} & a_{n-1,1} & \ldots & a_{n-1,n-1}\end{pmatrix}.$$

a) Suppose $T(w,z)=p(w)\,q(z)A(wr(w),zs(z))$, where p, q, r, and s are arbitrary power series. If the corresponding $n \times n$ matrices are denoted by T_n, prove that

$$\det T_n \;=\; p(0)^n q(0)^n r(0)^{n(n+1)/2} s(0)^{n(n+1)/2} \det A_n.$$

b) Use this technique to evaluate $\det A_n$ when $A(w,z)=e^{w+z}/(e^w+e^z-e^{w+z})$.

▸**Page 537** in the Stillingfleet quote ——— 24 May 2024

which first $\rightsquigarrow$ *which at first*

Page 538 new answer for Notes on the Exercises ——— 24 Nov 2023

2. You can check whether you understand something, by looking at the answer.

▸**Page 539** line 10 of answer 6 ——— 23 Dec 2024

Sitzungsberichte . . . 82–83 $\rightsquigarrow$
Sitzungsberichte Königl. preuß. Akad. Wiss. (Berlin: 1895), 164–165

Page 543 line 2 of answer 3 ——— 23 Dec 2024

m/d values. $\rightsquigarrow$ m/d values, modulo m.

Page 545 replacement for lines −5 and −4 of answer 11 ——— 23 Dec 2024

$$t \leftarrow 70317(x \bmod 30540) - 2467\lfloor x/30540\rfloor\,,$$
$$t \leftarrow 883(t \bmod 2432031) - 274\lfloor t/2432031\rfloor\,.$$

▸**Page 552** line 1 of answer 7 ——— 23 Dec 2024

(1897) $\rightsquigarrow$ (1895)

Page 559 line 5 of answer 6 ——— 18 Jun 2024

max $\rightsquigarrow$ sup

Page 560 lines 1–3 of answer 13 ——— 18 Jun 2024

(The “max” . . . notation.) For convenience $\rightsquigarrow$ For convenience

Page 561 in answer 16 ——— 18 Jun 2024

line −5: If we set $z\sqrt{2}=x_p$ and write $\rightsquigarrow$ If we now set
line −1: . $\rightsquigarrow$, as reported for $x_p=z\sqrt{2}$ and $\nu>30$ in Table 1.

Page 572 line 3 ——— 18 Jun 2024

$m \ll N$ $\rightsquigarrow$ m is a lot smaller than N

Page 586 line 9 of answer 10 ——— 30 Jun 2024

x_j is the root of the equation $\rightsquigarrow$ x_j satisfies

▸**Page 609** lines 4 and 5 of answer 30 ——— 03 Jul 2025

$2b_jb_k$ $\rightsquigarrow$ $2|b_jb_k|$

Page 610 in answer 32 ——— 22 Jul 2024

A more $\rightsquigarrow$ See exercise 1.2.6–11 and OEIS sequence A129508. A more

Page 628 line 9 of answer 39 ——— 30 Sep 2024

913.] $\rightsquigarrow$ 913. The stated identity (and much more) was derived by Euler in 1779, but forgotten for hundreds of years! See M. Ojanguren, *Elemente der Math.* **77** (2022), 94–100.]

▸**Page 629** bottom line ——— 29 Jul 2024

insert the factor $(-1)^r$ before '$\binom{m+1}{r}$'

▸**Page 645** in the Morris reference, beginning of answer 34 ——— 10 Nov 2022

73–145 $\rightsquigarrow$ 73–143

Page 650 line 23 ——— 29 Jul 2024

then $\rightsquigarrow$ where ${}_2F_0$ denotes a hypergeometric function, then

▸**Page 664** line 1 ——— 29 Jul 2024

55182 $\rightsquigarrow$ 54792, 55182

Page 667 in the last paragraph of answer 41 ——— 29 Dec 2022

Further refinements ... 768. $\rightsquigarrow$ Further refinements due to numerous other researchers have made it possible to go much higher; the current record, due to David Baugh and Kim Walisch in 2022, is $\pi(10^{29}) = 1520698109714272166094258063$. (See OEIS sequence A006880.)

Page 667 at the end of answer 41 ——— 30 Dec 2022

large.] $\rightsquigarrow$ large. See also the elementary method of D. Hirsch, I. Kessler, and U. Mendlovic, arXiv:2212.09857 [math.NT] (2022), 60 pages.]

▸**Page 673** bottom line ——— 08 Dec 2022

$\pm\ell_k^{n_{k-1}}/\prod_{1<j<k}\ell_j^{\delta_{j-1}(\delta_j-1)}$. $\rightsquigarrow$ $\pm\prod_{j=2}^{k}\ell_j^{e_j}$, where $e_j = \delta_{j-1}\prod_{i=j}^{k-1}(1-\delta_i)$, $\delta_i = \deg(u_i)-\deg(u_{i+1})$.

▸**Page 677** line -2 of answer 22 ——— 17 Aug 2024

coefficients y_j $\rightsquigarrow$ coefficients u_j

Page 679 in last paragraph of answer 2 ——— 17 Aug 2024

line -3: $v(x)w(x)^r$ $\rightsquigarrow$ $v(x)^r w(x)$
lines -2 and -1: $v(x)$ $\rightsquigarrow$ $w(x)$

Page 686 line 1 of answer 26 ——— 17 Aug 2024

$D \leftarrow (0\ldots01)$, representing $\rightsquigarrow$ $D \leftarrow 0\ldots01$, the n-bit vector that represents

▸**Page 693** line 1 of answer 15 ——— 17 Aug 2024

Diplomarbeit in Mathematics $\rightsquigarrow$ Diplomarbeit in Mathematik

Page 697 in answer 39 ——— 14 Aug 2022

line 8: 21.] $\rightsquigarrow$ 21.] In this argument $\{n_1, n_2, \ldots, n_m\}$ can be any *multiset*.
line 16: 2–3 $\rightsquigarrow$ 2–4

Page 698 line 2 ——— 22 Feb 2022

for any $(r+1) \times (s+1)$ $\rightsquigarrow$ for any nonzero $(r+1) \times (s+1)$

Page 698 bottom line of answer 41 ——— 22 Feb 2022

for $\{n_1, \ldots, n_m\}$ for odd numbers $\{n_1, \ldots, n_m\}$

Page 698 bottom line of answer 42 ——— 24 Sep 2024

. $\rightsquigarrow$. His website `https://www.additionchains.com/` provides extensive further details.

Page 700 near the end of answer 12 ——— 10 Nov 2022

arXiv:1211.6320 [cs.CC] (2013), 10 pages. $\rightsquigarrow$ *Linear Algebra and Its Applications* **438** (2013), 4500–4509.

Page 715 the end of answer 62 ——— 10 Nov 2023

235. $\rightsquigarrow$ 235. For a modern treatment, including proofs that $\underline{\mathrm{rank}}(T(2,2,3)) = 10$ and $\underline{\mathrm{rank}}(T(2,3,3)) = 14$, see A.Conner, A. Harper, and J. M. Landsberg, *Forum of Mathematics, Pi* **11** (2023), e17:1–30.

Page 715 last line of answer 64 ——— 10 Nov 2023

The border rank of $T(2,2,2)$ remains unknown.] $\rightsquigarrow$ The paper cited in answer 62 proves that $\underline{\mathrm{rank}}(T(2,2,2)) = 7$ and $\underline{\mathrm{rank}}(T(3,3,3)) \geq 17$.]

Page 725 new answer ——— 01 Apr 2024

29. (a) The result is clear when p, q, r, and s are constant. So it suffices to consider cases where $p(0) = q(0) = r(0) = s(0) = 1$.

Suppose $p(w) = 1 + p_1 w + p_2 w^2 + \cdots$ and $q(z) = r(w) = s(z) = 1$. Then $t_{m,n} = a_{m,n} + p_1 a_{m-1,n} + \cdots + p_m a_{0,n}$. So every row of T_n is obtained from A_n by adding multiples of previous rows (working bottom to top), leaving the determinant unchanged.

Similarly, suppose $r(w) = 1 + r_1 w + r_2 w^2 + \cdots$ and $p(w) = q(z) = s(z) = 1$. Then we have, for example, $t_{5,n} = a_{5,n} + 4r_1 a_{4,n} + (3r_2 + 3r_1^2) a_{3,n} + (2r_3 + 2r_2 r_1) a_{2,n} + r_4 a_{1,n}$; in general every row is again a linear combination of its predecessors.

(b) Setting $p(w) = 1/(1+w)$, $q(z) = 1/(1+z)$, $r(w) = \frac{1}{w}\ln(1+w)$, $s(z) = \frac{1}{z}\ln(1+z)$ gives $T(w,z) = 1/(1-wz)$. Hence T_n is the identity matrix, and $\det A_n = 1$.

[See *AMM* **86** (1979), 64–65; Ira M. Gessel and Guoce Xin, *Electronic Journal of Combinatorics* **13** (2006), #R53, 1–48, §4.]

Page 734 new entry (after $x \approx y$) ——— 05 Nov 2024

$\mathrm{round}(x)$	nearest representable approximation to x	4.2.2, 4.5.1

Page 737 and following ——— 01 Feb 2022

Miscellaneous changes to the existing index of Volume 2 are collected here, including corrections and amendments to the old entries as well as new entries that are occasioned by the new material. Thus, the lines of the full index that have changed serve also as an index to the present document. However, when a correction or amendment has caused an old index entry to be deleted, the deletion is usually not indicated.