☆ THE ART OF COMPUTER PROGRAMMING ☆

☆☆☆☆ ERRATA TO VOLUME 1 (after 2021) ☆☆☆☆

This document is a transcript of the notes that I have been making in my personal copy of *The Art of Computer Programming*, Volume 1 (third edition, 27th printing), since it was first printed in 2022. Previous errata are recorded in another file '`all1-pre.ps`'.

Four levels of updates — "errors," "amendments," "plans," and "improvements" — appear, indicated by four different typographic conventions:

▸**Page 666** line 1 ——— 04 Jul 1776

Technical or typographical errors (aka bugs) are the most critical items, so they are flagged with a '▸' preceding the page number. The date on which I first was told about the bug is shown; this is the effective date on which I paid the finder's fee. The necessary corrections are indicated in a straightforward way. If, for example, the book says 'n' where it should have said '$n+1$', the change is shown thus:

$n \rightsquigarrow n+1$

Page 666 line 2 ——— 14 Jul 1789

Amendments to the text appear in the same format as bugs, but without the '▸'. These are things I wish I had known about or thought of when I wrote the original text, so I added them later. The date is the date I drafted the new text.

Page 666 line 3 . . . . 20 Nov 1917

Plans for the future represent a third kind of item. In such notes I sketched my intentions about things that I wasn't ready to flesh out further when I wrote them down. You can identify these items because they're written in slanted type, and preceded by a bunch of dots '. . . . .' leading to the date on which I recorded the plan in my files.

Page 666 line 4 ——— 10 Jan 1938

The fourth and final category — indicated by page and line number in smaller, slanted type — consists of minor corrections or improvements that most readers don't want to know about, because they are so trivial. You wouldn't even be seeing these items if you hadn't specifically chosen to print the complete errata list in all its gory details. Are you sure you wanted to do that?

My shelves at home are bursting with preprints and reprints of significant research results that I want to digest and summarize, where appropriate, in the ultimate edition of Volume 1. I didn't do that in the third edition because I would surely have to do it over again later: New results continue to pour forth at a great rate, and I will have time to rewrite that volume only once. Volumes 4 and 5 need to be finished first. So I've put most of my effort so far into writing up those parts of the total picture that seem to have converged to their near-final form. It follows, somewhat paradoxically, that the updates in this document are most current in the areas where there has been least activity.

On the other hand I do believe that the changes listed here bring Volume 1 completely up to date in two respects: (1) All of the research problems in the previous edition — i.e., all exercises that were rated 46 and above — have received new ratings of 45 or less whenever I learned of a solution; and in such cases, the answer now refers to that solution. (2) All of the historical information about pioneering developments has been amended whenever new details have come to my attention.

The ultimate, glorious, 100% perfect editions of Volumes 1–4 are works in progress. Please let me know of any improvements that you think I ought to make. Send your comments either by snail mail to D. E. Knuth, Computer Science, Gates Building 1B, Stanford University, Stanford CA 94305-9015, or by email to `taocp@cs.stanford.edu`*. (Use email for book suggestions only, please — all other correspondence is returned unread to the sender, or discarded, because I have no time to read ordinary email.) Although I'm working full time on Volume 4 these days, I will try to reply to all such messages within a year of receipt. Current news about The Art of Computer Programming is posted on*

`http://www-cs-faculty.stanford.edu/~knuth/taocp.html`

and updated regularly.

— Don Knuth, January 2011

What happened?
The subject took the bit in its teeth and ran away with it,
that's what happened.
I know now how Sir James Frazer felt when,
after setting out to dash off a brief monograph
on a single obscure rite, he found himself
in the embarrassing possession of
the 12 volumes of "The Golden Bough."

— WAVERLEY ROOT (1974)

No matter how many times you proofread your book,
after publication there will be roughly one typo per page.

— JOSEPH H. SILVERMAN (2017)

☆☆☆☆☆☆ FUNDAMENTAL ALGORITHMS ☆☆☆☆☆☆

Last updated 11 September 2025

Most of these corrections have already been made in recent printings.

Page 8 lines 9–10 after (2) ——— 07 Sep 2024

The idea ⇝ When $n = 0$, this notation '$x_1x_2 \ldots x_n$' stands for the empty string; in general, it stands for a string of length n. The idea

Page 11 replacement for displayed equation (2) ——— 18 Jan 2025

$$1 + 3 + \cdots + (2n - 1) = n^2 \qquad \text{(a sum of } n \text{ terms).} \tag{2}$$

Page 12 replacement for line −14 ——— 29 Mar 2023

$$p(0) = 1, \quad p(1) = 1, \quad p(2) = 2, \quad p(3) = 3, \quad p(4) = 5, \quad p(5) = 7.$$

Page 13 add a sentence following line −7 ——— 05 Dec 2023

legitimate. ⇝ legitimate. [Actually $P(0)$ does happen to be true, in this particular context; but we weren't asked to prove it. So we didn't.]

Page 16 line 18 ——— 18 Feb 2022

the key inductive assertions. Those assertions either ⇝ the key assertions. Those assertions — which are often called "inductive assertions" because we prove them by induction on the length of computation — either

Page 17 lines 20 and 21 ——— 17 Nov 2023

The first European ... 1575. ⇝ Noteworthy early examples of inductive reasoning can be found in *Sefer Maasei Hoshev* by Levi ben Gershon in France (1321), and in *Arithmeticorum libri duo* by Francesco Maurolico in Italy (1557).

Page 21 line 2 after (1) ——— 07 Feb 2024

end with infinitely many 9s. ⇝ end with infinitely many 9s. (Instead of 999..., we can use 000... and add 1 to the digit at the left.)

Page 22 top lines ——— 07 Feb 2024

Throughout this section ... It is easy to prove ⇝

Throughout this section, let the letter b stand for a positive real number. If n is an integer, then b^n and 0^n and $(-b)^n$ are defined by the familiar rules

$$b^0 = 1, \qquad b^n = b^{n-1}b \quad \text{if} \quad n > 0, \qquad b^n = b^{n+1}/b \quad \text{if} \quad n < 0; \tag{4}$$

$$0^0 = 1, \qquad 0^n = 0 \quad \text{if } n > 0; \qquad (-b)^n = (-1)^n b^n. \tag{5}$$

It is easy to prove

Page 22 through 26 ——— 07 Feb 2024

[renumber equations formerly numbered (5) through (19) to be (6) through (20)]

Page 27 line 5 ——— 01 Mar 2022

following equivalent ⇝ following essentially equivalent

Page 27 line 11 after (2) ——— 07 Feb 2024

silly to interpret it as a sum on values of $n \geq j$; but $\leadsto$ silly to interpret it as a sum $a_j + a_j + a_j + \cdots$ over all indices n that satisfy $n \geq j$! But

Page 34 in exercise 8 ——— 24 Nov 2022

infinite series in which $\leadsto$ doubly infinite sequence $\langle a_{ij} \rangle$ of integers for which

Page 35 in exercise 17 ——— 05 Aug 2025

a set of integers $\leadsto$ a finite set of integers

Page 40 just after (5) ——— 08 Aug 2025

$x \bmod z = y \bmod z$; it is the same as saying that $x - y$ is an integer multiple of z. $\leadsto$

$x \bmod z = y \bmod z$. And property (c) above tells us that $x \bmod z = y \bmod z$ is the same as saying that $x - y$ is an integer multiple of z; that is, $x = y + qz$, for some integer q.

Page 57 six lines after (13) ——— 11 Aug 2022

since the terms in Eq. (13) are zero when $k < 0$ or $k > r$. $\leadsto$ since we consider $\binom{r}{k}$ to be "strongly zero" when $k < 0$ or $k > r = \lfloor r \rfloor$ — strong enough to cancel negative powers of 0.

Page 59 in (26) ——— 05 Sep 2025

$\displaystyle\sum_{k\geq 0} \leadsto \sum_{k}$

▸**Page 60** line −1 ——— 19 Jan 2023

assuming that $k \geq 0 \leadsto$ because $k \geq 0$

Page 62 and 63, throughout Example 4 ——— 05 Sep 2025

$nt \leadsto tn$ (seven times)
$kt \leadsto tk$ (eleven times)
$(n-k)t \leadsto t(n-k)$ (seven times)

▸**Page 63** line −10 ——— 11 Aug 2022

$\displaystyle\sum_{k} \leadsto \sum_{k\geq 0}$

▸**Page 63** lines −3 and −2 ——— 11 Aug 2022

$\displaystyle\sum_{n} \leadsto \sum_{n\geq 0}$

Page 64 in (35) ——— 05 Sep 2025

$kt \leadsto tk$

Page 70 in exercise 22 ——— 05 Sep 2025

$nt \leadsto tn$

Page 70 in exercise 25 — 05 Sep 2025

$jt \rightsquigarrow tj$ (twice)

▸**Page 70** append a new line to exercise 25 — 09 Aug 2022

where again the left-hand side is considered to be a polynomial in r.
[Also change '.' to ',' on the preceding line.]

Page 75 the line before (3) — 19 Mar 2025

as follows: $\rightsquigarrow$ as follows when $n > 0$:

Page 77 the line before (10) — 01 Mar 2022

$S_1 = x \rightsquigarrow S_0 = 0$

Page 77 line −7 — 01 Mar 2022

If $x > 0$, then $\rightsquigarrow$ *If $x > 0$ and $n \geq 0$, then*

▸**Page 79** line −3 — 29 Mar 2023

285 $\rightsquigarrow$ 284

▸**Page 80** second full paragraph, last line — 29 Mar 2023

126 $\rightsquigarrow$ 124

▸**Page 80** third full paragraph, line 4 — 06 Apr 2022

not greater than F_k, $\rightsquigarrow$ not greater than F_k, and $k > 1$,

Page 83 replacement for (13) — 07 Aug 2022

$$\hat{\phi} = 1 - \phi = -1/\phi = \tfrac{1}{2}(1 - \sqrt{5}). \tag{13}$$

▸**Page 85** in exercises 22 and 23 — 06 Apr 2022

number. $\rightsquigarrow$ number when $n \geq 0$.
[And exercise 25 should also specify that $n \geq 0$.]

Page 85 in exercise 29(a) — 29 Mar 2023

$0 \leq k \leq n \leq 6 \rightsquigarrow 0 \leq k, n \leq 6$

▸**Page 86** in exercise 31 — 06 Apr 2022

ϕ^{-2n-1}. $\rightsquigarrow$ ϕ^{-2n-1}, when $n \geq 0$.

Page 88 line −10 — 09 Aug 2022

simplest example $\rightsquigarrow$ simplest nontrivial example

Page 92 replacement for (33) — 14 Jan 2024

$$h_m = \sum_{1 \leq j_1 \leq \cdots \leq j_m \leq n} x_{j_1} \ldots x_{j_m}; \qquad h_0 = 1. \tag{33}$$

▸**Page 93** replacement for (39) — 12 Mar 2023

$$h_m = \frac{1}{m}(S_1 h_{m-1} + S_2 h_{m-2} + \cdots + S_m h_0), \qquad m \geq 1. \tag{39}$$

▸**Page 93** line −3 ——— 29 Mar 2023

picture writing ⇝ picture-writing

Page 94 replacement for second line of exercise 10 ——— 14 Jan 2024

$$e_m = \sum_{1\le j_1<\cdots<j_m\le n} x_{j_1}\ldots x_{j_m}; \qquad e_0 = 1.$$

Page 94 append new sentence to end of exercise 13 ——— 01 Jan 2024

(Assume that the real part of s is positive, so that the integral exists.)

Page 96 line 1 of Algorithm M ——— 24 Aug 2024

Given n elements ⇝ Given $n > 0$ elements

Page 100 line 10 ——— 04 Jan 2024

some event has a value ⇝ some random nonnegative integer has the value

Page 101 insert qualification in (17) ——— 25 Oct 2024

integer $n, k > 0$,

Page 102 line 1 ——— 04 Jan 2024

$G_1(z) = q + pz$ ⇝ $G_0(z) = 1$

Page 103 line 1 ——— 04 Jan 2024

replace z by $z + 1$ ⇝ replace z by $1 + z$

Page 104 line 1 ——— 04 Jan 2024

zero except ⇝ zero except possibly

Page 104 line 11 ——— 04 Jan 2024

random integer ⇝ random nonnegative integer

▸**Page 105** bottom line ——— 29 Mar 2023

Reading, Mass. ⇝ Cambridge, Mass.

Page 106 last line of exercise 21 ——— 25 Apr 2023

when $\epsilon \ge 0$, and obtain ⇝ when $q = 1 - p$ and $\epsilon \ge 0$. Also obtain

Page 107 line 2 of exercise 22 ——— 21 May 2023

$p_k + q_k = 1$ ⇝ $p_k + q_k = 1$ and $0 \le p_k \le 1$

Page 107 exercise 23 ——— 19 Sep 2024

line 1: [*HM23*] ⇝ [*HM24*]
line 2: $p + 1$. ⇝ $p + 1$ and $p, n > 0$.

Page 110 line 2 after (18) ——— 06 May 2024

where constants are ignored ⇝ where constant factors are ignored

Page 112 line before (4) ——— 04 Sep 2024

the coefficients in ⇝ the numbers B_0, B_1, B_2, … in

Page 112 line −2 ——— 04 Sep 2024

an even function $\leadsto$ an even function (unchanged when z becomes $-z$)

Page 114 shortly after (13) ——— 04 Sep 2024

the first discarded term $\leadsto$ the first nonzero discarded term

Page 114 last line of exercise 4 ——— 04 Sep 2024

0 replacing 1 $\leadsto$ 0 replacing the lower limit 1

Page 116 line 2 of exercise 6 ——— 04 Sep 2024

$x \ge a > 0 \leadsto x \to \infty$

Page 118 line 7 ——— 30 Dec 2024

Setting $w = \sqrt{2v}$, we $\leadsto$ Setting $w = \sqrt{2v}$, with $u, v, w \ge 0$, we

Page 118 line 17 ——— 30 Dec 2024

$|u| \le r$, $|v| \le r$, $|w| \le r$, for $\leadsto$ $0 \le u, v, w \le r$ for
[and also $|v| \leadsto v$ on line 19]

▸**Page 118** line −5 ——— 29 Mar 2023

1.2.11.3–(13) $\leadsto$ 1.2.11.1–(13)

Page 119 replacement for line 3 ——— 30 Dec 2024

Now y (and therefore u) can be negative. We find

▸**Page 119** line 6 ——— 02 May 2024

for $0 \le u \le y \leadsto$ for $0 \le u \le |y|$

Page 135 line 15 ——— 20 Dec 2023

instructions are $\leadsto$ instructions for rX and rIi are

Page 142 line 2 of exercise 18 ——— 16 Dec 2023

registers, toggles, and memory $\leadsto$ registers, memory, and other aspects of `MIX`'s state

Page 144 line 15 or 16 ——— 03 Nov 2022

always greater than 100 $\leadsto$ always 0100 or more

▸**Page 148** line *05* of Program P ——— 28 Aug 2022

`EQU -1` $\leadsto$ `EQU 99`

Page 174 lines 9–10 after Fig. 21 ——— 07 Dec 2024

rI3 $= n$, the number of distinct names seen. $\leadsto$ and rI3 $\equiv n$ is the number of distinct names that have been put into the table.

Page 178 replacement for the beginning of Program J ——— 23 Dec 2024

Program J (*Analogous to Program I*). $\mathrm{rI1}\equiv m$; $\mathrm{rI2}\equiv j$; $\mathrm{rI3}\equiv -i$; $\mathrm{rI4}\equiv k-n-1$.

```
INVERT ENN4 N                  1   J1. Negate all.
       ST4  X+N+1,4(0:0)       N   Set sign negative.
       INC4 1                  N
       J4N  *-2                N   More?
```

Page 181 replacement for line 3 ——— 23 Dec 2024

$$\frac{P_{n0}}{n!} = 1 - \frac{1}{1!} + \frac{1}{2!} - \cdots + (-1)^n \frac{1}{n!}. \qquad (25)$$

▸**Page 182** line 3 ——— 29 Mar 2023

1957 $\rightsquigarrow$ 1967

▸**Page 183** line −4 ——— 29 Mar 2023

for all n $\rightsquigarrow$ for all $n \ge 1$

Page 185 new exercise ——— 24 Mar 2023

▸**38.** [*M22*] (*Conjugate permutations.*) When π and τ are arbitrary permutations, the permutation

$$\pi^\tau \;=\; \tau^-\pi\tau$$

is called "the conjugate of π by τ."

a) Prove that $\pi = \pi^\tau$ if and only if π commutes with τ.
b) Suppose π has the cycle form $(a_{11}\ldots a_{1l_1})\ldots(a_{k1}\ldots a_{kl_k})$. Prove that π^τ has the cycle form $(b_{11}\ldots b_{1l_1})\ldots(b_{k1}\ldots b_{kl_k})$, where τ takes $a_{ij}\mapsto b_{ij}$.
c) Find all τ for which $\bigl((1\,2\,3)(4\,5)(6\,7\,8)\bigr)^\tau = (8\,7\,6)(5\,4)(3\,2\,1)$.
d) Find all τ for which $\bigl((1\,2\,3)(4\,5)(6\,7\,8)\bigr)^\tau = (8\,7\,6\,5)(4\,3\,2\,1)$.
e) True or false: Every permutation π is conjugate to its inverse, π^-.
f) True or false: $(\pi\sigma)^\tau = \pi^\tau\sigma^\tau$ and $\pi^{(\sigma\tau)} = (\pi^\sigma)^\tau$, for all π, σ, and τ.

Page 212 new exercise ——— 01 Jan 2025

9. [*20*] Write the `SPEC` routine, which is missing from the program in the text (operation code 5). It's OK to let `NUM` and `CHAR` simulate themselves.

▸**Page 229** line −14 ——— 29 Mar 2023

Reading, Mass. $\rightsquigarrow$ Cambridge, Mass.

Page 231 line −5 ——— 22 May 2024

FORTRAN $\rightsquigarrow$ Fortran

▸**Page 241** below Figure 3 ——— 07 Jul 2023

Fig. 3a $\rightsquigarrow$ Fig. 3(a)
Fig. 3b $\rightsquigarrow$ Fig. 3(b)
Fig. 3c $\rightsquigarrow$ Fig. 3(c)

Page 245 line 5 ——— 02 May 2024

`X[M]`. $\rightsquigarrow$ `X[M]` and `M` > 1.

▸**Page 268** line 5 before the exercises ——— 07 Jul 2023

Fundamenta Mathematica $\rightsquigarrow$ *Fundamenta Mathematicæ*

▸**Page 276** line 1 of step A4 ——— 01 Dec 2022

`LINK(Q1)` $\leftarrow$ `Q` $\leftarrow$ `LINK(Q)`, $\rightsquigarrow$ `Q` $\leftarrow$ `LINK(Q)`, `LINK(Q1)` $\leftarrow$ `Q`,

Page 277 top of page ——— 20 Sep 2024

[move Fig. 10 from page 279 to here]

▸**Page 286** second table entry for time 0180, column `D2` ——— 07 Jul 2023

X $\rightsquigarrow$ 0

▸**Page 297** line 1 of exercise 8 ——— 02 May 2024

277–292 $\rightsquigarrow$ 277–291

Page 300 line 1 ——— 07 Jul 2023

Formulas (5) and (6) $\rightsquigarrow$ When $c = 1$, formulas (5) and (6)

Page 310 at the "root" of Fig. 18(a) ——— 13 Sep 2022

Charles $\rightsquigarrow$ Charles III

▸**Page 330** third line of the quotation ——— 22 May 2024

have ever been $\rightsquigarrow$ *have been ever*

Page 336 lines 29–32 ——— 26 Apr 2024

title passes ... lineal chart $\rightsquigarrow$ title traditionally passed to the first son, then to descendants of the first son, and finally if they all died out it passed to other sons of the family in the same way. (Thank goodness such customs are gradually changing so as to be more fair to the fairer sex.) In theory, we could take a suitable lineal chart

Page 349 line −6 ——— 01 Dec 2022

complete the most recent instance of an incomplete `RLINK`. $\rightsquigarrow$ set the `RLINK` field of the most recent node whose `RTAG` was blank.

Page 355 rewording of step E3 ——— 07 Jul 2023

If `PARENT[`j`]` > 0, ... step. (After this operation, $\rightsquigarrow$
While `PARENT[`j`]` > 0, set $j \leftarrow$ `PARENT[`j`]`. Then, while `PARENT[`k`]` > 0, set $k \leftarrow$ `PARENT[`k`]`. (After these loops,

Page 371 line 1 of exercise 11 ——— 17 Nov 2023

delete '(R. C. Prim, ... 1389–1401.)'

Page 371 line −4 of exercise 11 ——— 07 Jul 2023

minimal cost tree $\rightsquigarrow$ minimum cost tree

Page 376 line 5 ——— 22 May 2024

for which $\rightsquigarrow$ *for which the following three conditions hold:*

▸**Page 376** line 2 of exercise 1 —— 07 Jul 2023

simple oriented path $\rightsquigarrow$ simple oriented path or cycle

Page 387 line −5 —— 20 May 2024

contain a centroid $\rightsquigarrow$ *contain a centroid of the overall free tree*

Page 388 line 4 after (7) —— 15 Jan 2023

$n - s_j \le s_j$ $\rightsquigarrow$ $n - s_j \le s_j = \max(s_1, s_2, \ldots, s_k)$

▸**Page 406** line −6 —— 07 Jul 2023

(1919), 7 $\rightsquigarrow$ (1919), 7:3–7:33

▸**Page 408** line 2 of exercise 2 —— 07 Jul 2023

r-sided polygon $\rightsquigarrow$ r-sided convex polygon

▸**Page 421** line −16 —— 07 Jul 2023

338–343 $\rightsquigarrow$ 338–346

Page 430 the line after (8) —— 20 May 2024

either α' or β' is an elementary item (not a group item). $\rightsquigarrow$ either α' or β' (or both) is an elementary item, not a group item.

▸**Page 433** line 14 —— 07 Jul 2023

(Syracuse, N.Y.: 1962) $\rightsquigarrow$ (Syracuse, N.Y.: 1962), 74–75

▸**Page 457** line 7 —— 29 Mar 2023

Reading, Mass. $\rightsquigarrow$ Cambridge, Mass.

▸**Page 458** line 7 —— 07 Jul 2023

61–70 $\rightsquigarrow$ 61–79

Page 460 line −6 —— 15 Oct 2024

introduction to IPL-V $\rightsquigarrow$ introduction to Information Processing Language V

Page 460 line −5 —— 22 May 2024

FORTRAN-compiled $\rightsquigarrow$ Fortran-compiled

Page 460 line −2 —— 22 May 2024

machine, I $\rightsquigarrow$ machine, Part I

▸**Page 461** line 24 —— 07 Jul 2023

578–588 $\rightsquigarrow$ 578–589

Page 462 line 9 —— 07 Jul 2023

Press, 1969) $\rightsquigarrow$ Press, 1969), 1–75

▸**Page 462** line 12 —— 07 Jul 2023

567–643 $\rightsquigarrow$ 547–643

▸**Page 462** line −4 ——— 28 Nov 2024

3–63 ⇝ 1–64

Page 465 date of the Sherlock Holmes quote ——— 07 Jul 2023

(1888) ⇝ (1914)

▸**Page 465** line 6 of the Lewis Carroll quote ——— 22 Sep 2024

A Genealogical *tree* ⇝ *A* Genealogical *"Tree"*

Page 466 new answer for Notes on the Exercises ——— 24 Nov 2023

2. They help you to lodge ideas in your brain.

▸**Page 467** replacement for lines 5–7 of answer 9 ——— 04 Sep 2025

b) If q is in Q_2 then $f_1(h(q)) = h(q')$, where $q' = f_2^{[j(q)]}(q)$, and $f_2^{[j(q)]}$ means that the function f_2 is to be iterated $j(q)$ times.

c) If q is in Q_2 then $h(q')$ is in Ω_1 if and only if q' is in Ω_2.

Page 469 and 470 ——— 07 Feb 2024

[renumber equations formerly numbered (5) through (19) to be (6) through (20)]

Page 471 append a new paragraph to answer 1 ——— 01 Mar 2022

Note: We might also say that $a_2+\cdots+a_0 = \sum_{j=2}^{0} a_j$, and that $\sum_{j=p}^{q} a_j + \sum_{j=q+1}^{r} a_j = \sum_{j=p}^{r} a_j$, using the left half of (1). On the other hand, we should *not* replace $\cdots$ using the right half of (1)! The sum $\sum_{2\le j\le 0} a_j$ is 0, by (2).

Page 472 line 2 of answer 25 ——— 05 Aug 2025

at once. ⇝ at once: Each index variable is "bound" to a particular $\sum$.

Page 472 new copy for end of answer 30 ——— 01 Jul 2022

exercise 46.) ⇝ exercise 46. Cauchy's sums were extended to integrals by V. Bouniakowsky [*Mém. Acad. St.-Pétersbourg* (7) **1** (1859), No. 9, 4] and H. A. Schwarz [*Acta Societatis Scientiarum Fennicæ* **15** (1885), 315–362].)

Page 473 last line of answer 35 ——— 07 Feb 2024

sup(sup ⇝ max(sup

Page 474 three more lines for answer 44 ——— 17 May 2024

[See D. Grinberg, *Math. Intelligencer* **46**, 1 (Spring 2024), 46–48, for an alternative proof using the identity $\sum_{i=1}^{m}\sum_{j=1}^{n}(x_i+y_j)A_{ij}B_{ji} = \sum_{i=1}^{m} x_i(AB)_{ii} + \sum_{j=1}^{n} y_j(BA)_{jj}$, where A is $m\times n$ and B is $n\times m$, and for related information.]

Page 481 in answer 16 ——— 22 Aug 2025

$1/e$. ⇝ $1/e$, and the other factor is unbounded.

▸**Page 481** line −2 of answer 20 ——— 19 Jan 2023

less than ⇝ at most

▸**Page 485** line 3 of answer 22 ——— 19 Jan 2023

$(-1)^{k-1}$ ⇝ $(-1)^k$

Page 485 line 2 of answer 24 ——— 05 Sep 2025

nt ⇝ tn

Page 485 in answers 25 and 26 ——— 05 Sep 2025

$jt \rightsquigarrow tj$ (nine times)

Page 486 replacement for answer 30 ——— 29 Mar 2023

30. Apply (6) and (19) to get

$$\sum_{k\ge 0}\binom{n+k}{n-m-k}\binom{-k-1}{k}\frac{1}{k+1}.$$

Now we can apply Eq. (26) with $(r,s,t,n)=(-1,2n-m,1,n-m)$, obtaining

$$\binom{n-1}{n-m}.$$

This result is the same as our previous formula, when n is positive; but when $n=0$ the answer we have obtained is correct while $\binom{n-1}{m-1}$ is not. Our derivation has a further bonus, since the answer $\binom{n-1}{n-m}$ is valid for $n\ge 0$ and *all* integers m.

▸**Page 486** line 2 of answer 31 ——— 29 Mar 2023

38–57 $\rightsquigarrow$ 37–57

▸**Page 489** line 1 of answer 58 ——— 29 Mar 2023

Arithmetik $\rightsquigarrow$ *Arithmetik* **2**

▸**Page 490** line 8 of answer 62 ——— 29 Mar 2023

285–289 $\rightsquigarrow$ 284–289

▸**Page 492** replacement for answer 9 ——— 01 Mar 2022

9. $(n=0?\ 0\!: -1/n)$.

Page 493 new answer for end of section 1.2.7 ——— 01 Mar 2022

25. $H_n^{(0,v)}=\sum_{k=1}^n k^{1-v}=H_n^{(v-1)}$ and $H_n^{(u,0)}=\sum_{k=1}^n H_k^{(u)}$; so the identity generalizes (8). [See L. Euler, *Novi Comment. Acad. Sci. Pet.* **20** (1775), 140–186, §2.]

Page 494 new answer for exercise 31 ——— 29 Mar 2023

31. Use exercise 11.

▸**Page 495** line 1 of answer 37 ——— 15 Nov 2023

1 (December $\rightsquigarrow$ **1**, 4 (December

▸**Page 495** replacement for last line of answer 37 ——— 29 Mar 2023

by A. J. Schwenk [*Fibonacci Quarterly* **8** (1970), 225–234, 241].

▸**Page 497** line 5 of answer 11 ——— 29 Mar 2023

Invention Nouvelle en Algébre $\rightsquigarrow$ *Invention Nouvelle en l'Algebre*

Page 497 in answer 17 ——— 29 Mar 2023

[change $_k$ to $_{k\ge 0}$ in the last two sums]

▸**Page 498** replacement for reference at end of answer 21 ——— 29 Mar 2023

[See K. Knopp, *Theory and Application of Infinite Series*, 2nd ed. (Glasgow: Blackie, 1951), Section 66.]

▸**Page 500** replacement for line 3 of answer 10 (because $B_1 = \frac{1}{2}$) ———— 29 Mar 2023

$$H_M - \sum_{m=1}^{M}\Bigl(1-\frac{m}{M}\Bigr)^{n} m^{-1} = H_n + \sum_{k=1}^{n}\biggl(\binom{n}{k}-1\biggr) B_k M^{-k} k^{-1} - \frac{n-1}{M}.$$

▸**Page 500** first line of answer 11 ———— 14 Mar 2024

term $\leadsto$ factor

Page 502 replacement for line 5 of answer 21 ———— 25 Apr 2023

$p \ge \frac{1}{2}$. Still further work yields the upper bound $\exp\bigl(-\sum_{k\ge 1} 2^{2k-1}\epsilon^{2k}/(2k^2-k)\bigr) \le \exp(-2\epsilon^2 n)$, valid for all p.)

Page 502 append new sentence to answer 22(b) ———— 22 May 2023

The slightly more complicated bound $e^{-\delta^2/(2+\delta)}$ can be used for all $\delta \ge 0$, because $2\delta/(2+\delta) \le \ln(1+\delta)$.

▸**Page 502** line 2 of answer 22(c) ———— 29 Mar 2023

it is $1/2$ $\leadsto$ it is $< 1/2$

Page 502 last line of answer 23 ———— 17 Sep 2024

It follows $\ldots \epsilon \le p$. $\leadsto$ If $0 \le \epsilon \le p$, it follows that $f'(\epsilon) \le -\epsilon/((p+\epsilon)q) \le -\epsilon/(2pq)$; hence $f(\epsilon) \le -\epsilon^2/(4pq)$.

Page 503 answer 2 ———— 04 Sep 2024

The function $\leadsto$ We need $B_1(x) = x - \frac{1}{2}$, by (3); and the function

Page 503 line 1 of answer 3 ———— 29 Mar 2023

$(m)!$ $\leadsto$ $m!$

▸**Page 504** line 6 of answer 7 ———— 29 Mar 2023

122–158 $\leadsto$ 122–138

Page 518 line 19 ———— 18 Jan 2025

all kinds.] $\leadsto$ kinds. Optimized algorithms for the most common calendrical calculations have been presented by C. Neri and L. Schneider, *Software Practice & Exper.* **53** (2023), 937–970.]

▸**Page 520** line −3 of answer 19 ———— 29 Mar 2023

Philomathique $\leadsto$ *philomatique*

▸**Page 523** in answer 13 ———— 29 Mar 2023

$X[i] = -j$ if and only if i goes $\leadsto$ $X[i] = -j$ if and only if no number $> m$ goes to i under π, but i goes

Page 524 append a sentence to answer 21 ———— 10 Apr 2023

(See also answer 1.2.5–21.)

▸**Page 525** last line of answer 23 ———— 10 Nov 2022

373 $\leadsto$ 343

Page 527 new answer ——— 24 Mar 2023

38. (a) Obviously $\pi = \tau^{-}\pi\tau \iff \tau\pi = \pi\tau$. (b) π^{τ} takes $b_{ij} \mapsto a_{ij} \mapsto a_{i(j+1)} \mapsto b_{i(j+1)}$, if we regard $i(l_i{+}1)$ as $i1$. (c) One of the $6 \cdot 3 \cdot 2$ solutions is $\tau = (1\,8)(2\,7)(3\,6)(4\,5)$. Others are $\tau = (2\,3)(7\,8)$; $\tau = (1\,6\,2\,8\,3\,7)$. (d) None. (e) True. (f) True. [See E. Netto, *The Theory of Substitutions and its Applications to Algebra* (1892), §46.]

Page 536 line 3 of answer 4 ——— 07 Jul 2023

divide a polygon $\rightsquigarrow$ divide a convex polygon

▶**Page 537** line 6 ——— 02 May 2024

$g_{n(m-1)} \rightsquigarrow g_{n(m-1)}[m>0]$

Page 537 line −4 ——— 07 Jul 2023

$\displaystyle\sum_{n,m} \rightsquigarrow \sum_{m,n\geq 0}$

Page 538 lines 3 and 4 of answer 5 ——— 07 Jul 2023

is impossible, since . . . yet $\rightsquigarrow$ is impossible: It means that p_i must go off before p_j is put on, and then p_k, yet

Page 542 (and top line of p. 543) new beginning for answer 12 ——— 20 Sep 2024

12. Let $k = \lfloor m/2 \rfloor$. The average is 2^{-m} times

$$\binom{m}{0}m + \binom{m}{1}(m-1) + \cdots + \binom{m}{k}(m-k) + \binom{m}{k+1}(k+1) + \cdots + \binom{m}{m}m$$
$$= m\left(\binom{m-1}{0} + \binom{m-1}{1} + \cdots + \binom{m-1}{k} + \binom{m-1}{k} + \cdots + \binom{m-1}{m-1}\right);$$

and, rather neatly, the latter sum turns out to be $2^{m-1} + \binom{m-1}{k}$, whether m is even or odd. Therefore the answer is

Page 554 lines 1 and 2 of answer 6 ——— 02 May 2024

Add the condition . . . and U4. $\rightsquigarrow$ In steps U2 and U4, the condition "`FLOOR` = `IN`" becomes "`FLOOR` = `IN` and ((`OUT` < `IN` and `STATE` ≠ `GOINGUP`) or (`OUT` > `IN` and `STATE` ≠ `GOINGDOWN`))."

Page 562 bottom line ——— 13 Sep 2022

Charles's $\rightsquigarrow$ Charles III's

▶**Page 565** line 4 of answer 13 ——— 07 Jul 2023

go on to T6 $\rightsquigarrow$ go on to T6′

Page 574 new answer ——— 12 Jun 2024

19. See T. Skolem, *Videnskapsselskapets skrifter I, Matematisk-naturvidenskabelig klasse, Videnskabsakademiet i Kristiania* **4** (1920), 1–36; P. Whitman, *Annals of Mathematics* (2) **42** (1941), 325–330; **43** (1942), 104–115.

▶**Page 576** beginning of answer 14, 15 ——— 01 Nov 2022

Hussain Zedan $\rightsquigarrow$ Hussein Zedan

Page 579 new paragraph at end of answer 11 ——— 17 Nov 2023

References: V. Jarník, *Acta Societatis Scientiarum Naturalium Moravicæ* **6** (1930), 57–61; R. C. Prim, *Bell System Technical Journal* **36** (1957), 1389–1401.

Page 586 in lines 3 and 4 of answer 4 07 Jul 2023

$n \times n$ solution(s) $\leadsto$ $n \times n$ tiling(s)

Page 587 replacing the the last lines of answer 5 31 Jan 2023

In 1974, ... Chapters 1–2]. $\leadsto$ After many decades of improvements, an optimum solution has finally been found: E. Jeandel and M. Rao [*Advances in Combinatorics* (2021) 1:1–1:39] constructed a set of eleven tetrad types, using only four colors, that tiles the plane only nontoroidally. They also proved that no solution with fewer types or fewer colors is possible.

Page 597 bottom line 11 Jun 2024

as follows: $\leadsto$ as follows, assuming that $n > 0$:

Page 598 line 16 13 Jun 2024

Narayana number. [T. V. ... 1189.] $\leadsto$ *Narayana number* $N(n,k)$. [T. V. ... 1189.] P. A. MacMahon [*Combinatory Analysis* **2** (1916), §495] enumerated $l \times n$ plane partitions with parts $\leq m$; the case $m = 2$ is the Narayana number $N(l+n+1, l+1)$.

▶**Page 598** line −6 of answer 3 10 Nov 2022

Discrete Math. **64** $\leadsto$ *Discrete Math.* **62**

▶**Page 600** line −9 07 Jul 2023

(1974) $\leadsto$ (1973)

▶**Page 605** in answer 6 18 Nov 2023

B3 $\leadsto$ B3$'$, B6 $\leadsto$ B6$'$, B2 $\leadsto$ B2$'$

▶**Page 606** in answer 12 07 Jul 2023

B2 $\leadsto$ B2$''$, B3 $\leadsto$ B3$''$, B4 $\leadsto$ B4$''$, B5 $\leadsto$ B5$''$

▶**Page 616** last line of answer 40 07 Jul 2023

96–97 $\leadsto$ 96–99

Page 625 add a definition of multinomial coefficient 20 May 2024

$n!/(n_1!\,n_2!\ldots n_m!)$

▶**Page 629** page numbers for Algorithms 2.3.5H and 2.3.5Z 07 Jul 2023

605 $\leadsto$ 604

Page 623 and continuing through page 627 07 Sep 2025

[All of the cross references in the 'Where defined' column are now identified by a "Section sign": 1.1 $\leadsto$ §1.1, ..., 1.3.2 $\leadsto$ §1.3.2]

Page 630 and following ______ 01 Feb 2022

Miscellaneous changes to the existing index of Volume 1 are collected here, including corrections and amendments to the old entries as well as new entries that are occasioned by the new material. Thus, the lines of the full index that have changed serve also as an index to the present document. However, when a correction or amendment has caused an old index entry to be deleted, the deletion is usually not indicated.