✩ THE ART OF COMPUTER PROGRAMMING ✩
EARLIEST ERRATA TO VOLUME 1 (3rd edition)

This document is a transcript of the notes that I had been making in my personal copy of *The Art of Computer Programming*, Volume 1 (third edition) since it was first printed in 1997.

Four levels of updates — "errors," "amendments," "plans," and "improvements" — appear, indicated by four different typographic conventions:

▸**Page 666** line 1 ——— 04 Jul 1776

Technical or typographical errors (aka bugs) are the most critical items, so they are flagged with a '▸' preceding the page number. The date on which I first was told about the bug is shown; this is the effective date on which I paid the finder's fee. The necessary corrections are indicated in a straightforward way. If, for example, the book says 'n' where it should have said '$n+1$', the change is shown thus:

$n \rightsquigarrow n+1$

Page 666 line 2 ——— 14 Jul 1789

Amendments to the text appear in the same format as bugs, but without the '▸'. These are things I wish I had known about or thought of when I wrote the original text, so I added them later. The date is the date I drafted the new text.

Page 666 line 3 . . . . 20 Nov 1917

Plans for the future represent a third kind of item. In such notes I sketched my intentions about things that I wasn't ready to flesh out further when I wrote them down. You can identify these items because they're written in slanted type, and preceded by a bunch of dots '. . . . .' leading to the date on which I recorded the plan in my files.

Page 666 line 4 ——— 10 Jan 1938

The fourth and final category — indicated by page and line number in smaller, slanted type — consists of minor corrections or improvements that most readers don't want to know about, because they are so trivial. You wouldn't even be seeing these items if you hadn't specifically chosen to print the complete errata list in all its gory details. Are you sure you wanted to do that?

My shelves at home are bursting with preprints and reprints of significant research results that I want to digest and summarize, where appropriate, in the ultimate edition of Volume 1. I didn't do that in the third edition because I would surely have to do it over again later: New results continue to pour forth at a great rate, and I will have time to rewrite that volume only once. Volumes 4 and 5 need to be finished first. So I've put most of my effort so far into writing up those parts of the total picture that seem to have converged to their near-final form. It follows, somewhat paradoxically, that the updates in this document are most current in the areas where there has been least activity.

On the other hand I do believe that the changes listed here bring Volume 1 completely up to date in two respects: (1) All of the research problems in the previous edition — i.e., all exercises that were rated 46 and above — have received new ratings of 45 or less whenever I learned of a solution; and in such cases, the answer now refers to that solution. (2) All of the historical information about pioneering developments has been amended whenever new details have come to my attention.

The ultimate, glorious, future editions of Volumes 1–3 are works in progress. Please let me know of any improvements that you think I ought to make. Send your comments either by snail mail to D. E. Knuth, Computer Science, Gates Building 1B, Stanford University, Stanford CA 94305-9015, or by email to `taocp@cs.stanford.edu`*. (Use email for book suggestions only, please — all other correspondence is returned unread to the sender, or discarded, because I have no time to read ordinary email.) Although I'm working full time on Volume 4 these days, I will try to reply to all such messages within a year of receipt. Current news about The Art of Computer Programming is posted on*

`http://www-cs-faculty.stanford.edu/~knuth/taocp.html`

and updated regularly.

— Don Knuth, April 1997

What happened?
The subject took the bit in its teeth and ran away with it,
that's what happened.
I know now how Sir James Frazer felt when,
after setting out to dash off a brief monograph
on a single obscure rite, he found himself
in the embarrassing possession of
the 12 volumes of "The Golden Bough."

— WAVERLEY ROOT (1974)

☆☆☆☆☆☆ FUNDAMENTAL ALGORITHMS ☆☆☆☆☆☆

Last updated 16 August 2022

All of these corrections have been made in subsequent printings.

Page iv line 4 of the Library of Congress Data ——— 07 Jan 2011

`xx,650p.` ⇝ `xx,652p.`

Page vii lines 13 and 14 ——— 05 Jan 2011

three separate books (Volumes 4A, 4B, and 4C) ⇝ several separate books (Volumes 4A, 4B, and so on)

Page vii line 15 ——— 30 Dec 1997

Volume 6, *The Theory of Languages*; Volume 7, *Compilers*. ⇝ Volume 6, *The Theory of Languages* (Chapter 11); Volume 7, *Compilers* (Chapter 12).

Page xi line −7 ——— 28 Oct 2008

cheerfully pay \$2.56 ⇝ cheerfully award \$2.56

▸**Page xii** step 2 of the flowchart ——— 16 Jul 1997

pp. xvii–xix ⇝ pp. xv–xvii

Page xv line 11 ——— 09 Oct 2010

This is sometimes unfortunate ⇝ A motley mixture is, however, often unfortunate

Page xvi line −6 ——— 09 Oct 2010

This is not ⇝ This distinction is not

Page xvii line 9 ——— 09 Oct 2010

Later editions ⇝ Later printings

Page xix replacement for the bottom line ——— 07 Jan 2011

▸**Page xviii** lines 12–19 ——— 01 Jul 1998

1.2.8 ⇝ 1.2.8. · · · 1.3 ⇝ 1.3.

▸**Page 1** line 11 ——— 26 Feb 2007

(1844) ⇝ (1843)

Page 1 line −5 ——— 10 Apr 2001

celebrated book ⇝ celebrated Arabic text

▸**Page 1** line −4 ——— 01 Aug 1997

Kitab al jabr ⇝ *Kitāb al-jabr*

Page 3 line 3 after Fig. 1 — 03 Feb 2000

Step E3 $\rightsquigarrow$ step E3

Page 4 line −15 — 09 Apr 2003

$\frac{544}{119} = 4\frac{68}{119}$ $\rightsquigarrow$ $544/119 = 4 + 68/119$

Page 6 lines 1 and 2 — 02 Mar 2000

set of divisors $\rightsquigarrow$ set of common divisors (twice)

▸**Page 8** three places in (3) — 05 Aug 2000

$f(\sigma, j) \rightsquigarrow f\big((\sigma, j)\big)$ [twice]
$f(\sigma, N) \rightsquigarrow f\big((\sigma, N)\big)$ [once]

▸**Page 9** line 1 of exercise 5 — 25 Dec 2010

appears in $\rightsquigarrow$ appears after

Page 9 line 2 of exercise 5 — 25 Dec 2010

three $\rightsquigarrow$ at least three

Page 9 lines 6 and 7 of exercise 9 — 02 May 2008

I_2 might be the initial state ... values of m and n. $\rightsquigarrow$ I_2 might be the set of initial states, each including the program that determines the greatest common divisor as well as the particular values of m and n.

Page 12 line −5 — 06 Aug 2002

8.] $\rightsquigarrow$ 8. See also Section 7.2.1.4.]

Page 13 line 23 — 19 Mar 2008

$\phi^0 = \phi^{n-1} \rightsquigarrow \phi^0 = \phi^{1-1}$

Page 13 line −3 — 21 May 2005

d and two integers a and b, $\rightsquigarrow$ d, and we also compute two not-necessarily-positive integers a and b

▸**Page 15** replacement for (7) — 26 Jul 1997

if an assertion attached to any arrow leading into the box is true before the operation in that box is performed, then all of the assertions on relevant arrows leading away from the box are true after the operation. (7)

▸**Page 16** line −7 — 24 Mar 1999

m, n, c, and r $\rightsquigarrow$ m, n, c, d, and r

Page 16 near the bottom — 24 Mar 1999

line −5: the algorithm with $\rightsquigarrow$ the method with
line −2: of the algorithm $\rightsquigarrow$ of the procedure
line −1: start the procedure with $\rightsquigarrow$ start with the values

Page 17 lines 26 and 27 — 29 Mar 2005

[See *AMM* **24** $\rightsquigarrow$ [See *The Penny Cyclopædia* **11** (1838), 465–466; *AMM* **24**

Page 19 add clarification to line 3 ——— 23 Nov 1997

(A prime number is considered to be the "product" of a single prime, namely itself.)

Page 19 lines 11–12 ——— 05 Aug 2004

suggested to the author by R. W. Floyd, is shown in Fig. 5. $\leadsto$ suggested by Warren Lushbaugh, is shown in Fig. 5; see *Math. Gazette* **49** (1965), 200.

Page 19 lines 18 and 20 ——— 20 Sep 2002

$(1 \leadsto \cdot(1$

▸**Page 19** in the Area part of Fig. 5 ——— 30 Jul 1997

$4 \cdot 3 \cdot 3^2 + 4 \cdot 5 \cdot 5^2 \leadsto 4 \cdot 3 \cdot 3^2 + 4 \cdot 4 \cdot 4^2 + 4 \cdot 5 \cdot 5^2$

▸**Page 19** line 4 of exercise 13 ——— 27 Nov 1997

"$m > 0$, $n = 0$, $T = 0$." $\leadsto$ "$m > 0$, $n > 0$, $T = 0$."

▸**Page 22** line 8 ——— 28 Jul 1997

mth root of v, $\leadsto$ *mth root* of u,

Page 22 line 10 ——— 28 Jul 1997

numbers $r \leadsto$ numbers $r = p/q$

Page 22 line −12 ——— 02 Oct 1998

$1^x = 1 \leadsto b^x = 1$

Page 23 line −1 ——— 08 Aug 2002

the return $\leadsto$ the annual return

Page 25 line 1 ——— 21 Apr 1999

$0.0100110100010000011\ldots \leadsto (0.0100110100010000011\ldots)_2$

Page 26 first line of exercise 26 ——— 20 Mar 2001

Determine upper bounds on the accuracy of $\leadsto$ Find a rigorous upper bound on the error made by

Page 27 new exercise for Section 1.2.2 ——— 28 Jun 2003

30. [*12*] Simplify the expression $(\ln n)^{\ln n/\ln\ln n}$, assuming that $n > 1$.

Page 27 the line following (1) ——— 04 Jan 2010

If n is zero, ... zero. In general, $\leadsto$
If n is zero, the value of $a_1 + a_2 + \cdots + a_n = \sum_{j=1}^{n} a_j = \sum_{1\le j\le n} a_j$ is defined to be zero. Our convention of using "three dots" in sums such as $a_1 + a_2 + \cdots + a_n$ therefore has some slightly peculiar, but sensible, behavior in borderline cases (see exercise 1).

In general,

Page 27 lines 9 and 10 from the bottom ——— 20 Mar 2001

The example in the preceding sentence $\leadsto$ A sum such as $\sum_{j\le k}\binom{j+k}{2j-k}$

▸**Page 27** line 4 from the bottom ——— 16 Jul 1997

$\sum_{j=1}^{\infty} = \leadsto \sum_{j=1}^{\infty} a_j =$

▸**Page 27** replacement for the bottom line ——— 16 Jul 1997

$$\sum_{R(j)} a_j = \Bigl(\lim_{n\to\infty} \sum_{\substack{R(j)\\ -n\le j<0}} a_j\Bigr) + \Bigl(\lim_{n\to\infty} \sum_{\substack{R(j)\\ 0\le j<n}} a_j\Bigr), \qquad (3)$$

Page 28 line −11 ——— 06 Aug 2001

the range $\leadsto$ the relevant values

Page 33 near the top ——— 06 Aug 2001

line 3: range $\leadsto$ relevant values
line 4: a permutation of the range $\leadsto$ such a permutation

▸**Page 33** line −5 ——— 29 Jul 1997

$[1 \le i \le n][1 \le j \le n] \leadsto [1 \le i \le n][1 \le j \le i]$

Page 34 replacements for exercises 1–3 ——— 04 Jan 2010

▸**1.** [*10*] The text says that $a_1 + a_2 + \cdots + a_0 = 0$. What, then, is $a_2 + \cdots + a_0$?
2. [*01*] What does the notation $\sum_{1\le j\le n} a_j$ mean, if $n = 3.14$?
▸**3.** [*13*] Without using the $\sum$-notation, write out the equivalent of

$$\sum_{0\le n\le 5} \frac{1}{2n+1},$$

and also the equivalent of

$$\sum_{0\le n^2\le 5} \frac{1}{2n^2+1}.$$

Explain why the two results are different, in spite of rule (b).

Page 34 line 1 of exercise 15 ——— 12 Jun 2002

$n2^n \leadsto n \times 2^n$

▸**Page 35** exercise 21 ——— 07 Feb 2004

from . . . (16). $\leadsto$ from (8) and (17).

▸**Page 37** line −2 ——— 15 Jul 1997

$a_{ij} = 1/(x_i + y_i) \leadsto a_{ij} = 1/(x_i + y_j)$

Page 39 line 6 of section 1.2.4 ——— 25 Feb 1999

because both $\leadsto$ because

▸**Page 40** line −12 ——— 02 Aug 2000

(modulo m), $\leadsto$ (modulo m).

Page 41 line 1 of exercise 6 ——— 12 Aug 1997

real numbers $\leadsto$ real numbers

▸**Page 42** line 3 of exercise 21 ——— 16 Jul 1997

factors. $\leadsto$ factors).

Page 44 line 1 of exercise 45 ——— 15 Oct 2007

implies that $\leadsto$ implies that, when m and n are positive integers,

▸**Page 45** line 9 of exercise 47 ——— 23 Jun 2010

(modulo 2). $\leadsto$ (modulo 2), unless q is a multiple of p.

▸**Page 45** line 1 of exercise 49 ——— 07 Jan 1998

the function $f(x)$ $\leadsto$ the integer-valued function $f(x)$

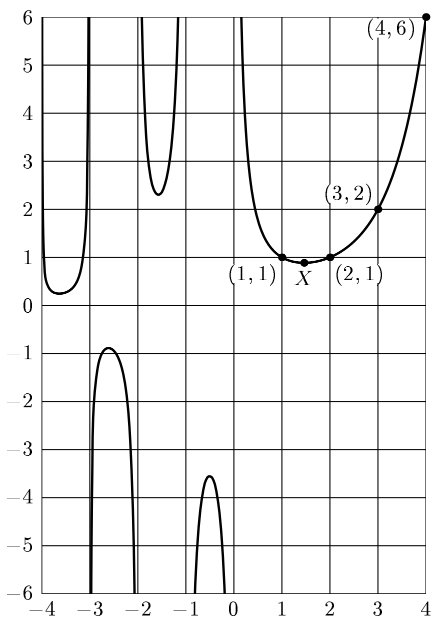
Page 48 line 8 ——— 27 Feb 2005

$n!$. $\leadsto$ $n!$. [See A. M. Legendre, *Essai sur la Théorie des Nombres*, second edition (Paris: 1808), page 8.]

Page 49 line 16 ——— 19 Jan 2005

1808) $\leadsto$ 1808), page 219

Page 50 better labels on Figure 7 ——— 12 Aug 1998



Page 50 in Eq. (19) ——— 01 Jan 2006

$(x + j)$; $\leadsto$ $(x + j)$.

Page 50 lines −6 and −5 ——— 12 Jun 2003

[This notation ... 313.] $\leadsto$ [The notations $x^{\overline{k}}$ and $x^{\underline{k}}$ are due respectively to A. Capelli, *Giornale di Mat. di Battaglini* **31** (1893), 291–313, and L. Toscano, *Comment. Accademia della Scienze* **3** (1939), 721–757.]

Page 53 near the bottom ——— 30 Nov 2000

line −5: Shih-Chieh Chu ⇝ Chu Shih-Chieh
line −4: invention. ⇝ invention. Yang Hui, in 1261, credited them to Chia Hsien, whose work (c. 1000) is now lost.

▸**Page 53** line −2 or −3, 12th thru 14th printings only ——— 28 Aug 2003

Halāyuddha ⇝ Halāyudha

Page 53 line −1 or −2 ——— 21 Sep 2004

Chandaḥ-sūtra ⇝ *Chandaḥśāstra*

Page 54 opening lines ——— 23 Jan 2005

In about 1150 ... were known ⇝ Another Indian mathematician, Mahāvīra, had previously explained rule (3) for computing $\binom{r}{k}$ in Chapter 6 of his *Gaṇita Sāra Saṅgraha*, written about 850; and in 1150 Bhāskara repeated Mahāvīra's rule near the end of his famous book *Līlāvatī*. For small values of k, binomial coefficients were known

Page 54 penultimate line of first paragraph ——— 23 Jan 2005

Ettingshausen in his book ⇝ Ettingshausen in §31 of his book

Page 54 caption to Figure 8 ——— 23 Jun 2001

[the second line should be in 9-point type, not 10-point]

Page 59 line −4 ——— 22 Aug 2002

Paris (1772) ⇝ (Paris, 1772)

Page 59 line −3 ——— 19 Jan 2005

489–498 ⇝ part 1, 489–498.

Page 59 line −3 ——— 30 Nov 2000

Shih-Chieh Chu's ⇝ Chu Shih-Chieh's

▸**Page 59** line −2 ——— 27 Oct 2004

Civilization ⇝ *Civilisation*

Page 60 line −13 ——— 30 Dec 2004

with $n = -1$ ⇝ with $m = 0$ and $n = -1$

▸**Page 61** line 13 ——— 19 Oct 1997

$\binom{n+1-(n+1)}{n+1-1+0}$ ⇝ $\binom{n+1-(n+1)}{n+1-1+0}\frac{1}{n+1}$

▸**Page 63** line 12 ——— 21 Jul 2003

233–248 ⇝ 233–247

▸**Page 64** line 11 ——— 18 Dec 1999

r of less ⇝ r or less

Page 69 line 6 of exercise 10 ——— 26 Jul 1997

(E. Lucas, 1877.) $\rightsquigarrow$ (É. Lucas, 1877.)

Page 70 line 1 of exercise 17 ——— 30 Nov 2000

Chu-Vandermonde $\rightsquigarrow$ Chu–Vandermonde

Page 71 line 3 of exercise 31 ——— 11 Oct 2001

nonnegative integers $\rightsquigarrow$ integers

Page 71 replacement for exercise 33 ——— 19 Jan 2005

33. [*M20*] (A. Vandermonde, 1772.) Show that the binomial formula is valid also when it involves factorial powers instead of the ordinary powers. In other words, prove that

$$(x+y)^{\underline{n}} = \sum_k \binom{n}{k} x^{\underline{k}} y^{\underline{n-k}}; \qquad (x+y)^{\overline{n}} = \sum_k \binom{n}{k} x^{\overline{k}} y^{\overline{n-k}}.$$

Page 73 line 2 of exercise 56 ——— 31 May 2002

$n = \binom{a}{1} + \binom{b}{2} + \binom{c}{3}$ and $0 \le a < b < c$. $\rightsquigarrow$ $n = \binom{a}{3} + \binom{b}{2} + \binom{c}{1}$ and $a > b > c \ge 0$.

Page 74 replacement for exercise 66 ——— 08 Nov 2005

66. [*HM30*] Suppose x, y, and z are real numbers satisfying

$$\binom{x}{n} = \binom{y}{n} + \binom{z}{n-1},$$

where $x \ge n-1$, $y \ge n-1$, $z > n-2$, and n is an integer ≥ 2. Prove that

$$\binom{x}{n-1} \le \binom{y}{n-1} + \binom{z}{n-2} \qquad \text{if and only if} \qquad y \ge z;$$
$$\binom{x}{n+1} \le \binom{y}{n+1} + \binom{z}{n} \qquad \text{if and only if} \qquad y \le z.$$

Page 74 line 1 of exercise 68 ——— 17 Mar 1998

De Moivre $\rightsquigarrow$ de Moivre

Page 75 lines 6–9 ——— 05 May 2007

(Besides H_n, ... the harmonic series.) $\rightsquigarrow$ Besides H_n, ... the harmonic series. Chinese bamboo strips written before 186 B.C. already explained how to compute $H_{10} = 7381/2520$, as an exercise in arithmetic. [See C. Cullen, *Historia Math.* **34** (2007), 10–44.]

Page 77 the line after (10) ——— 25 Feb 2000

the infinite series $\rightsquigarrow$ the infinite series 1.2.9–(17)

Page 80 line 3 ——— 26 Jul 1997

sequence F_n $\rightsquigarrow$ sequence $\langle F_n \rangle$

Page 80 line 7 ——— 20 Jul 2000

Hemachandra $\rightsquigarrow$ Hemacandra

Page 80 line 10 ——— 16 Dec 2001

Johann Kepler $\rightsquigarrow$ Johannes Kepler

▸**Page 80** line 20 — 04 Jan 2006

at most $k+1$ $\leadsto$ at most $k-1$

Page 81 line 8 — 18 Nov 2008

sculpture. $\leadsto$ sculpture. [See T. A. Cook, *The Curves of Life* (1914), 420.]

Page 81 line 16 — 22 Aug 2002

Paris **1** (1680) $\leadsto$ *Sci.* **1** (Paris, 1680)

Page 81 line −8 — 27 Jul 1999

multiple of F_k $\leadsto$ *multiple of* F_n

Page 82 line 7 — 26 Jul 1997

sequence, F_n, for which statement (8) holds $\leadsto$ sequence $\langle F_n\rangle$ for which statement (8) holds,

▸**Page 83** line 8 — 15 Oct 2007

quadratic equation $\leadsto$ quadratic polynomial

Page 83 lines 2–4 after (14) — 22 Sep 1999

by A. De Moivre ... essentially $\leadsto$ early in the eighteenth century. (See D. Bernoulli, *Comment. Acad. Sci. Petrop.* **3** (1728), 85–100, §7; see also A. de Moivre, *Philos. Trans.* **32** (1722), 162–178, who showed how to solve general linear recurrences in essentially

Page 86 bottom line — 07 Feb 2004

(The k's $\leadsto$ (See exercise 34. The k's

Page 87 line 15 — 17 Mar 1998

A. De Moivre $\leadsto$ A. de Moivre

▸**Page 87** line 22 — 27 Jul 1997

125–169) $\leadsto$ 125–169

▸**Page 88** line −3 — 27 Feb 1998

(a_0+b_0) $\leadsto$ (a_0b_0)

Page 89 on the left side of (13) — 15 Oct 2007

$\scriptstyle n \bmod m=r$ $\leadsto$ $\scriptstyle n\geq 0,\, n \bmod m=r$

▸**Page 90** in Eq. (15) — 07 Jan 1998

$$\sum_{k\geq 0} \leadsto \sum_{k\geq 1}$$

Page 94 exercise 10 — 18 Aug 2002

[Change the notation from 'a_m' to the more modern 'e_m', throughout this exercise.]

▸**Page 95** line 6 of exercise 19 — 17 Jul 1997

$$\Bigl(\frac{1}{n}-\frac{1}{n+x}) \leadsto \Bigl(\frac{1}{n}-\frac{1}{n+x}\Bigr)$$

Page 98 line −9 ——— 22 Aug 2002

Paris **37** (1853) ⇝ **37** (Paris, 1853)

▸**Page 98** line −2 ——— 12 Sep 1997

$(n-1)P_{(n-k)k}$ ⇝ $(n-1)P_{(n-1)k}$

Page 103 line −12 ——— 04 Oct 2005

by Thiele in 1903. ⇝ by T. N. Thiele in 1889 [see A. Hald, *International Statistical Review* **68** (2000), 137–153].

▸**Page 103** in (23) ——— 29 Jul 1997

$\kappa_1 t^2$ ⇝ $\kappa_2 t^2$

Page 104 line 2 before the exercises ——— 30 Oct 2007

for all $t \geq r$, ⇝ for all $t \geq r$, and if $f(t) \geq 0$ for all t in the domain of the random variable X,

Page 105 line −7 ——— 01 Jan 2006

probability ⇝ Pr

Page 106 line 1 of exercise 14 ——— 17 Mar 1998

De Moivre ⇝ de Moivre

Page 106 line 1 of exercise 18 ——— 01 Jan 2006

distinct values ⇝ values

▸**Page 106** replacement for line 6 of exercise 18 ——— 15 Oct 2007

$$\left(\frac{k_n z}{k_n}\right)\left(\frac{k_{n-1}z + k_n}{k_{n-1}+k_n}\right)\left(\frac{k_{n-2}z + k_{n-1} + k_n}{k_{n-2}+k_{n-1}+k_n}\right)\ldots\left(\frac{k_1 z + k_2 + \cdots + k_n}{k_1 + k_2 + \cdots + k_n}\right)\Big/ z,$$

▸**Page 106** lines 3 and 4 of exercise 20 ——— 24 Dec 2001

$a_1,\, a_2\, \ldots\, a_n$ ⇝ $a_1 a_2 \ldots a_n$

Page 107 the line after (1) ——— 07 Sep 2001

Oiler's constant ⇝ Euler's constant [pronounced 'Oiler's constant']

▸**Page 108** line 3 ——— 31 Jul 1997

a_1 ⇝ $a_1 n$

▸**Page 109** line −7 ——— 22 May 1998

all value ⇝ all values

Page 110 line −2 ——— 08 Nov 2005

Big Theta notation ⇝ Big Theta

Page 112 line −10 ——— 20 Mar 2006

introduced by Jacques Bernoulli in his ⇝ introduced to European mathematicians in James Bernoulli's

Page 113 line 13 ——— 04 Nov 2001

(9) ⇝ (9)

Page 114 line 10 ——— 10 Nov 2002

$f^{(m)} \rightsquigarrow f^{(m)}(x)$

Page 114 line 15 ——— 28 Nov 2002

$\pm\int_1^\infty \rightsquigarrow -\int_1^\infty$

▸**Page 115** line 1 of exercise 3 ——— 01 Jan 2006

$((-1)^m(B_m/m!) \rightsquigarrow (B_m/m!)$

Page 116 first line of exercise 8 ——— 01 Jan 2006

$\ln(an^2+bn)! \rightsquigarrow \ln\,(an^2+bn)!$

▸**Page 116** last line of exercise 8 ——— 03 May 1998

$(1+\epsilon) \rightsquigarrow (1+\epsilon).$

▸**Page 121** line 5 ——— 08 Feb 2000

$\frac{1}{8505} \rightsquigarrow \frac{16}{8505}$

Page 123 the displayed equation of exercise 17 ——— 27 Mar 1998

$$\sum_{0\le k} \rightsquigarrow \sum_{k\ge 0}$$

Page 125 line 1 ——— 17 Sep 2004

basic unit of information $\rightsquigarrow$ basic unit of `MIX` data

Page 125 footnote ——— 03 Dec 1997

binary bits $\rightsquigarrow$ binary digits

▸**Page 126** replacement for bottom of Fig. 13 ——— 15 Jul 1998

Magnetic tape units ⟷				Disks and drums ⟷				Card reader	Card punch	Line printer	Typewriter	Paper tape
U0	U1	⋯	U7	U8	⋯	U14	U15	U16	U17	U18	U19	U20

▸**Page 130** line 6 ——— 14 Feb 1999

right-hand of $\rightsquigarrow$ right-hand portion of

▸**Page 130** line −15 ——— 14 Feb 1999

(store X), C $\rightsquigarrow$ (store X). C

Page 131 line 13 ——— 03 Aug 1997

left of A $\rightsquigarrow$ left of rA

Page 131 line −6 ——— 28 Sep 1997

10-byte number $\rightsquigarrow$ 10-byte number rAX

▸**Page 131** line −2 ——— 28 Sep 1997

the quotient $\pm\lfloor|\mathrm{rA}/\mathrm{V}|\rfloor \rightsquigarrow$ the quotient $\pm\lfloor|\mathrm{rAX}/\mathrm{V}|\rfloor$
the remainder $\pm(|\mathrm{rA}| \bmod |\mathrm{V}|) \rightsquigarrow$ the remainder $\pm(|\mathrm{rAX}| \bmod |\mathrm{V}|)$

Page 133 line −3 ——— 22 Aug 2005

$C = 48 + i$ $\rightsquigarrow$ $\mathrm{C} = 48 + i$

Page 134 line 14 ——— 03 Aug 1997

field of A $\rightsquigarrow$ field of rA

Page 135 line 14 ——— 18 Aug 2010

These are $\rightsquigarrow$ These 42 instructions are

Page 135 line −7 ——— 24 Jan 2003

F = number. $\rightsquigarrow$ F = number, normally 1.

Page 135 line −2 ——— 03 Sep 1997

when the groups of locations involved overlap $\rightsquigarrow$ when there's overlap between the locations involved

Page 138 line 17 ——— 04 Apr 2003

40, . . . convert $\rightsquigarrow$ 40, . . . convert

Page 138 line 23 ——— 03 Aug 1997

register A and X $\rightsquigarrow$ registers A and X

▸**Page 138** line −5 ——— 11 Oct 1998

unspecified. $\rightsquigarrow$ specified in Section 4.2.1.

Page 139 line −4 in 1st or 2nd printing ——— 17 Apr 1998

appears on page 133 $\rightsquigarrow$ on page 133

▸**Page 139** line −4 in 3rd, 4th, 5th printings ——— 10 Jan 1999

appears on page ?? $\rightsquigarrow$ on page 133

▸**Page 142** line 3 of exercise 13 ——— 27 Feb 2000

"`JNOV 1001`, $\rightsquigarrow$ "`JNOV 1001`",

▸**Page 143** line 6 of exercise 23 ——— 19 Jul 1997

to load $\rightsquigarrow$ ability to load

▸**Page 144** last line of the exercise ——— 03 Sep 1997

cannot be used; $\rightsquigarrow$ instructions cannot be used;

Page 145 line −24 ——— 03 Aug 1997

"`LOC OP ADDRESS`" $\rightsquigarrow$ "`LOC`", "`OP`", and "`ADDRESS`"

▸**Page 145** line −8 ——— 07 Sep 1998

(page 1) $\rightsquigarrow$ (page 96)

Page 147 lines 16 and 19 ——— 18 Jul 2004

program $\rightsquigarrow$ algorithm (twice)

Page 149 line −6 ——— 13 Jul 2008

25 and 35 $\rightsquigarrow$ 09, 25, and 35

Page 150 line 15 05 May 2010

assembly procedure $\leadsto$ assembly procedure (possibly with con2 first)

Page 151 line 23 18 Apr 2010

"`ALF`" operation $\leadsto$ pseudo-operation `ALF`

Page 151 line −8 21 Sep 2007

42, and 48. $\leadsto$ 42, 48, and 50.

▸**Page 154** line 5 03 Aug 1997

that is., $\leadsto$ that is,

▸**Page 155** line 4 of point 10 19 Jul 1997

previous defined. $\leadsto$ previously defined.

Page 156 the line before the exercises 14 Oct 2008

`PL/MIX` will be described in Chapter 10. $\leadsto$

Page 156 lines 2–3 of exercise 1 13 Oct 2008

you wish to set $\leadsto$ the algorithm is supposed to set

▸**Page 157** line 1 of exercise 6 15 May 1999

$1 \le d \le \sqrt{n} \leadsto 1 < d \le \sqrt{n}$

Page 159 line 1 of exercise 12 24 Apr 1997

[*M47*] $\leadsto$ [*HM42*]

Page 160 lines 11 and 12 10 Mar 2002

[March $\bigl((-D) \bmod 7\bigr)$ actually will be a Sunday.] $\leadsto$ (March $\bigl((-D) \bmod 7\bigr)$ will actually be a Sunday.)

Page 162 line −4 of exercise 20 21 May 1999

this traffic light $\leadsto$ these lights

▸**Page 162** line 4 of exercise 21 12 Jul 1997

jumt below $\leadsto$ just below

Page 162 lines 10 and 11 of exercise 21 28 Mar 2000

Manuel Moschopoulos, who lived in Constantinople about 1300 $\leadsto$ Ibn al-Haytham, who was born in Basra about 965 and died in Cairo about 1040

▸**Page 163** line 3 after Fig. 18 10 Jul 1997

is what $\leadsto$ is white

Page 163 in the midst of the bottom paragraph 18 Jan 2008

card corresponding to the matrix $\leadsto$ card corresponding to the top row of the matrix

Page 166 line 3 02 Nov 2005

[the spacing in this formula should be more consistent]

Page 166 line 2 of step A1 30 Mar 1998

copy of the element $\leadsto$ copy of the input symbol

▸**Page 166** line −4 ——— 19 Jul 1997

[See `CURRENT`.] $\leadsto$ [Set `CURRENT`.]

Page 167 first line of Table 1 ——— 27 May 1998

The formula on this line should repeat the input formula (6).

Page 168 line 8 ——— 24 Oct 2005

cycles. $\leadsto$ cycles. But it doesn't catch errors in the input.

Page 168 line 30 of Program A ——— 30 Mar 1998

nonblank element $\leadsto$ nonblank input symbol

Page 170 lines 11 and 12 ——— 17 Sep 2005

and it has led to some horribly inefficient uses of computing machinery. $\leadsto$ and some horribly inefficient uses of computing machinery have arisen as a result.

Page 170 replacement for lines −11 to −4 ——— 30 Nov 2009

26, 38, 39	$A = 1 + (C - 1)$
33, 28, 29	$C = B + (A - B)$
41, 84, 86	$E = 1 + R$
42, 46, 48	$F = E + (G - 1)$
64, 43, 44	$H = F - G$
67, 70, 76	$J = H + (K - (L - J))$
75, 79, 80	$K = Q + (L - P)$
82, 72, 81	$R = P - Q$

▸**Page 172** line 5 ——— 31 May 2004

(it involves ...), $\leadsto$ (it involves $\cdots + 3F + 4G + \cdots + 3K + 4L + \cdots = \cdots + 7G + 7L + \cdots$),

Page 172 line −5 ——— 18 Aug 1997

homo sapiens $\leadsto$ *Homo sapiens*

▸**Page 174** line 26 of Program B ——— 13 Jul 2000

`ENT3 1` $\leadsto$ `ENT3 0`

▸**Page 176** step I5 of Algorithm I ——— 22 Feb 1998

$X[i]$ was $\leadsto$ $X[-i]$ was

▸**Page 178** line −5 ——— 24 Dec 2001

permutation(1 6 3)(4 5) $\leadsto$ permutation (1 6 3)(4 5)

Page 182 line 1 ——— 17 Mar 1998

A. De Moivre $\leadsto$ A. de Moivre

Page 182 line 4 ——— 18 Sep 2002

W. A. Whitworth $\leadsto$ I. Todhunter in his *Algebra* (second edition, 1860), §762, and by W. A. Whitworth

▸**Page 182** the rating of exercise 6 ——— 24 Oct 2005

M23 $\leadsto$ *M28*

Page 182 line 1 of exercise 11 ——— 02 Oct 1997

$\pi^{-1} \leadsto \pi^{-}$

▸**Page 182** line 4 of exercise 12 ——— 04 Aug 1997

$n \times n$ matrix $(b_{ij}) \leadsto n \times m$ matrix (b_{ij})

▸**Page 183** line 12 of exercise 22 ——— 09 Sep 1998

$P(n : k_1, k_2, k_3, \ldots) \leadsto P(n; k_1, k_2, k_3, \ldots)$

Page 184 in exercise 29 ——— 14 Oct 2008

lines 2–3: the "doubling" ... mod $2n + 1 \leadsto$ the "perfect shuffle" permutation of $\{1, 2, \ldots, 2n\}$, which takes $(1, 2, \ldots, 2n)$ into $(2, 4, \ldots, 2n, 1, 3, \ldots, 2n-1)$
line 4: the doubling permutation $\leadsto$ the perfect shuffle

▸**Page 184** line 2 of exercise 30 ——— 01 Jan 2006

$(2^{d-1} - 1)(2n + 1)/(2^d - 1) \leadsto (2^d - 1)(2n + 1)/(2^{d+1} - 1)$

▸**Page 184** in exercise 31 ——— 14 Oct 2008

line 1: *HM33* $\leadsto$ *HM38*
line 4: fixed $m \leadsto$ fixed $m > 1$
line 5: $(m^{k+1} - (m - 1))^k \leadsto (m^{k+1} - (m - 1)^k)$

Page 185 line 3 ——— 06 Aug 2001

this is the permutation on $\{0, 1, \ldots, m + n - 1\}$ that takes k into $(k + m) \bmod (m + n)$ $\leadsto$ each element x_k should be replaced by $x_{p(k)}$ for $0 \le k < m + n$, where $p(k) = (k + m) \bmod (m + n)$.

Page 185 new exercise ——— 29 Jan 2010

37. [*M26*] (*Even permutations.*) Let π be a permutation of $\{1, \ldots, n\}$. Prove that π can be written as the product of an even number of 2-cycles if and only if π can be written as the product of exactly two n-cycles.

Page 186 line 14 ——— 17 Sep 2005

by having less space $\leadsto$ by occupying less space

▸**Page 190** in (10) ——— 07 Jan 1999

`EXIT C` $\leadsto$ `EXITC`

Page 193 lines 4 and 5 of exercise 6 ——— 21 Aug 2001

the time to execute the subroutine will be somewhat longer. $\leadsto$ a subroutine naturally consumes more time and space than a hardware instruction does.

▸**Page 194** line −11 ——— 19 Dec 2001

stores J $\leadsto$ stores rJ

Page 195 in (3) ——— 13 Jun 1999

The spaces between three-character groups should be wider.

Page 196 line −6 ——— 14 Jun 1999

three-digit groups $\leadsto$ three-character groups

▸**Page 198** line 17 from the bottom ———— 09 Jul 1997

UNIX[Exit conditions (when] users $\leadsto$ UNIX® users

▸**Page 198** line 15 from the bottom ———— 22 Jul 1997

A, B, and C $\leadsto$ B, C, and D

Page 202 line 22 ———— 07 Aug 1997

TeX: $\leadsto$ *TEX:*

Page 202 line 25 ———— 11 Jul 1997

PostScript® [Adobe $\leadsto$ PostScript® [Adobe

Page 202 line −3 ———— 08 Nov 2005

simulator (or sometimes an *emulator*) $\leadsto$ *simulator* (or sometimes an *emulator*)

▸**Page 204** line 009 of the program ———— 14 Feb 1999

see line 033 $\leadsto$ (see line 033)

▸**Page 205** line 042 of the program ———— 03 Sep 2008

`SPEC(1)` $\leadsto$ `SPEC(10)`

Page 208 lines 161 and 164 of the program ———— 19 Dec 2001

Let rI1 indicate the A register. $\leadsto$ rI1 ← index of simulated rA.

▸**Page 211** line 11 ———— 30 Dec 1997

as does `ENTA -5,1`” $\leadsto$ as does “`ENTA -5,1`”

▸**Page 211** line −5 in 3rd, 4th, 5th printings ———— 16 Dec 1998

1.3.1–?? $\leadsto$ 1.3.1–26

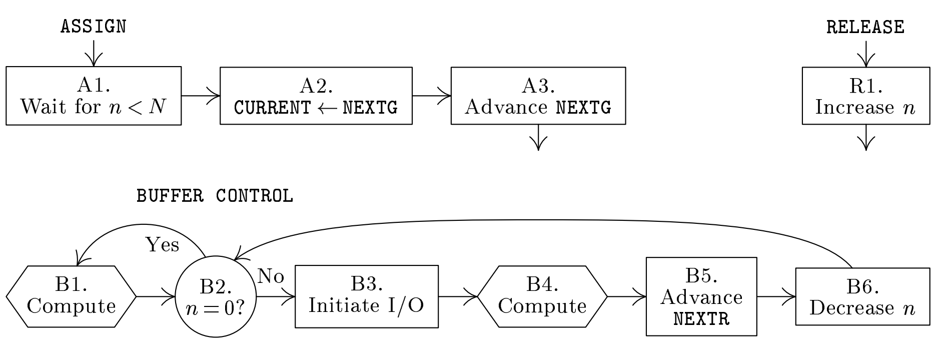
▸**Page 213** line 28 of the program ———— 24 Dec 2001

`JXL` $\leadsto$ `JXNP`

▸**Page 216** line 7 ———— 02 Aug 1997

52–53. $\leadsto$ 52–53).

Page 223 replacement for Fig. 25 ———— 08 Aug 1997



Page 225 exercise 1 ———— 19 Dec 2001

Would $\leadsto$ (a) Would
What if $\leadsto$ (b) What if

▸**Page 225** line −5 ______ 14 Feb 1999

necessary ⇝ necessary.

Page 230 line 12 ______ 27 Apr 2007

Illiac I ⇝ ILLIAC I

Page 230 line 16 ______ 07 Dec 2002

Nat. Conf. (1952) ⇝ (Toronto: 1952)

Page 230 line −4 ______ 05 Feb 2002

difficult. ⇝ difficult. See also the trace routine for IBM's System/360 architecture, presented in *A Compiler Generator* by W. M. McKeeman, J. J. Horning, and D. B. Wortman (Prentice–Hall, 1970), 305–363.

Page 231 line −21 ______ 19 Jun 2003

Nat. Conf. ⇝ *Nat. Meeting* **13**

Page 231 lines −12 through −9 of the final paragraph ______ 29 Nov 2009

X-1 computer ... paper tape ⇝ X1 computer [see *Comp. J.* **2** (1959), 39–43]. Dijkstra's doctoral thesis, "Communication with an Automatic Computer" (1959), discussed primitive synchronization operations by which users could create long chains of buffers with respect to paper tape

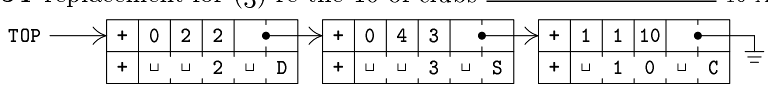
▸**Page 231** quotation at bottom ______ 15 Jul 1997

line −3: About 1000 ⇝ About 1,000
line −2: the problems new envisioned. ⇝ problems now envisioned.

▸**Page 233** line 19 ______ 28 Dec 1997

JAVA ⇝ Java

▸**Page 234** replacement for (3) re the 10 of clubs ______ 10 Aug 1997



(3)

▸**Page 242** line 4 of exercise 2 ______ 21 Jul 1997

require cards ⇝ require cars

▸**Page 242** exercise 3 ______ 05 Jan 1998

(a) through (h) ⇝ (i) through (viii)
[In the first and second printings, make similar changes to exercise 2.]

▸**Page 245** line 14 ______ 17 Jun 1998

(6)and (7) ⇝ (6) and (7)

Page 247 line 19 ______ 29 Oct 2006

to be manipulated ⇝ to be treated

▸**Page 247** line −5 ______ 06 Jan 2011

$\le \texttt{TOP}[i]$ ⇝ $< \texttt{TOP}[i]$

Page 248 line −6 — 20 Feb 2009

`NODE(TOP[`i`])` $\leadsto$ `CONTENTS(TOP[`i`])`

▸**Page 249** line −2 — 06 Mar 1998

`NEWBASE[`j`]` $>$ `NEWBASE[`j`]` $\leadsto$ `NEWBASE[`j`]` $>$ `BASE[`j`]`

▸**Page 252** last line of exercise 5 — 13 Aug 1997

exercise 4(iii) $\leadsto$ exercise 4(c)

Page 253 lines 7 and 9 of exercise 12 — 18 Dec 1999

max (k_1, k_2) $\leadsto$ $\max(k_1, k_2)$

▸**Page 256** replacement for line 26 — 08 Nov 1997

$$\boxed{\quad\texttt{INFO}\quad}\boxed{\texttt{LINK}}\ . \qquad (3)$$

Page 258 line −5 — 13 Aug 1997

17 cycles, compared to 12 cycles $\leadsto$ 17 units of time, compared to 12 units

Page 260 replacement for line −7 — 03 May 2004

an empty queue is represented by `F` $= \Lambda$ *and* `R` $=$ `LOC(F)`.

▸**Page 260** line 4 — 11 Aug 1997

handle empty lists $\leadsto$ to handle empty lists

Page 264 lines 1, 3, 6, 12 — 15 Aug 1997

pairs $\leadsto$ relations

Page 265 line 1 — 15 Aug 1997

inputs pairs of relations $\leadsto$ inputs a sequence of relations

Page 265 formerly line 3, now line 4 — 15 Aug 1997

in linear order $\leadsto$ in a linear order

Page 266 line 22 — 03 May 2004

directly to Algorithm T. $\leadsto$ directly to Algorithm T but with slight changes for efficiency.

▸**Page 266** line −20 — 25 May 1999

`COUNT[`j`]` $\leadsto$ `COUNT[`k`]`

▸**Page 268** line −2 — 10 Aug 1997

`UNDERFLOW` $\leadsto$ `UNDERFLOW`;

▸**Page 270** line 2 of exercise 12 — 09 Mar 1998

Given two $\leadsto$ Give two

Page 270 line 2 of exercise 15 — 15 Aug 1997

irredundant pairs of relations $\leadsto$ irredundant relations

Page 273 line −3 of exercise 28 — 26 Jul 1997

E. Lucas $\leadsto$ É. Lucas

Page 273 line −1 of exercise 28 ——— 08 Jul 2005

Winning Ways **2** (Academic Press, 1982) ⤳ *Winning Ways* **3** (A. K. Peters, 2003)

▸**Page 274** line 12 ——— 17 Nov 1997

"`INFO(P)` ← `Y`″ ⤳ "`INFO(P)` ← `Y`"

▸**Page 275** second line after (4) ——— 12 Aug 1997

don't have a ⤳ there is no

Page 277 lines 2 and 3 of step M2 ——— 06 Dec 1999

"if `ABC(P)` < 0 then −1, otherwise `ABC(P)` + `ABC(M)`" ⤳ "(if `ABC(P)` < 0 then −1, otherwise `ABC(P)` + `ABC(M)`)"

▸**Page 279** exercise 12 ——— 19 Dec 2001

Algorithm A ⤳ Program A

Page 281 lines 9 and 10 ——— 03 May 2004

NODE(X) ⤳ `NODE(X)` and *X* ⤳ `X`

Page 285 step E3 ——— 20 Aug 1997

[Open door.] ⤳ [Open doors.]

Page 285 step E5 ——— 20 Aug 1997

[Close door.] ⤳ [Close doors.]

Page 287 step D2 ——— 20 Aug 1997

[Should door open?] ⤳ [Should doors open?]

Page 290 lines 047 and 049 ——— 20 Aug 1997

Indicates door open ⤳ Indicates doors open

Page 292 program line 098 ——— 25 Jan 2003

Compute `IN`, `OUT`, ⤳ Set `INFLOOR`, `OUTFLOOR`,

▸**Page 292** program lines *103* and *105* ——— 08 Jul 1998

`POOLMAX` ⤳ `POOLMAX(0:2)`

Page 293 line −14 ——— 24 Dec 2001

coroutine E ⤳ Coroutine E

Page 294 program line 201 ——— 20 Aug 1997

E3. Open door. ⤳ *E3. Open doors.*

Page 295 program line 237 ——— 20 Aug 1997

E5. Close door. ⤳ *E5. Close doors.*

▸**Page 296** line 7 ——— 08 Jul 1998

`CON` ⤳ `NOP`

▸**Page 296** line 20 ——— 14 Feb 1999

by performed ⤳ be performed

Page 296 line 22 — 08 Oct 2005

quasi-parallel $\rightsquigarrow$ *quasiparallel*

▸**Page 296** line −4 — 31 Aug 1998

216–218 $\rightsquigarrow$ 217–219

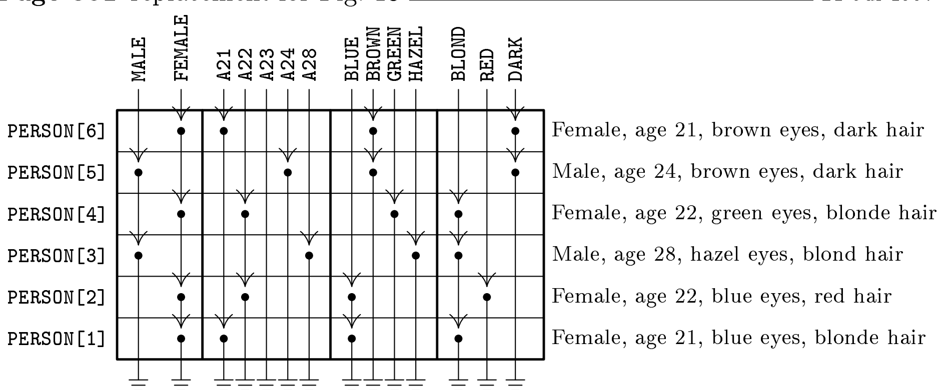
Page 298 last line of exercise 11 — 14 Feb 1999

on the same step $\rightsquigarrow$ in the same step

Page 299 line 6 — 09 Jun 2004

an array $\rightsquigarrow$ an array like (1)

▸**Page 301** replacement for Fig. 13 — 21 Jul 1997



Page 305 last line before the exercises — 06 Jan 2011

in Chapter 7 $\rightsquigarrow$ in Chapter 7 (for example, Algorithm 7B)

▸**Page 305** line 1 of exercise 2 — 01 Feb 2004

Formula (6) has $\rightsquigarrow$ Formulas (5) and (6) have

▸**Page 305** line 2 of exercise 2 — 24 Dec 2001

$l_r \le \mathtt{I} \le u_r$ $\rightsquigarrow$ $l_r \le \mathtt{I}_r \le u_r$

▸**Page 310** among the sons of Japheth — 30 Jul 1997

Meschech $\rightsquigarrow$ Meshech

Page 311 line 3 — 14 Mar 2004

was being $\rightsquigarrow$ was first being

Page 311 lines 26 and 27 — 05 Dec 2000

represents, not a person ... so-and-so." $\rightsquigarrow$ represents "a person in the role of mother or father of so-and-so," not simply a person as an individual.

Page 311 near the bottom — 29 Sep 2010

the ancestors of G are E, C, and A. $\rightsquigarrow$ the ancestors of G are E, C, and A. Sometimes, especially when talking about "nearest common ancestors," we consider a node to be an ancestor of itself (and a descendant of itself); the *inclusive ancestors* of G are G, E, C, and A, while its *proper ancestors* are just E, C, and A.

▸**Page 313** line 1 after Fig. 21 ______ 29 Nov 1997

looks very much the $\rightsquigarrow$ looks very much like the

▸**Page 313** line −3 ______ 19 Dec 2001

parent($F[a,b,c,d]$) $\rightsquigarrow$ the parent of $F[a,b,c,d]$

Page 315 line 8 before (3) ______ 28 Sep 2002

list structure $\rightsquigarrow$ *list structure*

Page 316 line 9 ______ 16 Feb 2003

lower case letters $\rightsquigarrow$ lowercase letters

Page 317 exercise 17 ______ 13 Feb 2001

If F is $\rightsquigarrow$ If Z stands for
$F[1,2,2]$ $\rightsquigarrow$ $Z[1,2,2]$

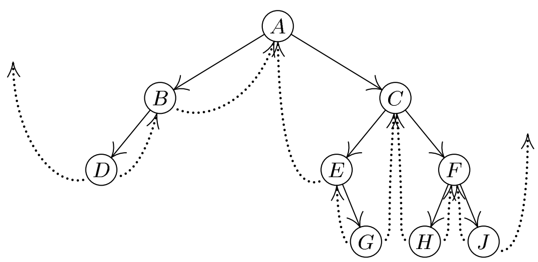
Page 317 line 2 of exercise 19 ______ 20 Aug 1997

this list? $\rightsquigarrow$ this List?

Page 318 exercise 22 ______ 13 Oct 1997

line 1: A2, ... $\rightsquigarrow$ A2, ..., An, ...
line 2: $\sqrt{2}$ to 1. $\rightsquigarrow$ $\sqrt{2}$ to 1 and whose areas are 2^{-n} square meters.

▸**Page 322** replacement for (7), has arrow $E \to G$ ______ 15 Jul 1997



(7)

▸**Page 328** line −17 ______ 31 Aug 1998

$u'_1, u'_2, \ldots, u'_n$ $\rightsquigarrow$ $u'_1, u'_2, \ldots, u'_{n'}$

▸**Page 328** line −9 ______ 24 Aug 1997

contents of $\rightsquigarrow$ complements of

▸**Page 329** equation (15) ______ 07 Jul 1998

$f(u_{l+1})$ $\rightsquigarrow$ $f(u_{n_l+1})$

Page 331 new wording for exercises 7 and 8 ______ 04 Dec 2005

7. [*22*] Show that if we are given the preorder and the inorder of the nodes of a binary tree, the binary tree structure may be constructed. (Assume that the nodes are distinct.) Does the same result hold true if we are given the preorder and postorder, instead of preorder and inorder? Or if we are given the inorder and postorder?

8. [*20*] Find all binary trees whose nodes appear in exactly the same sequence in both (a) preorder and inorder; (b) preorder and postorder; (c) inorder and postorder. (As in the previous exercise, we assume that the nodes have distinct labels.)

▸**Page 333** line 2 of exercise 31 ——— 21 Jun 2006

the tree node $\leadsto$ the tree nodes

Page 335 line 1 ——— 29 Nov 1999

45° $\leadsto$ 45° clockwise

Page 337 line 9 ——— 12 Mar 2004

tree, $\leadsto$ tree or forest,

▸**Page 342** line 5 ——— 26 Dec 2002

rI1 $\equiv$ `P` $\leadsto$ rI2 $\equiv$ `P`

▸**Page 345** line *180* of the program ——— 01 Feb 2004

`COPY2` $\leadsto$ `COPYP2`

▸**Page 346** line 1 of exercise 7 ——— 21 Jul 1997

as a , $\leadsto$ as a partial ordering,

Page 348 line 1 of exercise 20 ——— 29 Sep 2010

an ancestor $\leadsto$ a proper ancestor

▸**Page 348** line 15 from the bottom ——— 15 Jul 1997

LLINK in tree There are $\leadsto$ There are

▸**Page 349** replacement for (1) ——— 08 Aug 1997

$$\bigl(A(B,C(K)),\ D(E(H),F(J),G)\bigr) \tag{1}$$

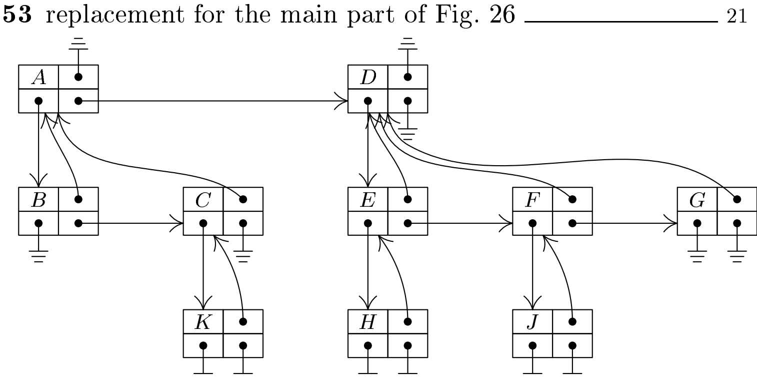
Page 351 line -2 ——— 29 Aug 1997

`P` $\leftarrow$ `P` $+ 1 \leadsto$ `P` $\leftarrow$ `P` $+ c$

▸**Page 352** lower middle ——— 26 Dec 2002

line -14: from 6 bytes to 3 $\leadsto$ from 5 bytes to 3
line -12: the latter has 60 $\leadsto$ the latter has 50

▸**Page 353** replacement for the main part of Fig. 26 ——— 21 Aug 1997



Page 354 line 1 ——— 29 Aug 1997

pairs of equivalences $\leadsto$ pairs of equivalent elements

▸**Page 355** line 7 —— 29 Aug 1997

the roots to two trees $\rightsquigarrow$ the roots of two trees

Page 360 line 2 of exercise 8 —— 29 Aug 1997

equivalences, $\rightsquigarrow$ equivalent elements,

Page 360 line 2 of exercise 9 —— 29 Aug 1997

pairs of equivalences $\rightsquigarrow$ equivalences

Page 360 line 3 of exercise 11 —— 29 Aug 1997

pairs of relations $\rightsquigarrow$ a set of relations

▸**Page 360** line 10 of exercise 11 —— 25 Jul 1997

`ARRAY X[[0:10]` $\rightsquigarrow$ `ARRAY X[0:10]`

Page 363 updated terminology for lines 5–9 of §2.3.4.1 —— 25 Jul 2008

$(V_0, V_1, \ldots, V_n)$ is a *path* ... vertex to itself. $\rightsquigarrow$ $(V_0, V_1, \ldots, V_n)$ is a *walk* of length n from V to V' if $V = V_0$, V_k is adjacent to V_{k+1} for $0 \le k < n$, and $V_n = V'$. The walk is a *path* if vertices V_0, V_1, ..., V_n are distinct; it is a *cycle* if V_0 through V_{n-1} are distinct, $V_n = V_0$, and $n \ge 3$. Sometimes we are less precise, and refer to a cycle as "a path from a vertex to itself." We often speak of a "simple path" to emphasize the fact that we're talking about a path instead of an arbitrary walk. A graph is *connected* if there is a path between any two vertices of the graph.

Page 368 lines −21, −7, −2 —— 25 Jul 2008

path $\rightsquigarrow$ walk

Page 369 lines 9, −14, −12 —— 25 Jul 2008

path $\rightsquigarrow$ walk

Page 364 line 6 —— 02 Sep 1997

edge $V_{k-1}V_k$ $\rightsquigarrow$ edge $V_{k-1} — V_k$

Page 364 line 14 —— 02 Sep 1997

For we take an arbitrary $\rightsquigarrow$ This follows because we can find some

Page 364 lines 18 and 20 —— 01 Jun 2006

a tree $\rightsquigarrow$ a free tree

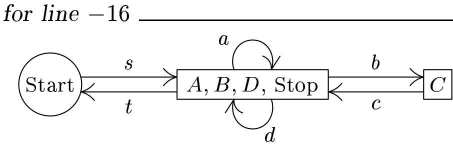
Page 366 line 1 below the caption to Fig. 32 —— 01 Jun 2006

reach a subtree $\rightsquigarrow$ reach a free subtree

▸**Page 367** line −2 —— 14 May 1999

for which e_j $\rightsquigarrow$ for which E_j

Page 370 replacement for line −16 —— 22 Jun 2004



▸**Page 371** line 4 of exercise 10 —— 02 Dec 1998

to that there $\rightsquigarrow$ so that there

Page 372 lines 19–23 —— 25 Jul 2008

is an *oriented path* ... vertex to itself. $\rightsquigarrow$ is an *oriented walk* of length n from V to V' if $V = \mathrm{init}(e_1)$, $V' = \mathrm{fin}(e_n)$, and $\mathrm{fin}(e_k) = \mathrm{init}(e_{k+1})$ for $1 \le k < n$. An oriented walk $(e_1, e_2, \ldots, e_n)$ is called *simple* if $\mathrm{init}(e_1)$, ..., $\mathrm{init}(e_n)$ are distinct and $\mathrm{fin}(e_1)$, ..., $\mathrm{fin}(e_n)$ are distinct; such a walk is an *oriented cycle* if $\mathrm{fin}(e_n) = \mathrm{init}(e_1)$, otherwise it's an *oriented path*.

Page 373 lines 18–25 —— 02 Sep 1997

Furthermore, if ... *an oriented* $\rightsquigarrow$ Furthermore, it has no cycles. For if $(V_0, V_1, \ldots, V_n)$ is an undirected cycle ... to the cycle

$$(V_1, V_0, V_{n-1}, \ldots, V_1),$$

because $V_n = V_0$. Therefore *an oriented*

Page 374 and the next few pages too —— 15 Jul 2004

[Starting with the 17th printing, the term 'Eulerian circuit' was changed to 'Eulerian trail', and 'Hamiltonian circuit' was changed to 'Hamiltonian cycle', in accordance with what has now become established usage. About thirty such changes were made, affecting pages 374, 375, 376, 379, 380, 581, and 584.]

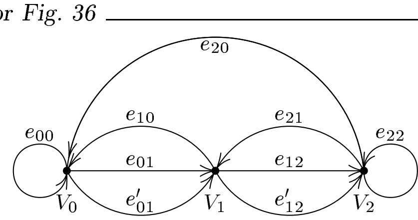
Page 374 and subsequent pages too —— 25 Jul 2008

[Starting with the 25th printing, the term 'walk' is often used formally instead of 'path', according to the definition now given on page 372, in order to conform more carefully to trends in the graph theory literature. In particular, 'path' should become 'walk' on page 374, line −24; page 375, lines −26, −24, −19, −18; page 376, lines 5, 17, −7; page 380, lines 18, −3; page 381, lines 2, 4, −24, −19, −15, −13, −11, −9; page 578, lines 4, 5; page 580, lines 1 and 2 of answer 1; page 584, lines 13 and 29.]

Page 374 lines −7 and −6 —— 25 Jul 2008

oriented path, simple oriented path $\rightsquigarrow$ oriented walk, oriented path

Page 375 replacement for Fig. 36 —— 31 Dec 1997



Page 376 lines 4, 8, 10, and 11 —— 01 Jan 2006

init $\rightsquigarrow$ init (thrice) and *fin* $\rightsquigarrow$ fin (once)

Page 376 line 3 of exercise 4 —— 22 Sep 1999

all edges e $\rightsquigarrow$ all arcs e

▸**Page 377** line −2 of exercise 10 ——— 14 May 1999

the F table $\leadsto$ the P table

Page 377 line 2 of exercise 14 ——— 25 Jul 2008

all paths $\leadsto$ all oriented walks

Page 379 in exercise 21 ——— 06 Sep 2001

line 4: $n+1$ vertices V_0, V_1, $\ldots$, V_n $\leadsto$ n vertices V_0, V_1, $\ldots$, V_{n-1}
line 5: $(n+1)m$ arcs $\leadsto$ mn arcs
line 6: $(n+1)m$ vertices $\leadsto$ mn vertices
line 10: $m^{(n+1)(m-1)}$ $\leadsto$ $m^{(m-1)n}$

Page 379 line 5 of exercise 21 ——— 26 Dec 2002

the graph $\leadsto$ the digraph

▸**Page 381** replacement for the bottom line ——— 01 Jan 2006

$$x_j = \sum_{\substack{\mathrm{init}(e)=V_i\\ \mathrm{fin}(e)=V_j}} p(e)\,x_i, \qquad \text{for } 1 \le j \le n.$$

Page 387 lines −11, −10, −9 ——— 25 Jan 2003

The weight of Y_1 . . . we have $\leadsto$ If Y is any node in the Y_1 subtree, its weight must be at least $n - s_1 = 1 + s_2 + \cdots + s_k$, since when Y is the assumed root there are at least $n - s_1$ vertices in its subtree containing X. Thus if Y is a centroid we have

Page 387 line −8 ——— 16 May 1999

weight(X) $\leadsto$ weight (X)

Page 395 line 2 after (30) ——— 26 May 2000

some of this function's $\leadsto$ this function's history and some of its

▸**Page 395** line 11 after (30) ——— 19 Mar 1998

1997 $\leadsto$ 1998

Page 395 line 2 of exercise 1 ——— 14 Aug 1997

$\dfrac{1}{2}A(z^2)$ $\leadsto$ $\frac{1}{2}A(z^2)$

Page 396 line 9 of exercise 4 ——— 14 Aug 1997

$\dfrac{1}{2}A(z^2)$ $\leadsto$ $\frac{1}{2}A(z^2)$

Page 397 line 5 ——— 14 Feb 1999

prove there $\leadsto$ prove that there

▸**Page 398** line 2 of exercise 25 ——— 27 Nov 1999

$r(n,\,n-1) = n^{n-1}$ $\leadsto$ $r(n,\,n-1) = n^{n-2}$

▸**Page 399** line 3 ——— 27 Nov 1999

$(x + \epsilon_1 z_1 + \cdots + \epsilon_n z_n)^{e_1+\cdots+e_n-1}$ $(x + \epsilon_1 z_1 + \cdots + \epsilon_n z_n)^{\epsilon_1+\cdots+\epsilon_n-1}$

▸**Page 399** line 1 of exercise 32 — 11 Dec 2002

(1940) $\rightsquigarrow$ (1941)

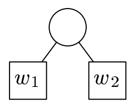
Page 399 line 2 of exercise 32 — 08 Nov 2005

7–12). $\rightsquigarrow$ 7–12.)

▸**Page 402** line −4 — 11 Sep 1998

(1951) $\rightsquigarrow$ (1952)

Page 403 replacement for (10) — 14 Sep 1997



. (10)

Page 404 line 4 and 5 — 22 Oct 2008

of such a tree is minimized if and only if the tree $\rightsquigarrow$ of a tree for the weights w_1, …, w_m that contains (10) as a subtree is minimized if and only if that tree

▸**Page 406** line 9 — 19 Aug 2002

Messenger of Math. **58** (1939) $\rightsquigarrow$ *Messenger of Math.* **58** (1929)

Page 406 line −9 — 24 Mar 2008

Kőnig $\rightsquigarrow$ König [to agree with the German spelling in the works cited]

Page 407 lines 2–4 — 29 Jan 2005

by J. von Segner … it was the subject $\rightsquigarrow$ by L. Euler, who mentioned his results in a letter to C. Goldbach on 4 September 1751 [see J. von Segner and L. Euler, *Novi Commentarii Academiæ Scientiarum Petropolitanæ* **7** (1758–1759), summary 13–15, 203–210]. Euler's problem was the subject

Page 407 lines 14 and 15 — 29 Jan 1999

to appear $\rightsquigarrow$ 1999

▸**Page 411** line −6 — 28 Aug 1997

contain a `TYPE` field $\rightsquigarrow$ contains a `TYPE` field

▸**Page 415** line 3 of step A2 — 17 Sep 1997

not at atom $\rightsquigarrow$ not an atom

▸**Page 415** line 5 of step A2 — 17 Sep 1997

`K` ← min(`K1`,`BLINK(K)`) $\rightsquigarrow$ `K1` ← min(`K1`,`BLINK(K)`)

Page 415 line 22 — 03 May 2004

"*NODE(Λ)*" $\rightsquigarrow$ "`NODE(Λ)`"

Page 418 line 3 of step E4 — 17 Sep 1997

the list $\rightsquigarrow$ the List

▸**Page 421** line 11 — 17 Sep 1997

garbage is $\rightsquigarrow$ total memory is

▸**Page 421** line −15 ——— 01 Jan 2002

499–506 $\rightsquigarrow$ 499–507

▸**Page 423** line −4 of exercise 11 ——— 08 Sep 1997

$(a\colon(b\colon D), b, a\colon E))$ $\rightsquigarrow$ $(a\colon(b\colon D), b, a\colon E)$

Page 426 line 8 ——— 26 Oct 1998

`MONTH`, `DAY` and `YEAR` $\rightsquigarrow$ `MONTH`, `DAY`, and `YEAR`

▸**Page 426** line 9 ——— 26 Dec 2002

`DAY`, `MONTH`, `YEAR` $\rightsquigarrow$ `MONTH`, `DAY`, and `YEAR`

▸**Page 428** line −9 ——— 20 Sep 1997

memory location $\rightsquigarrow$ memory allocation

▸**Page 430** line 6 ——— 08 May 2000

set `P` = `PREV(P)` $\rightsquigarrow$ set `P` ← `PREV(P)`

▸**Page 430** line −9 ——— 15 Oct 2007

$0 \le j < n$; $\rightsquigarrow$ $0 \le j < n-1$ and that α and β are parents of A_{n-1};

Page 431 line 16 ——— 31 Jan 2008

qualifications $\rightsquigarrow$ complete qualifications

▸**Page 432** line −11 ——— 20 Sep 1997

"`NAME(S)` = P_k", `NAME(P)` = `NAME(Q)`" $\rightsquigarrow$ "`NAME(S)` = P_k" and "`NAME(P)` = `NAME(Q)`"

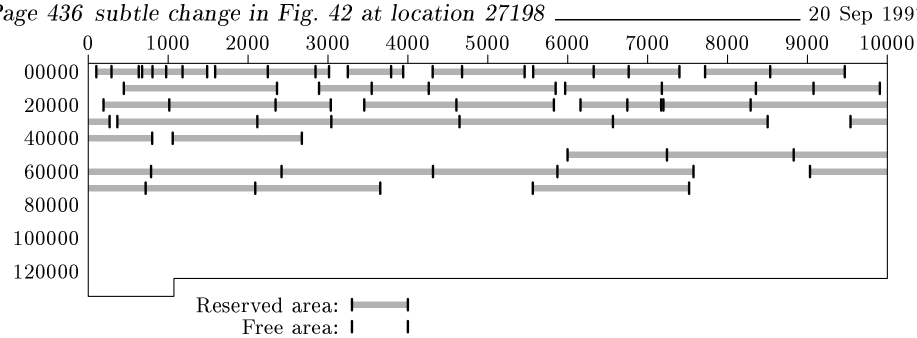
▸**Page 433** bottom line ——— 22 Jan 1999

A_{i+1} $\rightsquigarrow$ A_{j+1}

Page 435 line −11 ——— 21 Aug 2007

for a while $\rightsquigarrow$ for awhile

Page 436 subtle change in Fig. 42 at location 27198 ——— 20 Sep 1997



Page 443 line 1 ——— 10 Nov 2002

present in all blocks, $\rightsquigarrow$ present in all blocks, and which must not be tampered with by the users who reserve blocks, *Page 443 step R2* — 06 Nov 2006

`L` $\leftarrow$ `AVAILF[`j`]`, `P` $\leftarrow$ `LINKF(L)`, `AVAILF[`j`]` $\leftarrow$ `P`, `LINKB(P)` $\leftarrow$ `LOC(AVAIL[`j`])` $\rightsquigarrow$
`L` $\leftarrow$ `AVAILB[`j`]`, `P` $\leftarrow$ `LINKB(L)`, `AVAILB[`j`]` $\leftarrow$ `P`, `LINKF(P)` $\leftarrow$ `LOC(AVAIL[`j`])`

Page 444 line 16 — 20 Sep 1997

deleted $\rightsquigarrow$ *freed*

Page 444 line 22 — 03 May 2004

K $\rightsquigarrow$ `K`

▸**Page 444** bottom line — 15 Oct 2007

$1 - p \rightsquigarrow -1 + p.$

▸**Page 445** line 15 — 25 Mar 2001

BIT 20 $\rightsquigarrow$ *BIT* **20**

Page 445 lines −15 through −9 — 06 Jan 2011

[Reformat steps P1–P4 to make them like steps of a normal algorithm.]

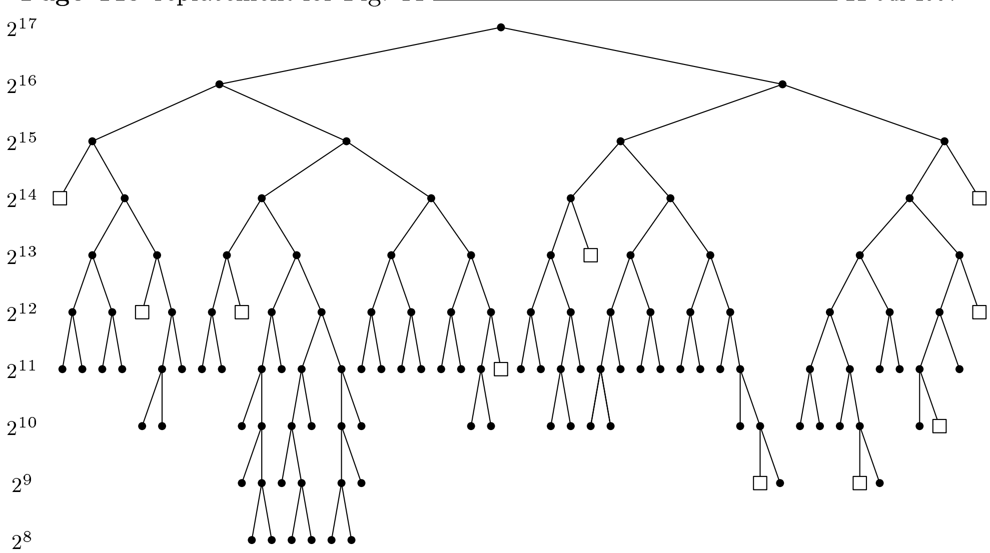
Page 447 line after the Figure — 19 Dec 2001

time distribution chosen randomly between 1 and 100 $\rightsquigarrow$ times distributed uniformly in $\{1, \ldots, 100\}$

Page 447 line −7 — 20 Sep 1997

about four thirds $\rightsquigarrow$ more than four thirds

▸**Page 448** replacement for Fig. 44 — 22 Jul 1997



▸**Page 450** line −15 — 28 Sep 2002

38, $\rightsquigarrow$ **38**

Page 450 line −15 — 11 Nov 1998

"distributed fit" method ⇝ "distributed-fit method"

Page 455 refinements to exercise 32 — 09 Nov 2002

line 1: *HM47* ⇝ *HM46*
line 2: $1 \le k < n$ ⇝ $0 \le k < n$
line 4: $f(t_1 - (n-1))$ ⇝ $f(t_0 - n)$

▸**Page 456** line 2 of exercise 39 — 10 Oct 1997

$\lim_{m\to\infty}$ ⇝ $\lim_{n\to\infty}$

▸**Page 456** line 1 of exercise 40 — 06 Nov 1997

$\lim_{n\to\infty}$ ⇝ $\lim_{m\to\infty}$

▸**Page 457** line −7 — 04 Dec 2000

insertion and deletions ⇝ insertion and deletion

Page 466 line 8 — 08 Oct 2010

Annals of Mathematics ⇝ *Annals of Mathematics* (2)

Page 467 changes to answer 9 — 01 Jan 2002

line 2: taking Ω_2 into Ω_1 ⇝
replacement for lines 4 and 5: a) If x is in I_1 then $h(g(x)) = x$.
new line to follow line 7: c) If q is in Q_2 then $h(q)$ is in Ω_1 if and only if q is in Ω_2.

Page 468 line 2 of answer 2 — 12 Oct 2008

a^{-1} ⇝ $a^{(n-1)-1} = a^{-1}$

Page 468 opening lines of answer 8 — 28 Mar 2005

(a) We will . . . an inductive proof ⇝ (a) We must show that $(n^2-n+1)+(n^2-n+3)+\cdots+(n^2+n-1)$ equals n^3: And indeed, the sum is $n(n^2-n)+(1+3+\cdots+(2n-1)) = n^3 - n^2 + n^2$, by Eq. (2). But an inductive proof

Page 468 line 1 of answer 10 — 06 Jan 1998

$(1+\frac{1}{10})^3$ ⇝ $(1+1/n)^3$

▸**Page 469** line 4 of answer 15(e) — 16 Dec 1997

$(x_1 \ldots, x_n) \prec (y_1, \ldots, y_n)$ ⇝ $(x_1, \ldots, x_n) \prec (y_1, \ldots, y_n)$

Page 470 line 1 of answer 25 — 10 Oct 2001

$z = 2^{-p}\lfloor 2^{p-k}x \rfloor$ ⇝ $z = 2^{-p}\lfloor 2^{p-k}x \rfloor > 0$

▸**Page 470** line 3 of answer 28 — 03 Feb 2002

$x \leftarrow 1-\epsilon$ ⇝ $x \leftarrow 1-\epsilon-x$

Page 471 line 6 — 13 Aug 1997

METAFONT: The Program ⇝ METAFONT*: The Program*

Page 471 new answer following line 8 — 28 Jun 2003

30. n.

Page 471 replacements for answers 1–3 ——— 04 Jan 2010

1. $-a_1$; and $a_2 + \cdots + a_1 = 0$. In general, sums with '$\cdots$' are defined so that $(a_p + \cdots + a_q) + (a_{q+1} + \cdots + a_r) = a_p + \cdots + a_r$ for arbitrary integers p, q, and r.

2. $a_1 + a_2 + a_3$.

3. $\frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11}$; $\frac{1}{9} + \frac{1}{3} + \frac{1}{1} + \frac{1}{3} + \frac{1}{9}$. The rule for $p(j)$ is violated: In the first case $n^2 = 3$ occurs for no n, and in the second case $n^2 = 4$ occurs for *two* n. [See Eq. (18).]

Page 471 answer 19 ——— 01 May 2007

$a_n - a_{m-1} \leadsto (a_n - a_{m-1})[m \le n]$

Page 472 replacement for lines 7–8 of answer 33 ——— 06 Jan 2009

where $P(x_j)$ is a polynomial of degree $n-2$ in x_j whose coefficients are symmetric functions of $\{x_1, \ldots, x_n\}$ that don't depend on j. (See exercise 1.2.9–10.) We obtain the desired answer from the solutions for $r = 0, 1, \ldots, n-1$.

Page 473 replacement for lines 7 and 8 ——— 06 Jan 2009

$$\sum_{\substack{j_1+\cdots+j_n=r-n+1\\ j_1,\ldots,j_n\ge 0}} x_1^{j_1}\ldots x_n^{j_n} = \sum_{1\le j_1\le\cdots\le j_{r-n+1}\le n} x_{j_1}\,\ldots\,x_{j_{r-n+1}};$$

see Eq. 1.2.9–(33). [J. J. Sylvester, *Quart. J. Math.* **1** (1857), 141–152.]

Page 474 new copy for the beginning of answer 40 ——— 08 Jan 2005

[A. de Moivre, *The Doctrine of Chances*, 2nd edition (London: 1738), 197–199.] We have

▸**Page 475** lines 10 and 11 ——— 15 Oct 2007

are Cauchy matrices $\leadsto$ are determinants of Cauchy matrices
not Hilbert matrices $\leadsto$ not determinants of Hilbert matrices

Page 475 last line of answer 45 ——— 13 Jan 2003

22. $\leadsto$ 22; Cauchy, *Œuvres* (2) **12**, 173–182. [Beginning with the 24th printing, the latter reference was changed to: A. Cauchy, *Exercices d'analyse et de physique mathématique* **2** (1841), 151–159.]

▸**Page 475** line 10 of answer 46 ——— 15 Dec 2001

$$\sum_{1\le k_1,\ldots,k_m\le m} \leadsto \sum_{1\le k_1,\ldots,k_m\le n}$$

▸**Page 475** bottom line ——— 15 Oct 2007

$n-1 \leadsto n-2$

Page 476 line 3 of answer 7 ——— 13 May 2005

$> 1.$ $\leadsto$ > 1; that is, if and only if $(-x) \bmod 1 + (-y) \bmod 1 < 1$.

▸**Page 477** line 7 of answer 34 ——— 16 Oct 1997

i satisfied $\leadsto$ is satisfied

▸**Page 478** line −2 of answer 37 ——— 08 Nov 2005

$0 \le k \le n \leadsto 0 < k \le n$

▸**Page 478** bottom line ——— 08 Mar 2002

$e^{2\pi y} \rightsquigarrow e^{2\pi i y}$

Page 479 displayed formula in answer 42 ——— 11 Aug 2005

$k{+}1$ is a power of b $\rightsquigarrow$ $k{+}1$ is a power of b

▸**Page 480** line 2 of answer 13 ——— 08 Nov 2005

Law 1.2.4D $\rightsquigarrow$ Law 1.2.4B

▸**Page 481** line 9 of answer 20 ——— 02 Dec 1997

$\dfrac{1}{m}\displaystyle\int_0^m t^{x+1}e^{-t} \rightsquigarrow \dfrac{1}{m}\int_0^m t^{x+1}e^{-t}\,dt$

▸**Page 483** line 1 of answer 25 ——— 21 Jan 2001

$x^{\overline{m+n}} = x^{\overline{m}}(x-m)^{\overline{n}} \rightsquigarrow x^{\overline{m+n}} = x^{\overline{m}}(x+m)^{\overline{n}}$

Page 484 last line of answer 10 ——— 09 Jan 2003

N. J. Fine $\rightsquigarrow$ L. E. Dickson, *Quart. J. Math.* **33** (1902), 383–384; N. J. Fine

▸**Page 485** replacement for answer 21 ——— 07 Jun 2003

21. The left-hand side is a polynomial of degree $\le n$; the right-hand side is a polynomial of degree $m+n+1$. The polynomials agree at $n+1$ points, but that isn't enough to prove them equal. [In fact, the correct formula in general is

$$\sum_{k=0}^{r}\binom{r-k}{m}\binom{s+k}{n} = \binom{r+s+1}{m+n+1} - \sum_{k=0}^{m}\binom{r+1}{k}\binom{s}{m+n+1-k}$$

when m, n, and r are nonnegative integers.]

▸**Page 485** line 1 of answer 22 ——— 24 May 2000

The kth term is $\rightsquigarrow$ The kth term is r times

▸**Page 486** line 2 ——— 24 May 2000

$tf(r-t-1,\ s+t,\ t,\ n-1) \rightsquigarrow tf(r+t-1,\ s-t,\ t,\ n-1)$

Page 486 line 1 of answer 33 ——— 18 Jan 2005

[*Giornale* ... may therefore be $\rightsquigarrow$ [*Mém. Acad. Roy. Sci.* (Paris, 1772), part 1, 492; C. Kramp, *Élémens d'Arithmétique Universelle* (Cologne: 1808), 359; *Giornale di Mat. Battaglini* **33** (1895), 179–182.] Since $x^{\overline{n}} = n!\binom{x+n-1}{n}$, the equation may be

Page 486 line −3 ——— 14 Jul 1997

$\sum_k \binom{n}{k} x(x-kz-1)^{\underline{k-1}}(y+kz)^{\underline{n-k}}.$ $\rightsquigarrow$ $\sum_k \binom{n}{k} x(x-kz-1)^{\underline{k-1}}(y+kz)^{\underline{n-k}}$, an equivalent formula of Rothe [*Formulæ de Serierum Reversione* (Leipzig: 1793), 18].

Page 487 line 9 of answer 38 ——— 11 Oct 2001

formula. $\rightsquigarrow$ formula. [See *Crelle* **11** (1834), 353–355.]

▸**Page 489** answer 52 ——— 26 Jan 1998

line 4: $f(x,y,r) \rightsquigarrow f(x,y,n)$
line 5: $rf(x,\ y,\ n-1) \rightsquigarrow nf(x,\ y,\ n-1)$

Page 489 replacement for answer 56 ——— 31 May 2002

56. 210 310 320 321 410 420 421 430 431 432 510 520 521 530 531 532 540 541 542 543 610. With a fixed, b and c run through the combinations of a things two at a time; with a and b fixed, c runs through the combinations of b things one at a time.

Similarly, we could express all numbers in the form $n = \binom{a}{4} + \binom{b}{3} + \binom{c}{2} + \binom{d}{1}$ with $a > b > c > d \geq 0$; the sequence begins 3210 4210 4310 4320 4321 5210 5310 5320 . . . . We can find the combinatorial representation by a "greedy" method, first choosing the largest possible a, then the largest possible b for $n - \binom{a}{4}$, etc. [Section 7.2.1.3 discusses further properties of this representation.]

Page 489 replacement for first three lines of answer 58 ——— 12 Jun 2003

58. [*Systematisches Lehrbuch der Arithmetik* (Leipzig: 1811), xxix.] Use induction and

$$\binom{n}{k}_q = \binom{n-1}{k}_q + \binom{n-1}{k-1}_q q^{n-k} = \binom{n-1}{k}_q q^k + \binom{n-1}{k-1}_q.$$

Therefore [F. Schweins, *Analysis* (Heidelberg: 1820), §151] the q-generalization of (21) is

Page 490 replacement for lines 5–7 ——— 12 Jun 2003

For further information, see G. Gasper and M. Rahman, *Basic Hypergeometric Series* (Cambridge Univ. Press, 1990). The q-nomial coefficients were introduced by Gauss in *Commentationes societatis regiæ scientiarum Gottingensis recentiores* **1** (1808), 147–186; see also Cauchy [*Comptes Rendus Acad. Sci.* **17** (Paris, 1843), 523–531], Jacobi [*Crelle* **32** (1846), 197–204], and Heine [*Crelle* **34** (1847), 285–328].

▶**Page 491** line 2 ——— 28 Jan 1998

$q^{m-r+s-k)(n-k)}$ ⇝ $q^{(m-r+s-k)(n-k)}$

Page 491 replacement for answer 66 ——— 08 Nov 2005

66. Let $X = \binom{x}{n}$, $\underline{X} = \binom{x}{n-1} = \frac{n}{x-n+1}X$, $\overline{X} = \binom{x}{n+1} = \frac{x-n}{n+1}X$, with similar notations for Y and Z. We may assume that $y > n-1$ is fixed, so that x is a function of z.

Let $F(z) = \overline{X} - \overline{Y} - \overline{Z}$, and suppose that $F(z) = 0$ for some $z > n-2$. We will prove that $F'(z) < 0$; therefore $z = y$ must be the only root $> n-2$, proving the second inequality. Since $F(z) = \frac{x-n}{n+1}(Y+Z) - \frac{y-n}{n+1}Y - \frac{z-n+1}{n}Z = 0$ and $x > y$ and $Y, Z > 0$, we must have $\frac{x-n}{n+1} < \frac{z-n+1}{n}$. Setting $X' = dX/dx$ and $Z' = dZ/dz = dX/dz$, we have

$$\frac{X'}{X} = \frac{1}{x} + \frac{1}{x-1} + \cdots + \frac{1}{x-n+1} > \frac{n}{n+1}\left(\frac{1}{z} + \cdots + \frac{1}{z-n+2}\right) = \frac{n}{n+1}\frac{Z'}{Z},$$

since $\frac{x-n+1}{n+1} < \frac{z-n+2}{n}$, . . . , $\frac{x-1}{n+1} < \frac{z}{n}$. Thus $dx/dz = Z'/X' < \frac{n+1}{n}(Z/X)$, and

$$F'(z) = \frac{X}{n+1}\frac{dx}{dz} + \frac{x-n}{n+1}Z' - \frac{Z}{n} - \frac{z-n+1}{n}Z' < \left(\frac{x-n}{n+1} - \frac{z-n+1}{n}\right)Z' < 0.$$

To prove the first inequality, we may assume that $n > 2$. Then if $\underline{X} = \underline{Y} + \underline{Z}$ for some $z > n-2$, the second inequality tells us that $z = y$.

References: L. Lovász, *Combinatorial Problems and Exercises*, Problem 13.31(a); R. M. Redheffer, *AMM* **103** (1996), 62–64.

Page 491 line −2 of that answer ——— 03 Nov 2008

Exercises, ⇝ *Exercises* (1993),

▸**Page 493** line 2 of answer 12 ———— 13 Feb 1998

$\mathcal{F}(z) \leadsto \mathcal{G}(z)$

Page 493 answer 14 ———— 25 Jun 2006

$F_{m+1}F_{n-1} + (F_{m+2}+1)F_n \leadsto F_{m+n+1} + F_n$

Page 493 last line of answer 19 ———— 21 Jun 2007

$\sin 72^\circ = \frac{1}{2}5^{1/4}\phi^{1/2}$.) $\leadsto$ $\sin 72^\circ = \frac{1}{2}5^{1/4}\phi^{1/2}$. Another interesting angle is $\alpha = \arctan\phi = \frac{\pi}{4} + \frac{1}{2}\arctan\frac{1}{2}$, for which we have $\sin\alpha = 5^{-1/4}\phi^{1/2}$, $\cos\alpha = 5^{-1/4}\phi^{-1/2}$.)

Page 494 replacement for line 4 ———— 26 Jan 2004

$$(n+1-x^nF_{n+1})/(2x+1).$$

Page 495 line 6 of answer 34 ———— 05 Feb 2005

182]; generalizations $\leadsto$ 182]; but Section 7.2.1.7 points out that it was implicitly known in 14th-century India. Generalizations

Page 495 bottom line of answer 34 ———— 12 Oct 2006

exercise 5.4.2–10 $\leadsto$ exercise 5.4.2–10 and in Section 7.1.3.

▸**Page 495** line 7 of answer 35 ———— 16 Nov 1997

$m - \phi^{2n}$, $\leadsto$ $m - \phi^{-2n}$,

Page 495 line 8 of answer 35 ———— 16 Nov 1997

$\phi^{k-1} + \cdots + \phi^{k-2j+1} \leadsto \phi^{k-1} + \phi^{k-3} + \cdots + \phi^{k-2j+1}$

Page 495 line 7 of answer 36 ———— 23 Aug 2002

Jean Bernoulli $\leadsto$ John Bernoulli

▸**Page 496** line 2 of answer 5 ———— 01 Jan 2006

$(n-1)! \leadsto \dfrac{(n-1)!}{k!}$

Page 496 answer 10 ———— 18 Aug 2002

[Change the notation from 'a_m' to the more modern 'e_m', throughout this exercise and also in the answer to exercise 11 on page 497.]

▸**Page 498** line 3 ———— 01 Jan 2006

$f(x) \leadsto \lim_{x\to 1-} f(x) = f(1)$

▸**Page 498** replacement for line 8 ———— 01 Jan 2006

$$\frac{i}{2} + \frac{i}{(\omega^{-p}-1)} = \frac{i}{2}\left(\frac{1+\omega^p}{1-\omega^p}\right) = -\frac{i}{2}\left(\frac{\omega^{p/2}+\omega^{-p/2}}{\omega^{p/2}-\omega^{-p/2}}\right) = -\frac{1}{2}\cot\frac{p}{q}\pi.$$

▸**Page 498** line 2 of answer 21 ———— 15 Apr 2002

$E_1(x) \leadsto E_1(z)$

▸**Page 498** line 1 of answer 23 ———— 16 Dec 2001

When $m=1 \leadsto$ (a) When $m=1$

▸**Page 500** replacement for line 2 of answer 6 ———— 01 Aug 1997

$$\ln(q+pe^t) = \ln\left(1+pt+\frac{pt^2}{2}+\frac{pt^3}{6}+\cdots\right) = pt + p(1-p)\frac{t^2}{2} + p(1-p)(1-2p)\frac{t^3}{6} + \cdots$$

▸**Page 500** line 6 of answer 6 ——— 11 Jan 2002

$H'(z) = e^t/(e^t-1) - 1/t \leadsto H'(t) = e^t/(e^t-1) - 1/t$

▸**Page 500** line 1 of answer 8 ——— 30 Jul 1999

$M^{\underline{n}}/((M-n)!\,M^n) \leadsto M^{\underline{n}}/M^n$

▸**Page 501** last line of answer 13 ——— 01 Aug 1997

n and x.] $\leadsto$ n and x.

Page 501 line 1 of answer 14 ——— 01 Aug 1997

$e^{-itpn/\sqrt{pqn}}(q+pe^{it/\sqrt{pqn}})^n = (qe^{-itp/\sqrt{pqn}} + pe^{itq/\sqrt{pqn}})^n \leadsto$
$e^{-itpn/\sqrt{pqn}}(q+pe^{it/\sqrt{pqn}})^n = (qe^{-itp/\sqrt{pqn}} + pe^{itq/\sqrt{pqn}})^n$

▸**Page 501** line 2 of answer 15 ——— 25 Mar 2001

$+-t^2/2np \leadsto -t^2/(2np)$

Page 502 line 1 of answer 21 ——— 10 Sep 1997

[There's slightly too much space before the word "The".]

▸**Page 502** line 2 of answer 21 ——— 01 Aug 1997

$\Pr(X \ge n(p+\epsilon) \leadsto \Pr(X \ge n(p+\epsilon))$

▸**Page 502** line 3 of answer 21 ——— 05 Jul 1999

$-\frac{1}{2q}\epsilon^2 \leadsto \epsilon - \frac{1}{2q}\epsilon^2$

▸**Page 502** line 2 of answer 22 ——— 22 Jan 1999

493–509 $\leadsto$ 493–507

▸**Page 502** line 4 of answer 22 ——— 05 Jul 1999

$\le e^{\delta^2/2}$ when $\delta \le 0$ and $\le e^{\delta^2/3} \leadsto \le e^{-\delta^2/2}$ when $\delta \le 0$ and $\le e^{-\delta^2/3}$

Page 502 line 1 of answer 23 ——— 10 Sep 1997

[There's slightly too much space before the word "Setting".]

▸**Page 502** line 4 of answer 23 ——— 27 Oct 1998

$f(\epsilon) \le -\epsilon^2/(6pq) \leadsto$ that $f(\epsilon) \le -\epsilon^2/(6pq)$ if $0 \le \epsilon \le p$.

Page 503 answer 8 ——— 26 May 1998

the previous $\leadsto$ the method of the previous

▸**Page 503** line 1 of answer 9 ——— 05 May 2003

$f(x) \leadsto f(z)$

▸**Page 503** line 1 of answer 9 ——— 26 Feb 2008

$e^{f(z)} \leadsto |e^{f(z)}|$

▸**Page 503** line 2 of answer 12 ——— 30 Jul 1999

$O(n^k/(n-1)!) \leadsto O(n^k/(k-1)!)$

▸**Page 503** answer 1 ——— 01 Aug 1997

$B_0 + B_1z + B_2z^2/2! + \cdots)e^z \leadsto (B_0 + B_1z + B_2z^2/2! + \cdots)e^z$

Page 504 lines 2 and 3 ———— 01 Jan 2006

$\mathrm{ln}(n-1)! \rightsquigarrow \ln\,(n-1)!$

Page 504 line 5 of answer 7 ———— 20 May 2005

The number A is "Glaisher's constant" 1.2824271 . . . [*Messenger* $\rightsquigarrow$ In these formulas, A is the "Kinkelin–Glaisher constant" 1.2824271 . . . [*Crelle* **57** (1860), 122–158; *Messenger*

▶**Page 504** line 3 of answer 8 ———— 01 Jan 2006

$+\,\sigma \rightsquigarrow +\,\frac{1}{2}\ln a+\sigma$

Page 504 line 4 of answer 8 ———— 01 Jan 2006

$\mathrm{ln}(cn^2)! - \mathrm{ln}(cn^2-n)! - n\ln c - \ln n^2! + \mathrm{ln}(n^2-n)! \rightsquigarrow$
$\ln\,(cn^2)! - \ln\,(cn^2-n)! - n\ln c - \ln n^2! + \ln\,(n^2-n)!$

Page 504 line 1 of answer 9 ———— 01 Jan 2006

$\mathrm{ln}(2n)! \rightsquigarrow \ln\,(2n)!$ and $\mathrm{ln}(n!)^2 \rightsquigarrow \ln\,(n!)^2$

▶**Page 504** line 3 of answer 9 ———— 24 Mar 1998

$O(n^{-3}) = \rightsquigarrow O(n^{-3})) =$

▶**Page 505** second display in answer 9 ———— 17 Oct 2007

$x^z \rightsquigarrow x^x$

▶**Page 506** lines 3 and 4 ———— 10 Aug 2008

$\displaystyle\sum_{k,m} \rightsquigarrow \sum_{k,m\geq 0}$ (twice)

Page 506 line −1 of answer 11 ———— 10 Aug 2008

$(-1)^n n!/e^n n^n \rightsquigarrow (-1)^n n!/(e^n n^n)$

▶**Page 507** line 4 of answer 20 ———— 01 Jan 2006

$\sum_{k=1}^{m-1} c_k(2u)^{k/2-1} \rightsquigarrow \sum_{k=1}^{m-1} kc_k(2u)^{k/2-1}$

Page 507 line 2 of answer 6 ———— 20 Apr 1998

into an index $\rightsquigarrow$ into an index

▶**Page 507** line 2 of answer 7 ———— 28 Sep 2002

$|V| \rightsquigarrow |\mathrm{V}|$

Page 508 line 4 of answer 14 ———— 25 Mar 2003

`JSJ` $\rightsquigarrow$ `STJ *(0:0)`, `STZ *(0:0)`, and `STZ *(3:3)`; `JSJ`

Page 508 line 2 of answer 21 ———— 20 Apr 1998

only by jumping $\rightsquigarrow$ only by jumping

▶**Page 509** line 3 ———— 03 May 1999

a sign $\rightsquigarrow$ the sign

Page 509 line 18 ———— 28 Sep 1997

time $= 54u$. $\rightsquigarrow$ time $= 54u$, not counting the `HLT`.

Page 509 lines −12 through −9 —— 17 Nov 2007

`(3495)` $(-5)^{13}$ [This line needed only when $b > 65$.]
`(3496)` $(-4)^{13}$
⋮
`(3505)` $(+5)^{13}$ [This line needed only when $b > 65$.] ▮

▶**Page 510** line 03 of the program —— 21 Nov 1998

`␣C␣04` $\rightsquigarrow$ `␣C␣04`

▶**Page 510** line −2 —— 30 Dec 1997

`␣␣␣B.` $\rightsquigarrow$ `␣J␣B.`

▶**Page 512** right column, line 12 of the program —— 25 Aug 2002

`GOOD` $\rightsquigarrow$ `MEMORY`

Page 512 right column, program lines 18–19 and 24–25 —— 06 Dec 2008

`CMPA =5=; JG BAD` $\rightsquigarrow$ `DECA 5; JAP BAD`
`CMPX =3999=; JLE GOOD` $\rightsquigarrow$ `DECX 3999; JXNP GOOD`

Page 513 changes for consistency —— 29 Oct 1997

line 1: *Solution 1:* $\rightsquigarrow$ *Solution 1:*
line −12: *Solution 2:* $\rightsquigarrow$ *Solution 2:*

▶**Page 513** line −8 —— 22 Mar 1999

$R(j) \le a_{ij}$ $\rightsquigarrow$ $R(i) \le a_{ij}$

Page 514 end of the program —— 18 May 1999

program lines −3 and −2: interchange '`ENT1 0`' with '`J3P 3B`'
line 6 after the program: $540u$ $\rightsquigarrow$ $530u$

Page 514 line −6 —— 25 Aug 2002

we can solve $\rightsquigarrow$ and $m \ge n$, we can solve

▶**Page 515** line 14 —— 31 Jul 1997

in 924744796234036231 our case, $\rightsquigarrow$ in our case,

Page 515 new answer —— 24 Apr 1997

12. M. Hofri and P. Jacquet [*Algorithmica* **22** (1998), 516–528] have analyzed the case when the $m \times n$ matrix entries are distinct and in random order. The running times of the two `MIX` programs are then respectively $\bigl(6mn+5mH_n+8m+6+5(m+1)/(n-1)\bigr)u+O\bigl((m+n)^2/\binom{m+n}{m}\bigr)$ and $(5mn+2nH_m+7m+7n+9H_n)u+O(1/n)+O\bigl((\log n)^2/m\bigr)$, as $m \to \infty$ and $n \to \infty$, assuming that $(\log n)/m \to 0$.

Page 516 answer 14 —— 31 Aug 1998

[Udo Wermuth has indeed squeezed out an additional $9u$, $10u$, or $12u$, depending on the year. But details are omitted here because `MIX` will soon be replaced.]

Page 518 lines 18 and 19 —— 10 Jan 2003

Calendrical Calculations ... 1997) $\rightsquigarrow$ *Calendrical Calculations* by E. M. Reingold and N. Dershowitz (Cambridge Univ. Press, 2001)

▸**Page 519** line 2 of answer 17 — 20 May 1999

$H_n \rightsquigarrow H_N$ and $H_{2m} \rightsquigarrow 2H_{2m}$

▸**Page 520** line −3 of answer 19 — 25 Nov 1997

Sociéte $\rightsquigarrow$ *Société*

▸**Page 521** replacement for line 11 — 20 Jan 2008

```
LDA  *     4  Waste 2u of time.
```

▸**Page 522** lines 1 and 2 of answer 6 — 24 Oct 2005

The tital time is increased ... name. $\rightsquigarrow$ The total time decreases by $8u$ for every blank word following a "(", because lines 30–32 cost $4u$ while lines 26–28, 33–34, 36–38 cost $12u$. It decreases by $2u$ for every blank word following a name, because lines 68–71 cost $5u$ while 42–46 or 75–79 cost $7u$.

Page 524 line 1 of answer 17 — 26 Feb 2008

The probability $\rightsquigarrow$ The probability p_m

Page 524 line 2 of answer 17 — 24 Dec 2001

$(m/mH_n) \rightsquigarrow m/(mH_n)$

▸**Page 524** answer 19 — 15 Sep 1999

$1/ \rightsquigarrow n!/$ (in three places)

▸**Page 524** lines 1, 3, and 9 of answer 22 — 01 Jan 2006

${}_{j\geq 0} \rightsquigarrow {}_{j>0}$ (four times)

Page 525 lines 4 and 5 of answer 23 — 04 Nov 2003

[to appear] $\rightsquigarrow$ [Ph.D. thesis, École Polytechnique (Paris, 1996)]

▸**Page 525** answer 24 — 24 Dec 2001

Proc. IFIP Congress (1971) $\rightsquigarrow$ *Proc. IFIP Congress* (1971)

Page 526 new answer — 14 Oct 2008

28. See I. N. Herstein and I. Kaplansky, *Matters Mathematical* (1974), §3.5.

Page 526 new sentence for answer 29 — 07 Apr 2003

Incidentally, the original number of the kth man executed is $2n+1-(2n+1-2k)2^{\lfloor \lg(2n/(2n+1-2k))\rfloor}$. [Armin Shams, *Proc. Nat. Computer Conf. 2002*, English papers section, **2** (Mashhad, Iran: Ferdowsi University, 2002), 29–33.]

Page 526 new answer 31 — 14 Oct 2008

See *CMath*, Section 3.3. Let $x_0 = jm$ and $x_{i+1} = (m(x_i - n) - d_i)/(m-1)$, where $1 \leq d_i < m$. Then $x_k = j$ if and only if $a_k j = b_k n + t_k$, where $a_k = m^{k+1} - (m-1)^k$, $b_k = m(m^k - (m-1)^k)$, and $t_k = \sum_{i=0}^{k-1} m^{k-1-i}(m-1)^i d_i$. Since $a_k \perp b_k$ and the $(m-1)^k$ possibilities for t_k are distinct, the average number of k-step fixed elements is $(m-1)^k/a_k$.

Page 527 last line of answer 34 — 26 Oct 1997

efficient. $\rightsquigarrow$ efficient. [W. Fletcher and R. Silver, *CACM* **9** (1966), 326.]

▸**Page 527** lines −3 and −2 of answer 35 — 01 Jan 2006

interchange γ'' with $\beta\gamma'$ $\rightsquigarrow$ interchange $\gamma''\beta$ with γ'

Page 527 new answer ——— 29 Jan 2010

37. The result is clear when $n \le 2$. Otherwise we can find $a, b < n$ such that π takes a to b. Then $(n\,a)\,\pi\,(b\,n) = (\alpha\,a)(b\,\beta)$ for $(n-1)$-cycles $(\alpha\,a)$ and $(b\,\beta)$ if and only if $\pi = (n\,\alpha\,a)(b\,\beta\,n)$. [See A. Jacques, C. Lenormand, A. Lentin, and J.-F. Perrot, *Comptes Rendus Acad. Sci.* **266** (Paris, 1968), A446–A448.]

▸**Page 529** lines 2–7 of answer 3 ——— 17 Jul 2006

But if … program.) $\rightsquigarrow$ But the comparison indicator isn't initialized; and if the final period is preceded by a replication digit, it won't be noticed. [*Note:* The most nitpickingly efficient program would probably remove lines 40, 44, and 48, and would insert "`CMPA PERIOD`" between lines 26 and 27, "`CMPX PERIOD`" between lines 59 and 60. The state of the comparison indicator should then become part of the coroutine characteristics in the program documentation.]

▸**Page 529** line −4 ——— 28 Dec 1997

is not to invent $\rightsquigarrow$ is not hard to invent

▸**Page 531** in the right-hand column of answer 2 ——— 03 Sep 2008

```
..............lines 14–31
    LD1  I1REG
..............lines 32–35
```

$\rightsquigarrow$

```
    LD1  I1REG
..............lines 14–35
```

▸**Page 532** lines 3–8 of answer 3, right column ——— 04 Sep 2008

```
BUFMAX  CON  ENDBUF1
* BUFFER AREAS
OUTBUF1 ORIG *+100
ENDBUF1 CON  ENDBUF2
OUTBUF2 ORIG *+100
ENDBUF2 CON  ENDBUF1  ▌
```

$\rightsquigarrow$

```
* BUFFER AREAS
OUTBUF1 ORIG *+100
ENDBUF1 CON  *+101  (ENDBUF2)
OUTBUF2 ORIG *+100
ENDBUF2 CON  ENDBUF1
BUFMAX  CON  ENDBUF1  ▌
```

▸**Page 532** line 1 of answer 7 ——— 16 Jul 1998

One way $\rightsquigarrow$ One way is

▸**Page 534** new instruction to follow '`J1P 1B`' in answer 15 ——— 05 Sep 2008

`JBUS *(U)`

▸**Page 534** line 7 of answer 18 ——— 11 Mar 1998

goint $\rightsquigarrow$ going

Page 534 in answer 18 ——— 06 Jan 2011

[The revised algorithms are now called A′, R′, and B′, to distinguish them from Algorithms A, R, and B of the text.]

▸**Page 534** line 10 of answer 19 ——— 24 Dec 2001

1339–1342 $\rightsquigarrow$ 1338–1342

▸**Page 536** line 1 of answer 4 ——— 01 Jan 2009

D. André (1878): $\rightsquigarrow$ J. Aebly and D. Mirimanoff [*L'Enseignement Math.* **23** (1923), 185–189]:

Page 536 line −13 ——— 17 Mar 1998

Abraham De Moivre $\rightsquigarrow$ Abraham de Moivre

▸**Page 537** line −4 ——— 01 Jan 2006

$A_m(n,-2)$ ⇝ $A_m(n+1,-2)$

▸**Page 539** answer 12 ——— 17 Nov 1999

line 3: We have ⇝ We have, for sufficiently small α,
replacement for line 7:

$$(-1)^n\binom{k+1/2}{n} = \binom{n-k-3/2}{n} = \frac{\Gamma(n-k-1/2)}{\Gamma(n+1)\Gamma(-k-1/2)} = \frac{(-1/2)^{\underline{k+1}}}{\sqrt{\pi}n}n^{\overline{-k-1/2}},$$

replacement for line 11:

$$c_j = \sqrt{\frac{1-\alpha}{\pi}}\sum_{k=0}^{j}\binom{1/2}{k}(-1/2)^{\underline{k+1}}\left\{ {j+1/2 \atop k+1/2} \right\}\frac{\alpha^k}{(1-\alpha)^k}.$$

▸**Page 540** line 4 of answer 3 ——— 18 Jun 1998

`LDA 2,7:1` ⇝ `LDA X2,7:1`

▸**Page 540** answer 4 ——— 13 Aug 1997

(i) (ii) (iii) (iv) (v) ⇝ (a) (b) (c) (d) (e)

Page 541 line 11 ——— 05 Oct 1999

`N`$\leftarrow 0$ ⇝ `N` $\leftarrow 0$

▸**Page 542** line 2 of answer 10 ——— 10 Sep 2008

$\sum_{1\le j\le k\le n}$ ⇝ $\sum_{1\le j<k\le n}$

▸**Page 542** new answer for exercise 11 ——— 04 Oct 2005

11. Counting as before, we find that the expected number is

$$E_{mnt} = \frac{1}{n^m}\binom{n}{2}\sum_{k=1}^{m}\sum_{r\ge t}(k-1)\binom{k-2}{r}(n-1)^{k-2-r}n^{m-k};$$

here r is the number of entries in $a_1, a_2, \ldots, a_{k-1}$ that equal a_k. This quantity can also be expressed in the simpler form

$$E_{mnt} = \frac{1}{n^m}\binom{n}{2}\sum_{k>t}\binom{m}{k}(n-1)^{m-k}\left(\binom{k}{2}-\binom{t+1}{2}\right), \quad \text{for } t\ge 0.$$

Is there a simpler way yet to give the answer? Apparently not, since the generating function for given n and t is

$$\sum_m E_{mnt}z^m = \frac{n-1}{2n}\frac{z}{(1-z)^3}\left(\frac{z}{n-(n-1)z}\right)^{t+1}\bigl(z+(1-z)n(t+1)\bigr).$$

▸**Page 543** line 1 of answer 14 ——— 18 Nov 1999

$n/m+\sqrt{n}\,x_j$ ⇝ $m/n+\sqrt{m}\,x_j$

▸**Page 543** line 5 of answer 14 ——— 11 Sep 2008

$(\sqrt{m})^{1-m}$ ⇝ $(\sqrt{m})^{1-n}$

▸**Page 543** line −2 ——— 18 Nov 1999

$I_1 = I_2/n$ ⇝ $I_1 = I_2/n^2$

▶**Page 544** line 6 ——— 18 Nov 1999

$3 \le j \le m \leadsto 3 \le j \le n$

▶**Page 544** line 3 of answer 17 ——— 20 Feb 2009

less $\leadsto$ more

▶**Page 547** line 3 of answer 13 ——— 13 May 2001

$2^{2^n(n+O(1))}$ by a factor of $2^{2^n+O(n)}$ $\leadsto$ $2^{2^n(n+O(\log n))}$ by a factor of $e^{2^n+O(n)}$

Page 549 line 12 ——— 28 Sep 2002

alpha. $\leadsto$ alphameric.

▶**Page 550** answer 27 ——— 26 Feb 2008

line 10: `J2Z B1` $\leadsto$ `J2Z 2F`
line 18: `LD1 N` $\leadsto$ `2H LD1 N`

▶**Page 550** line −14 ——— 26 Jul 1997

those move $\leadsto$ those moves

Page 555 line 04 of answer 9 ——— 20 Aug 1997

D2. Should door open? $\leadsto$ *D2. Should doors open?*

Page 555 line 14 of answer 9 ——— 04 Jul 2001

are zero $\leadsto$ are zero.

Page 555 last line of answer 9 ——— 03 Aug 1998

subroutine. $\leadsto$ subroutine. ▮

▶**Page 556** line 1 of answer 5 ——— 28 Sep 2002

`A0` $\leadsto$ `A0`

▶**Page 557** replacement for lines 2 and 3 ——— 25 Oct 1997

in `X[I`$_{a_1} + B_1$`,I`$_{a_2} + B_2$`, ..., I`$_{a_m} + B_m$`]`, where $B_1B_2 \ldots B_m$ is an inversion table for $a_1a_2 \ldots a_m$ as defined in exercise 5.1.1–7.)

▶**Page 557** line 1 of answer 15 ——— 24 Dec 2001

rI1 ≡ `PIVOT`, J, rI2 ≡ `P0`, rI3 ≡ `Q0`, rI4 ≡ `P`, rI5 ≡ `P1`, `X`; $\leadsto$ rI1 ≡ `PIVOT`, `J`; rI2 ≡ `P0`; rI3 ≡ `Q0`; rI4 ≡ `P`; rI5 ≡ `P1`, `X`;

▶**Page 560** line 4 of answer 21 ——— 01 Jan 2006

$\mathtt{L}_{r-1} \leadsto \mathtt{L}_{r-1} + (\mathtt{M}_r - 1)^r$

Page 561 line 1 of answer 6 ——— 30 Aug 1997

[There's slightly too much space before the word "Let".]

▶**Page 561** line −2 ——— 19 Dec 2001

$Z_1 \cap Z_2 \ne 0 \leadsto Z_1 \cap Z_2 \ne \emptyset$

Page 562 line 1 ——— 29 Sep 2010

descendant of X, if $\leadsto$ descendant of X (that is, a *proper* ancestor or descendant), if

▶**Page 562** new answer 13 ——— 15 Oct 2007

13. $a_1.a_2.\cdots.a_k,\ \ a_1.a_2.\cdots.a_{k-1},\ \ \ldots,\ \ a_1.a_2,\ \ a_1.$

Page 563 answer 17 ——— 13 Feb 2001

$F \rightsquigarrow Z$ (three times)

Page 564 line 6 ——— 03 Feb 1998

2-D trees ⇝ 2-d trees

Page 564 line 8 of answer 5 ——— 03 Sep 1997

level order; ⇝ *level order*;

▸**Page 565** line 9 of answer 11 ——— 13 Nov 1999

$+\dfrac{11}{24}\sqrt{\dfrac{\pi}{n}} \rightsquigarrow -\dfrac{13}{24}\sqrt{\dfrac{\pi}{n}}+\dfrac{1}{2n}$

Page 565 line 1 of answer 12 ——— 14 Jan 2000

between step T2 and T3 ⇝ between steps T2 and T3
between step T4 and T2 ⇝ in step T5

Page 565 line 6 of answer 13 ——— 20 Apr 2001

"Visit" ⇝ Visit

Page 565 in answer 13 ——— 06 Jan 2011

[The revised algorithm is now called T′, to distinguish it from Algorithm T of the text.]

Page 568 line 6 ——— 01 Jan 2006

will triple traverse the tree ⇝ will traverse the tree in triple order

Page 568 line 12 ——— 21 Mar 2004

7.2.1 ⇝ 7.2.1.6

Page 568 line −4 of answer 22 ——— 30 Jul 1998

[insert '▮' in the rightmost column, to end the program.]

▸**Page 568** line −3 ——— 15 Sep 1999

`RLINK(P)` ← `Q` ⇝ `RLINKT(Q)` ← `RLINKT(P)`, `RLINK(P)` ← `Q`

▸**Page 569** line 5 of answer 26 ——— 26 Dec 2002

$1 \le j \le n \rightsquigarrow 1 \le j \le 2n.$

Page 570 line 5 of answer 31 ——— 30 Jul 1998

$\Lambda. \rightsquigarrow \Lambda.$ ▮

Page 571 lines 2 and 4 of answer 37 ——— 28 Sep 2002

`NODE(Q+1)` ⇝ `NODE(Q + 1)`

▸**Page 572** line 2 of answer 12 ——— 15 Oct 2007

`R` ← `TREE("↑",TREE` ⇝ `R` ← `TREE("↑",R,TREE`

Page 575 line 6 of answer 2 ——— 29 Aug 1997

`P` ← `P` − 1 ⇝ `P` ← `P` − c

Page 577 line 5 of answer 17 ——— 29 Aug 1997

`P` ← `P` − 1 ⇝ `P` ← `P` − c

▸**Page 577** line 9 of answer 18 ——— 24 Dec 2001

`INFO2[`k`]` $\leftarrow$ `INFO[`i`]` ⇝ `INFO2[`k`]` $\leftarrow$ `INFO1[`i`]`

Page 579 line −2 of answer 8 ——— 21 Aug 2001

D. E. Knuth ⇝ A. Nahapetian, *Acta Informatica* **3** (1973), 37–41; D. E. Knuth

Page 579 line 4 of answer 11 ——— 14 Feb 1999

"common", ⇝ "common,"

Page 580 lines 3 and 6 of answer 13 ——— 23 Oct 2006

a tree ⇝ a free tree

▸**Page 581** replacement for answer 15 ——— 01 Feb 2004

15. True for finite graphs: If it is connected and balanced and has more than one vertex, it has an Eulerian trail that touches all the vertices. (But false in general.)

▸**Page 581** lines 3 and 4 of answer 16 ——— 21 Aug 2004

tracing out ... (this graph is balanced). ⇝ tracing out an Eulerian trail in G, because the game ends when the fourth arc to V_{13} is encountered (namely, when the fourth king turns up).

Page 581 line 3 of answer 17 ——— 20 Apr 1998

methods of Section ⇝ methods of Section

Page 581 bottom line and beginning of page 582 ——— 05 Sep 2006

2.3.4.4; it also ... direct proof: ⇝ 2.3.4.4. But such a simple answer deserves a simple, direct proof, and indeed there is one [see Tor B. Staver, *Norsk Matematisk Tidsskrift* **28** (1946), 88–89].

▸**Page 582** line 13 ——— 10 Dec 1998

See Section ⇝ See

▸**Page 583** line 3 ——— 28 Dec 1997

oriented trees is ⇝ oriented trees

Page 583 better answer for exercise 21 ——— 06 Sep 2001

Replace the last several lines on page 583, beginning with 'Add the indeterminate λ ...', by the following new copy:

Add the indeterminate λ to every diagonal element of A and A^*. If $t(G)$ and $t(G^*)$ are the numbers of oriented subtrees of G and G^*, we then have $\det A = \lambda t(G) + O(\lambda^2)$, $\det A^* = \lambda t(G^*) + O(\lambda^2)$. (The number of oriented subtrees of a balanced graph is the same for any given root, by exercise 22, but we do not need that fact.)

If we group vertices V_{jk} for equal k the matrix A^* can be partitioned as shown above. Let $B_{kk'}$ be the submatrix of A^* consisting of the rows for V_{jk} and the columns for $V_{j'k'}$, for all j and j' such that V_{jk} and $V_{j'k'}$ are in G^*. By adding the 2nd, ..., mth columns of each submatrix to the first column and then subtracting the first row of each submatrix from the 2nd, ..., mth rows, the matrix A^* is transformed so that

$$B_{kk'} = \begin{pmatrix} a_{kk'} & * & \ldots & * \\ 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 \end{pmatrix} \quad \text{for } k \neq k', \qquad B_{kk} = \begin{pmatrix} \lambda{+}a_{kk} & * & \ldots & * \\ 0 & \lambda{+}m & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \lambda{+}m \end{pmatrix}.$$

The asterisks in the top rows of the transformed submatrices turn out to be irrelevant, because the determinant of A^* is now seen to be $(\lambda+m)^{(m-1)n}$ times

$$\det\begin{pmatrix} \lambda+a_{00} & a_{01} & \ldots & a_{0(n-1)} \\ a_{10} & \lambda+a_{11} & \ldots & a_{1(n-1)} \\ \vdots & \vdots & \ddots & \vdots \\ a_{(n-1)0} & a_{(n-1)1} & \ldots & \lambda+a_{(n-1)(n-1)} \end{pmatrix} = \lambda t(G) + O(\lambda^2).$$

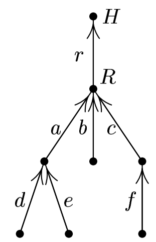
Page 584 replacement for the top four lines ———— 06 Sep 2001

Notice that when $n = 1$ and there are m arcs from V_0 to itself, we find in particular that exactly m^{m-1} oriented trees are possible on m nodes labeled. This result will be obtained by quite different methods in Section 2.3.4.4.

▶**Page 584** line 8 ———— 30 Jul 1997

314.) $\rightsquigarrow$ 314.

Page 584 replacement for figure at bottom left ———— 31 Dec 1997



▶**Page 585** last line of answer 27 ———— 14 Feb 1999

is combination $\rightsquigarrow$ is a combination

▶**Page 585** in answer 28 ———— 01 Jan 2006

line 9: is the determinant $\rightsquigarrow$ is $(*)$ times the determinant
line 17: column j_0 of the left section $\rightsquigarrow$ row j_0 of the bottom section

Page 586 last lines of answer 4 ———— 08 Oct 2005

(But the stated assumption ... set of types.) $\rightsquigarrow$
[But the stated assumption ... set of types. On the other hand, if such a tiling does exist, there is always a tiling that is *quasitoroidal*, in the sense that each of its $n \times n$ blocks occurs at least once in every $f(n) \times f(n)$ block, for some function f. See B. Durand, *Theoretical Computer Science* **221** (1999), 61–75.]

▶**Page 587** line 7 ———— 22 Jul 2005

$\gamma ST \rightsquigarrow \gamma SJ$

Page 589 line 3 of answer 5 ———— 30 Jun 2008

$\dfrac{1}{2} \rightsquigarrow \tfrac{1}{2}$

Page 589 last line of answer 6 ———— 08 Oct 2010

Annals of Math. $\rightsquigarrow$ *Annals of Math.* (2)

▶**Page 590** line 1 of answer 16 ———— 08 Nov 2005

$1 \le j \le n \rightsquigarrow 1 \le j < n$

▸**Page 592** line 4 of answer 29 — 07 Jan 1999

$\sum_{j,k} \frac{z^k}{k!} \sum_j \rightsquigarrow \sum_k \frac{z^k}{k!} \sum_j$

▸**Page 592** line 5 of answer 29 — 08 Nov 2005

$\sum_k \frac{z^k}{k!} \binom{k-1}{j} \rightsquigarrow \sum_k \frac{z^k}{k!} \sum_j \binom{k-1}{j}$

Page 593 line 3 — 03 Dec 2004

344–350. $\rightsquigarrow$ 344–350; F. Acerbi, *Archive for History of Exact Sciences* **57** (2003), 465–502.

▸**Page 593** line −6 — 08 Nov 2005

$\sum_{j=1}^{k}(1-d_j) \rightsquigarrow \sum_{j=1}^{k-1}(1-d_j)$

▸**Page 594** denominator of display on line 12 — 10 Sep 1997

$n_1!\, n_2! \rightsquigarrow n_1!\, n_2! \ldots$

▸**Page 596** line 8 of answer 15 — 21 Sep 1997

at each step. $\rightsquigarrow$ at each step as in part (b).

▸**Page 596** line 2 of answer 16 — 13 Jan 1999

$(l'_j - l'_{j+1}) \rightsquigarrow (l'_k - l'_{k+1})$ (twice)

Page 596 line −2 — 31 Oct 1997

5.2.3–20 and 21. $\rightsquigarrow$ 5.2.3–20 and 5.2.3–21.

Page 597 append a sentence to answer 1 — 01 Mar 2004

[See H. G. Forder, *Mathematical Gazette* **45** (1961), 199–201.]

Page 597 lines 1 and 2 of answer 3 — 28 Nov 1997

from j to k if they are consecutive elements of the same part and $j < k$; $\rightsquigarrow$ from i to j if i and j are consecutive elements of the same part and $i < j$;

▸**Page 598** line 12 — 14 Jan 1999

$(n-1, n+1, 1) \rightsquigarrow (n-1, n+1, k)$

Page 598 line 15 — 12 Mar 2004

$(k-1)!$. $\rightsquigarrow$ $(k-1)!$, a so-called *Narayana number* [T. V. Narayana, *Comptes Rendus Acad. Sci.* **240** (Paris, 1955), 1188–1189].

▸**Page 598** line 18 — 17 Sep 1997

different from the discussed $\rightsquigarrow$ different from the one discussed

▸**Page 598** line 19 — 01 Dec 1997

(1970) $\rightsquigarrow$ (1971)

Page 598 line 21 — 07 Apr 2006

171–179. $\rightsquigarrow$ 171–179; N. Dershowitz and S. Zaks, *Discrete Math.* **64** (1986), 215–218.

▸**Page 598** line −2 — 14 Feb 1999

these the new $\rightsquigarrow$ these new

▸**Page 600** line 3 following (∗∗∗) — 08 Nov 2005

$(t,w)(v,u) \rightsquigarrow (t,w)(u,v)$

Page 603 changes for consistency — 29 Oct 1997

line −20: Solution 1: $\rightsquigarrow$ *Solution 1:*
line −9: Solution 2: $\rightsquigarrow$ *Solution 2:*

Page 606 in answer 12 — 06 Jan 2011

[The revised algorithm is now called B′, to distinguish it from Algorithm B of the text.]

Page 606 in answer 14, continuing to page 607 — 06 Jan 2011

[The revised algorithm is now called C′, to distinguish it from Algorithm C of the text.]

▸**Page 607** line 3 of answer 6 — 19 Oct 1998

`ROVER` ← `LINK(Q)` $\rightsquigarrow$ `ROVER` ← `LINK(Q)` if `LINK(Q)` $\neq \Lambda$, otherwise set `ROVER` ← `LOC(AVAIL)`.

Page 608 in answer 8 — 06 Jan 2011

[The revised algorithm is now called A″, to distinguish it from Algorithms A and A′ of the text.]

▸**Page 608** replacement for lines 2–4 of answer 10 — 15 Oct 2003

At the beginning of step B3, insert "If `P0` + `N` > `P` and `P` $\neq \Lambda$, set `N` ← max(`N`, `P` + `SIZE(P)` − `P0`), `P` ← `LINK(P)`, and repeat step B3."

Step B4, for "`Q` + `SIZE(Q)` = `P0`", read "`Q` + `SIZE(Q)` ≥ `P0`"; and for "`SIZE(Q)` ← `SIZE(Q)` + `N`" read "`SIZE(Q)` ← max(`SIZE(Q)`, `P0` + `N` − `Q`)".

Page 608 in answer 12 — 06 Jan 2011

[The revised algorithm is now called A*, to distinguish it from Algorithm A and A′ of the text and Algorithm A″ of answer 10. This change extends to page 609, and also affects answer 19 on pages 610–611.]

Page 610 in answers 15, 16 — 06 Jan 2011

`D1` $\rightsquigarrow$ `C1`, `D2` $\rightsquigarrow$ `C2`, ..., `D8` $\rightsquigarrow$ `C8`

Page 610 line −3 — 02 Feb 1999

`LINK(P2+1)` $\rightsquigarrow$ `LINK(P2 + 1)`

Page 612 program lines 17, 18, 20 in answer 27 — 06 Nov 2006

`LINKF` $\rightsquigarrow$ `LINKB`, `LINKB` $\rightsquigarrow$ `LINKF`, `AVAILF` $\rightsquigarrow$ `AVAILB` (six changes altogether)

▸**Page 612** program line *04* in answer 28 — 20 Aug 1997

buddy_k`L)` $\rightsquigarrow$ buddy_k`(L)`

▸**Page 612** lines 12 and 13 of answer 28 — 04 Sep 2003

```
11  S2 LD2  0,5(LINKB)    S   S2. Combine with buddy.
12     LD3  0,5(LINKF)    S
```

Page 613 new answer ______ 10 Nov 2002

32. Steven Crain points out that the method always frees all blocks and starts afresh before 16667 units of time have elapsed; hence the stated limit certainly exists. *Proof:* Let $u_n = n + t_n$, so that $g_n = \lfloor \frac{5}{4} \min(10000, f(u_{n-1} - n), f(u_{n-2} - n), \ldots, f(u_0 - n)) \rfloor$. Let $x_0 = 0$ and $x_1 = u_0$, and $x_{k+1} = \max(u_0, \ldots, u_{x_k - 1})$ for $k \geq 1$. If $x_k > x_{k-1}$ then

$$u_n \leq n + \frac{5}{4} f(x_k - n) = \frac{5}{4} x_k - \frac{1}{4} n \leq \frac{5}{4} x_k - \frac{1}{4} x_{k-1} \qquad \text{for } x_{k-1} \leq n < x_k;$$

therefore $x_{k+1} - x_k \leq \frac{1}{4}(x_k - x_{k-1})$, and we must have $x_k = x_{k-1}$ before reaching time $12500 + \lfloor 12500/4 \rfloor + \lfloor 12500/4^2 \rfloor + \cdots$.

Page 620 Table 2 . . . . 06 May 1999

In the next edition I plan to give these constants to 36 hexadecimal places, instead of 45 octal places.

▸**Page 624** entry for Kronecker delta ______ 14 Aug 1998

1.2.6 $\leadsto$ 1.2.3

▸**Page 625** new entry (just above $[a\,..\,b]$) ______ 20 Mar 2006

$a_1 + a_2 + \cdots + a_n$ | n-fold sum: $\sum_{j=1}^{n} a_j$ | 1.2.3

▸**Page 626** entry for $\exp x$ ______ 09 Aug 1999

1.2.2 $\leadsto$ 1.2.9

Page 628 a new appendix ______ 06 Jan 2011

Beginning with the 27th printing, Volume 1 now has an "Appendix C" analogous to Appendix C of Volume 4A, which lists the page numbers on which named algorithms, theorems, programs, definitions, . . . , can be found. A copy of this two-page appendix can be found online in the author's website. The main index therefore begins now on page 630.

Page 629 quotation for bottom of the page ______ 07 Jan 2011

Numerical experimentations are necessary
to fully understand the algorithms and theorems in this book.
— STÉPHANE MALLAT, *A Wavelet Tour of Signal Processing* (1998)

Page 628 and following ________ 24 Apr 1997

Miscellaneous changes to the existing index of Volume 1 are collected here, including corrections and amendments to the old entries as well as new entries that are occasioned by the new material. Thus, the lines of the full index that have changed serve also as an index to the present document. However, when a correction or amendment has caused an old index entry to be deleted, the deletion is usually not indicated.