A Polynomial Time Algorithm for Log-Concave Maximum Likelihood via Locally Exponential Families

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Log-concave MLE

- Non-parametric
- Parameter Free
- log-concavity is a [relatively] light assumption
- Non-optimal sample complexity

Optimization Formulation

**Original Formulation:** \( \max_\theta \sum \log p_\theta(x_i) - f(p_\theta(x)) \)

**New Formulation:** \( \max_\theta \sum \log p_\theta(x_i) - n \log f(p_\theta(x)) \)

- Equivalent to minimizing \( \log f(p_\theta(x)) \) for a slightly more restrictive definition of \( p \).
- More robust to initialization and easier to implement.

Connection To Exponential Families

- Log-concave MLE has \( R^2(p') = 1 \).
- Consider a parameter region where the faces of the polytope don’t change (where the log part is constant).
- Call RLA with these properties “locally” exponential.
- “Locally” exponential family \( p(x) = \exp(\theta^T x - A(\theta)) \) when \( A(\theta) \) is convex.

References